TP1

gar

Find Optimal Distribution

- Find an optimal distribution such that the transportation cost is minimum

	W	X	Y	Z	Supply
A	6	5	8	8	30
B	5	11	9	7	40
C	8	9	7	13	50
Demand	35	28	32	25	

Find an Initial Basic Feasible Solution

- Find an initial BFS using any of the methods studied
- For this problem, the IBFS is obtained using the North West Corner Rule

	W		X		Y		Z		Supply
A	6	30	5		8		8		30
B	5	05	11	28	9	07	7		40
C	8		9		7	25	13	25	50
Demand	35		28		32		25		

- Now, we have the allocations, for which we are required to check for optimality

Iteration 0

- Make a new column u_i and a new row v_j
- Count the number of allocations in every row. Row A has 1, Row B has 3 and Row C has 2 allocations
- The row which has the highest number of allocations is assigned $u_i = 0$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j										

Iteration 0

- For the basic variable x_{ij}, we must have the cost $c_{ij} = u_i + v_j$
- Hence, from Row B costs,
  $c_{21} = u_2 + v_1 \implies 5 = 0 + v_1 \implies v_1 = 5$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5									

Iteration 0

- For the basic variable x_{ij}, we must have the cost $c_{ij} = u_i + v_j$
- Similarly, $c_{22} = u_2 + v_2 \implies 11 = 0 + v_2 \implies v_2 = 11$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5		11							

Iteration 0

- For the basic variable x_{ij}, we must have the cost $c_{ij} = u_i + v_j$
- $v_3 = 9$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5		11		9					

Iteration 0

- For the basic variable x_{ij}, we must have the cost $c_{ij} = u_i + v_j$
- $c_{11} = u_1 + v_1 \implies 6 = u_1 + 5 \implies u_1 = 1$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	1
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	
Demand	35		28		32		25			
v_j	5		11		9					

Iteration 0

- For the basic variable x_{ij}, we must have the cost $c_{ij} = u_i + v_j$
- $c_{33} = u_3 + v_3 \implies 7 = u_3 + 9 \implies u_3 = -2$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	1
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	-2
Demand	35		28		32		25			
v_j	5		11		9					

Iteration 0

- For the basic variable x_{ij}, we must have the cost $c_{ij} = u_i + v_j$
- $c_{34} = u_3 + v_4 \implies 13 = -2 + v_4 \implies v_4 = 15$

	W		X		Y		Z		Supply	u_i
A	6	30	5		8		8		30	1
B	5	05	11	28	9	07	7		40	0
C	8		9		7	25	13	25	50	-2
Demand	35		28		32		25			
v_j	5		11		9		15			

Iteration 0

- After filling u_i and v_j, compute the value $c_{ij} - u_i - v_j$ for all the nonbasic variables
- The obtained solution is optimal only if all the computed values are non-negative
- For the problem, there are several negative values, so, proceed with an iteration
- Determine the least value (break ties arbitrarily), and put a $\boxed{+}$ sign above the value
- It indicates that it is going to be allocated some value after adjusting with the adjacent allocations
- The chain reaction that occurs is indicated by drawing a loop, where the corners must be the allocated cells
- Assign alternating $+$ and $-$ signs to the corners of the loop
- The cells having a $+$ are called the *recipient cells* and those having $-$ are called *donor cells*

Iteration 0

- Compute the value $c_{ij} - u_i - v_j$ for all the nonbasic variables
- Determine the least value (break ties arbitrarily), and put a $\boxed{+}$ sign above the value

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	-2	8	-8	30	1
B	5	05	11	28	9	07	7	$\boxed{+}$ -8	40	0
C	8	5	9	0	7	25	13	25	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		15			

Iteration 0

- Starting from $\boxed{+}$, check all the 4 possible directions for an available allocation
- Then, draw an edge to any one of it. Here, an edge is from x_{24} to x_{34}

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	-2	8	-8	30	1
B	5	05	11	28	9	07	7	$\boxed{+}$ -8	40	0
C	8	5	9	0	7	25	13	$-$ 25	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		15			

Iteration 0

- Draw another edge from x_{34} to x_{33}
- and continue till one loop is drawn

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	-2	8	-8	30	1
B	5	05	11	28	9	07	7	$\boxed{+}$ -8	40	0
C	8	5	9	0	7	$+$ 25	13	$-$ 25	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		15			

Iteration 0

- Draw another edge from x_{34} to x_{33}
- and continue till one loop is drawn

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	-2	8	-8	30	1
B	5	05	11	28	9	− 07	7	$\boxed{+}$ −8	40	0
C	8	5	9	0	7	+ 25	13	− 25	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		15			

Iteration 0

- Draw another edge from x_{34} to x_{33}
- and continue till one loop is drawn

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	-2	8	-8	30	1
B	5	05	11	28	9	− 07	7	$\boxed{+}$ −8	40	0
C	8	5	9	0	7	+ 25	13	− 25	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		15			

Iteration 0

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells

	W		X		Y		Z		Sup.
A	6	30	5		8		8		30
B	5	05	11	28	9		7	07	40
C	8		9		7	32	13	18	50
Dem.	35		28		32		25		

Iteration 0

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells
- The recipient cell that had $\boxed{+}$ symbol is called the entering basic variable

	W	X	Y	Z	Sup.
A	6 30	5	8	8	30
B	5 05	11 28	9	7 07	40
C	8	9	7 32	13 18	50
Dem.	35	28	32	25	

Iteration 0

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells
- The donor cell which loses all its allocation is called the leaving basic variable

	W		X		Y		Z		Sup.
A	6	30	5		8		8		30
B	5	05	11	28	9		7	07	40
C	8		9		7	32	13	18	50
Dem.	35		28		32		25		

Iteration 0

- Among the donor cells, determine the minimum value of the allocations
- Subtract that value from all the donor cells and add that value to all the recipient cells
- If there are multiple leaving basic variables, choose one arbitrarily and assign 0 allocation to the remaining cells

	W		X		Y		Z		Sup.
A	6	30	5		8		8		30
B	5	05	11	28	9		7	07	40
C	8		9		7	32	13	18	50
Dem.	35		28		32		25		

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W	X	Y	Z	Sup.	u_i
A	6 30	5	8	8	30	
B	5 05	11 28	9	7 07	40	0
C	8	9	7 32	13 18	50	
Dem.	35	28	32	25		
v_j						

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W		X		Y		Z		Sup.	u_i
A	6	30	5		8		8		30	
B	5	05	11	28	9		7	07	40	0
C	8		9		7	32	13	18	50	
Dem.	35		28		32		25			
v_j	5									

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W	X	Y	Z	Sup.	u_i
A	6 30	5	8	8	30	
B	5 05	11 28	9	7 07	40	0
C	8	9	7 32	13 18	50	
Dem.	35	28	32	25		
v_j	5	11				

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W		X		Y		Z		Sup.	u_i
A	6	30	5		8		8		30	
B	5	05	11	28	9		7	07	40	0
C	8		9		7	32	13	18	50	
Dem.	35		28		32		25			
v_j	5		11				7			

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W		X		Y		Z		Sup.	u_i
A	6	30	5		8		8		30	1
B	5	05	11	28	9		7	07	40	0
C	8		9		7	32	13	18	50	
Dem.	35		28		32		25			
v_j	5		11				7			

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W		X		Y		Z		Sup.	u_i
A	6	30	5		8		8		30	1
B	5	05	11	28	9		7	07	40	0
C	8		9		7	32	13	18	50	6
Dem.	35		28		32		25			
v_j	5		11				7			

Iteration 1

- Fill in u_i and v_j again and check for optimality

	W		X		Y		Z		Sup.	u_i
A	6	30	5		8		8		30	1
B	5	05	11	28	9		7	07	40	0
C	8		9		7	32	13	18	50	6
Dem.	35		28		32		25			
v_j	5		11		1		7			

Iteration 1

- Since it's not optimal, again draw a loop and redistribute the allocations

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	6	8	0	30	1
B	5	05	11	28	9	8	7	7	40	0
C	8	-3	9	$\boxed{+}$ -8	7	32	13	18	50	6
Dem.	35		28		32		25			
v_j	5		11		1		7			

Iteration 1

- Since it's not optimal, again draw a loop and redistribute the allocations

	W		X		Y		Z		Sup.	u_i
A	6	30	5	-7	8	6	8	0	30	1
B	5	05	11	− 28	9	8	7	+ 7	40	0
C	8	-3	9	+ −8	7	32	13	− 18	50	6
Dem.	35		28		32		25			
v_j	5		11		1		7			

Iteration 1

	W		X		Y		Z		Sup.
A	6	30	5		8		8		30
B	5	05	11	10	9		7	25	40
C	8		9	18	7	32	13		50
Dem.	35		28		32		25		

Iteration 2

	W		X		Y		Z		Sup.	u_i
A	6	30	5	$\boxed{+}$ -7	8	-2	8	0	30	1
B	5	5	11	10	9	0	7	25	40	0
C	8	5	9	18	7	32	13	8	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		7			

Iteration 2

	W		X		Y		Z		Sup.	u_i
A	6	$-$ 30	5	$\boxed{+}$ -7	8	-2	8	0	30	1
B	5	$+$ 5	11	$-$ 10	9	0	7	25	40	0
C	8	5	9	18	7	32	13	8	50	-2
Dem.	35		28		32		25			
v_j	5		11		9		7			

Iteration 2

	W		X		Y		Z		Sup.
A	6	20	5	10	8		8		30
B	5	15	11		9		7	25	40
C	8		9	18	7	32	13		50
Dem.	35		28		32		25		

Iteration 3

	W		X		Y		Z		Sup.	u_i
A	6	20	5	10	8	5	8	0	30	0
B	5	15	11	7	9	7	7	25	40	-1
C	8	$\boxed{+}$ -2	9	18	7	32	13	1	50	4
Dem.	35		28		32		25			
v_j	6		5		3		8			

Iteration 3

	W		X		Y		Z		Sup.	u_i
A	6	$\overset{-}{20}$	5	$\overset{+}{10}$	8	5	8	0	30	0
B	5	15	11	7	9	7	7	25	40	-1
C	8	$\boxed{+}$ -2	9	$\overset{-}{18}$	7	32	13	1	50	4
Dem.	35		28		32		25			
v_j	6		5		3		8			

Iteration 3

	W		X		Y		Z		Sup.
A	6	2	5	28	8		8		30
B	5	15	11		9		7	25	40
C	8	18	9		7	32	13		50
Dem.	35		28		32		25		

Iteration 4

	W		X		Y		Z		Sup.	u_i
A	6	2	5	28	8	3	8	0	30	0
B	5	15	11	7	9	5	7	25	40	-1
C	8	18	9	2	7	32	13	3	50	2
Dem.	35		28		32		25			
v_j	6		5		5		8			

All the difference values are non-negative, hence the current BFS is optimal.