TP1

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Find Optimal Distribution

► Find an optimal distribution such that the transportation cost is minimum

	W	X	Υ	Z	Supply
Α	6	5	8	8	30
В	5	11	9	7	40
C	8	9	7	13	50
Demand	35	28	32	25	_

Find an Initial Basic Feasible Solution

- Find an initial BFS using any of the methods studied
- ► For this problem, the IBFS is obtained using the North West Corner Rule

	W		Χ		Υ		Z		Suppl	у
Α	6	30	5		8		8		30	
В	5	05	11	28	9	07	7		40	
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			

Now, we have the allocations, for which we are required to check for optimality

- ▶ Make a new column u_i and a new row v_j
- ► Count the number of allocations in every row. Row A has 1, Row B has 3 and Row C has 2 allocations
- ► The row which has the highest number of allocations is assigned $u_i = 0$

	W		Χ		Υ		Z	Ç	Supply	ui
Α	6	30	5		8		8		30	
В	5	05	11	28	9	07	7		40	0
С	8		9		7	25	13	25	50	
Demand	35		28		32		25			
V_j										

- ▶ For the basic variable x_{ij} , we must have the cost $c_{ij} = u_i + v_j$
- ► Hence, from Row B costs, $c_{21} = u_2 + v_1 \implies 5 = 0 + v_1 \implies v_1 = 5$

	W		Χ		Υ		Z	:	Supply	ui
Α	6	30	5		8		8		30	
В	5	05	11	28	9	07	7		40	0
С	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5									

- ▶ For the basic variable x_{ij} , we must have the cost $c_{ij} = u_i + v_j$
- ► Similarly, $c_{22} = u_2 + v_2 \implies 11 = 0 + v_2 \implies v_2 = 11$

	W		X		Υ		Z	9	Supply	иį
Α	6	30	5		8		8		30	
В	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_i	5		11							

$$v_3 = 9$$

	W		X		Υ		Z	:	Supply	ui
Α	6	30	5		8		8		30	
В	5	05	11	28	9	07	7		40	0
С	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5		11		9					

$$c_{11} = u_1 + v_1 \implies 6 = u_1 + 5 \implies u_1 = 1$$

	W		Χ		Υ		Z	:	Supply	иi
Α	6	30	5		8		8		30	1
В	5	05	11	28	9	07	7		40	0
С	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5		11		9					

$$c_{33} = u_3 + v_3 \implies 7 = u_3 + 9 \implies u_3 = -2$$

	W		Χ		Υ		Z	9	Supply	Иį
Α	6	30	5		8		8		30	1
В	5	05	11	28	9	07	7		40	0
С	8		9		7	25	13	25	50	-2
Demand	35		28		32		25			
v_j	5		11		9					

$$ightharpoonup c_{34} = u_3 + v_4 \implies 13 = -2 + v_4 \implies v_4 = 15$$

	W		X		Υ		Z	9	Supply	ui
Α	6	30	5		8		8		30	1
В	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	-2
Demand	35		28		32		25			
v_j	5		11		9		15			

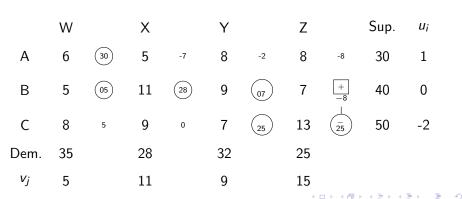
- After filling u_i and v_j , compute the value $c_{ij} u_i v_j$ for all the nonbasic variables
- ► The obtained solution is optimal only if all the computed values are non-negative
- ► For the problem, there are several negative values, so, proceed with an iteration
- Determine the least value (break ties arbitrarily), and put a
 + sign above the value
- It indicates that it is going to be allocated some value after adjusting with the adjacent allocations
- ► The chain reaction that occurs is indicated by drawing a loop, where the corners must be the allocated cells
- ▶ Assign alternating + and − signs to the corners of the loop
- ► The cells having a + are called the *recipient cells* and those having are called *donor cells*



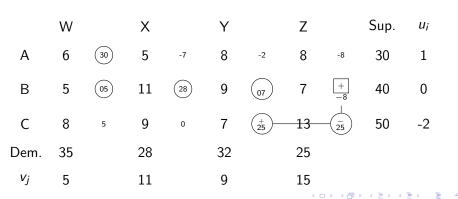
- ▶ Compute the value $c_{ij} u_i v_j$ for all the nonbasic variables
- ► Determine the least value (break ties arbitrarily), and put a + sign above the value

	W		Χ		Υ		Z		Sup.	иi
Α	6	30	5	-7	8	-2	8	-8	30	1
В	5	05)	11	28	9	07	7	+ -8	40	0
С	8	5	9	0	7	25	13	25	50	-2
Dem.	35		28		32		25			
Vj	5		11		9		15			

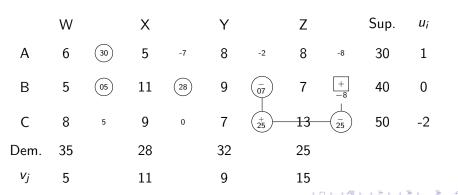
- ► Starting from +, check all the 4 possible directions for an available allocation
- ▶ Then, draw an edge to any one of it. Here, an edge is from x_{24} to x_{34}



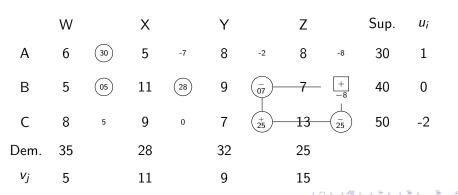
- ▶ Draw another edge from x_{34} to x_{33}
- ▶ and continue till one loop is drawn



- ▶ Draw another edge from x_{34} to x_{33}
- and continue till one loop is drawn



- ▶ Draw another edge from x_{34} to x_{33}
- and continue till one loop is drawn



- ► Among the donor cells, determine the minimum value of the allocations
- ► Subtract that value from all the donor cells and add that value to all the recipient cells

	W		Χ		Υ		Z		Sup.	
Α	6	30	5		8		8		30	
В	5	05	11	28	9		7	07	40	
C	8		9		7	32	13	18	50	
Dem.	35		28		32		25			

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells
- ► The recipient cell that had + symbol is called the entering basic variable

Sup.
30
40
50

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells
- ► The donor cell which loses all its allocation is called the leaving basic variable

	W		Χ		Υ		Z		Sup.	
Α	6	30	5		8		8		30	
В	5	05	11	28	9		7	07	40	
С	8		9		7	32	13	18	50	
Dem.	35		28		32		25			

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells
- ▶ If there are multiple leaving basic variables, choose one arbitrarily and assign 0 allocation to the remaining cells

	W		Χ		Υ		Z		Sup.	
Α	6	30	5		8		8		30	
В	5	05	11	28	9		7	07	40	
С	8		9		7	32	13	18	50	
Dem.	35		28		32		25			

Fill in u_i and v_j again and check for optimality

	W	X		Υ		Z		Sup.	иį
Α	6	30 5		8		8		30	
В	5	©5) 11	28	9		7	07	40	0
C	8	9		7	32	13	18	50	
Dem.	35	28		32		25			
v_j									

Fill in u_i and v_j again and check for optimality

	W	Χ	Υ	Z	Sup.	Иį
Α	6	30 5	8	8	30	
В	5	05) 11	28 9	7	07 40	0
C	8	9	7	(32) 13	18 50	
Dem.	35	28	32	25		
v_i	5					

Fill in u_i and v_i again and check for optimality

	W	X	Υ	Z	Sup.	иį
Α	6	30 5	8	8	30	
В	5	05) 11	28 9	7	07 40	0
C	8	9	7	(32) 13	18 50	
Dem.	35	28	32	25		
v_i	5	11				

Fill in u_i and v_j again and check for optimality

	W	X	Υ	Z	Sup.	uį
Α	6	30 5	8	8	30	
В	5	05 11	28 9	7	07) 40	0
C	8	9	7	³² 13	18) 50	
Dem.	35	28	32	25		
v_j	5	11		7		

Fill in u_i and v_i again and check for optimality

	W	X		Υ		Z		Sup.	Иį
Α	6	30 5		8		8		30	1
В	5	05 11	28	9		7	07	40	0
C	8	9		7	32	13	18	50	
Dem.	35	28		32		25			
V_j	5	11				7			

Fill in u_i and v_i again and check for optimality

	W	X	Υ	Z	Sup.	и
Α	6	30 5	8	8	30	1
В	5	05) 11	28 9	7	07) 40	0
C	8	9	7	32) 13	18) 50	6
Dem.	35	28	32	25		
v_j	5	11		7		

Fill in u_i and v_j again and check for optimality

	W	Χ	Υ	Z	Sup.	u,
Α	6	30 5	8	8	30	1
В	5	05) 11	28 9	7	07) 40	0
C	8	9	7	³² 13	18) 50	6
Dem.	35	28	32	25		
v_j	5	11	1	7		

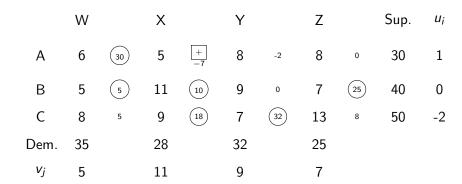
Since it's not optimal, again draw a loop and redistribute the allocations

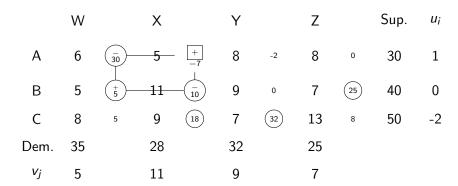
	W		Χ		Υ		Z		Sup.	ui
Α	6	30	5	-7	8	6	8	0	30	1
В	5	05	11	28	9	8	7	7	40	0
С	8	-3	9	+ -8	7	32	13	18	50	6
Dem.	35		28		32		25			
v_j	5		11		1		7			

► Since it's not optimal, again draw a loop and redistribute the allocations

	W		Χ		Υ		Z		Sup.	ui
Α	6	30	5	-7	8	6	8	0	30	1
В	5	05	11	28	9	8	7	- +	40	0
С	8	-3	9	+ -8	7	32	13	-(<u>18</u>)	50	6
Dem.	35		28		32		25			
v_j	5		11		1		7			

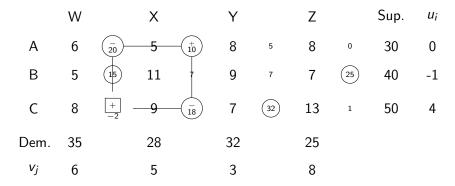
	W		Χ		Υ		Z		Sup.
Α	6	30	5		8		8		30
В	5	05	11	10	9		7	25	40
C	8		9	18	7	32	13		50
Dem.	35		28		32		25		





	W		Χ		Υ		Z		Sup.
Α	6	20	5	10	8		8		30
В	5	15	11		9		7	25	40
C	8		9	18	7	32	13		50
Dem.	35		28		32		25		

	W		Χ		Υ		Z		Sup.	ui
Α	6	20	5	10	8	5	8	0	30	0
В	5	15	11	7	9	7	7	25)	40	-1
С	8	+ -2	9	18	7	32	13	1	50	4
Dem.	35		28		32		25			
V_j	6		5		3		8			



	W		Χ		Υ		Z		Sup.
Α	6	2	5	28	8		8		30
В	5	15	11		9		7	25	40
C	8	18	9		7	32	13		50
Dem.	35		28		32		25		

	W		Χ		Υ		Z		Sup.	ui
Α	6	2	5	28	8	3	8	0	30	0
В	5	15	11	7	9	5	7	25	40	-1
C	8	18	9	2	7	32	13	3	50	2
Dem.	35		28		32		25			
v_j	6		5		5		8			

All the difference values are non-negative, hence the current BFS is optimal.