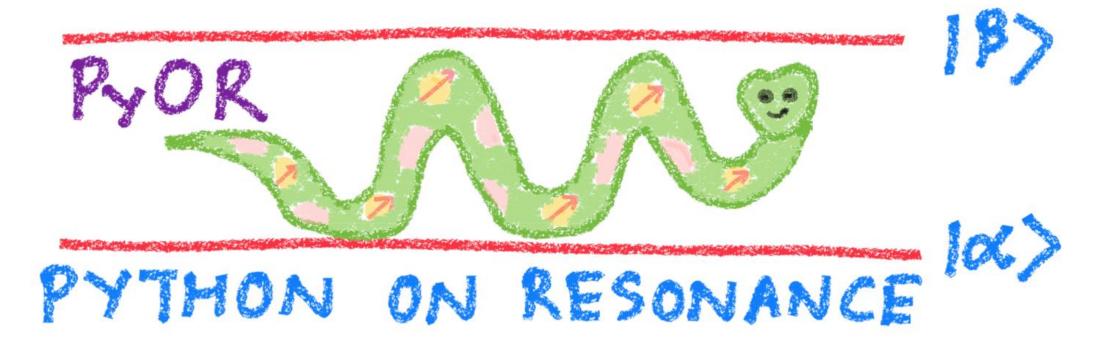
PyOR: A Versatile NMR Simulator for the Beginners and Teaching



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PyOR (Python On Resonance)

Motto: "Everybody can simulate NMR"**

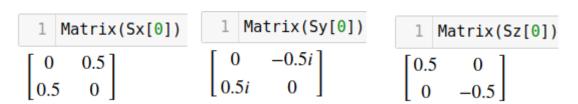
- Python based NMR Simulator
- Why Python? It's free, easy to learn, a lot packages (Numpy, Scipy, Sympy, Matplotlib,...), great online support.
- What makes PyOR unique? Great readability of the source code (single file) unlike other popular simulators available.
- Versatile: From basic to specialized NMR experiments
- Programming (user): Jupyter Notebook or text file
- https://github.com/VThalakottoor/PyOR-Jeener-Beta
- ** Required basic knowledge of spin operators

Features: Spin Operators (Heart of PyOR)

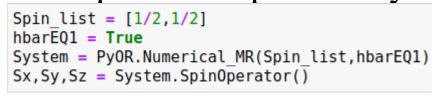
- Particle with any spin quantum number
 - ½, 1, 3/2, 2,
- Any number of particles
 - ½ (Single spin system)
 - ½, ½ (Two spin system)
 - ½, ½, ½ (Three spin system)
 - ½, 1 (Two spin mixed system)
- Example: Spin half system

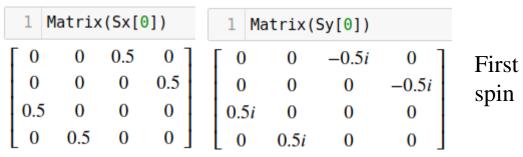
```
Spin_list = [1/2]
hbarEQ1 = True

System = PyOR.Numerical_MR(Spin_list,hbarEQ1)
Sx,Sy,Sz = System.SpinOperator()
```



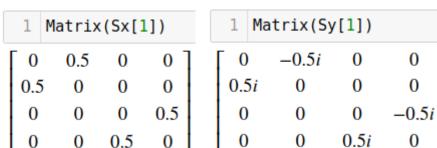
• Example: Two spin half system





Second

spin



Features: Hamiltonians

• Zeeman Hamiltonian

```
# Gyromagnetic Ratio
Gamma = [System.gammaH1,System.gammaH1]
# B0 Field in Tesla, Static Magnetic field (B0) along Z
B0 = 9.4
# Rotating Frame Frequency
OmegaRF = [-System.gammaH1*B0,-System.gammaH1*B0]
# Offset Frequency in rotating frame (Hz)
Offset = [10.0,20.0]
# generate Larmor Frequencies
LarmorF = System.LarmorFrequency(Gamma,B0,Offset)
# Lab Frame Hamiltonian
Hz_lab = System.Zeeman(LarmorF,Sz)
# Rotating Frame Hamiltonian
Hz = System.Zeeman_RotFrame(LarmorF,Sz,OmegaRF)
```

1 Matrix(System.Matrix_Round(Hz/(2.0*np.pi), 2))

```
\begin{bmatrix} -15.0 & 0 & 0 & 0 \\ 0 & 5.0 & 0 & 0 \\ 0 & 0 & -5.0 & 0 \\ 0 & 0 & 0 & 15.0 \end{bmatrix}
```

• J coupling Hamiltonian

- B1 Hamiltonian
- Dipole Hamiltonian
- Define your own Hamiltonian

Features: Superoperator

- Liouville-von Neumann Eq.
 - Form 1

$$\frac{d}{dt}\rho = \frac{-i}{\hbar} [H_0, \rho]$$
$$\rho(t) = e^{iH_0t} \rho(0) e^{-iH_0t}$$

■ Form 2

$$\frac{d}{dt}\tilde{\rho} = \frac{-i}{\hbar} \hat{H}_0\tilde{\rho}$$
$$\tilde{\rho}(t) = e^{-i\hat{H}_0t}\tilde{\rho}(0)$$

 Commutation Hamiltonian Superoperator

$$\begin{split} \hat{H}_0 &= H_0 \otimes 1_d - 1_d \otimes H_0^\top \\ \text{Hz_L = System.CommutationSuperoperator(Hz)} \end{split}$$

Vectorize density matrix

```
rho_in_L = System.Vector_L(rho_in)
rhoeq_L = System.Vector_L(rhoeq)
```

Features: Relaxation

- Redfield Master Equation (special case: Auto Corr. Dipolar relaxation)
 - **■** Form 1

$$rac{d}{dt}
ho = rac{-i}{\hbar}[H_0,
ho] + \sum_{q=-2}^2 \sum_p S(\omega_p^q) [A_{2p}^q,[(A_{2p}^q)^\dagger,
ho-
ho_{eq}]]$$

■ Form 2

$$rac{d}{dt} ilde{
ho}=rac{-i}{\hbar}(\hat{H}_0+i\hat{R}) ilde{
ho}$$

$$\hat{R} = \sum_{q=-2}^2 \sum_p S(\omega_p^q) \hat{A}_{2p}^q (\hat{A}_{2p}^q)^\dagger$$

- Phenomenological relaxation
- Random field fluctuation
- Homonuclear and Heteronuclear dipolar relaxation (any number of spin pairs)
- Lindblad master equation: Homonuclear and Heteronuclear dipolar relaxation
- Compute relaxation rates
- Define relaxation mechanism

Features: Solve Liouville-von Neumann Equation

- State of the system: Density matrix
 - Form 1 (Hilbert space)

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{bmatrix}$$

■ Form 2 (Liouville space)

$$\tilde{\rho} = [\rho_{11}, \rho_{12}, \rho_{13}, \rho_{14}, \dots, \rho_{44}]^{\top}$$

- Solve Liouville-von Neumann Eq.
 - Unitary propagation

$$\rho(t) = e^{iH_0t} \rho(0)e^{-iH_0t}$$

$$\tilde{\rho}(t) = e^{-i\hat{H}_0 t} \tilde{\rho}(0)$$

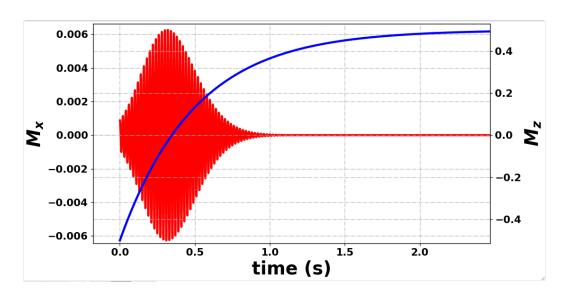
Solve Ordinary Diff. Eq

$$\frac{d}{dt}\rho = \frac{-i}{\hbar} \left[H_0, \rho \right]$$

$$\frac{d}{dt}\tilde{\rho} = \frac{-i}{\hbar} \quad \hat{H}_0\tilde{\rho}$$

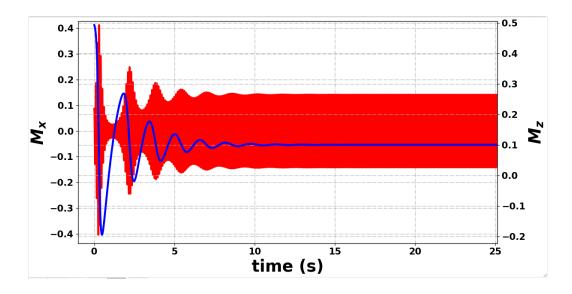
Features: Radiation Damping (not in beta version)

Radiation Damping



```
RDgain = [30]
RDphase = [0]
Rdamping = True
System.RDparameters(RDgain,RDphase,Rdamping)
```

Maser



Features: Product Operators Basis

- Decomposition of density matrix in terms of product operator basis
- Example: Two spin half (spherical)

$$\rho = \sum_{i} a_i B_i$$

```
sort = 'negative to positive'
Index = False
Normal = True
Basis_PMZ, coh_PMZ, dic_PMZ = System.ProductOperators_SpinHalf_PMZ(sort,Index,Normal)

1 print(dic_PMZ)
['ImlIm2', 'ImlIz2', 'IzlIm2', 'Iml', 'Im2', 'ImlIp2', 'IplIm2', 'IzlIz2', 'Izl', 'Iz2', 'ID', 'IplIz2', 'Ipl', 'Ip2', 'IplIp2']
```

- Spherical tensor product operator
 - Any spin quantum number
 - Any number of spins
- Special case: Spin half
 - Cartesian
 - Zeeman
 - Spherical

Documentation

```
Spin list = [1/2, 1/2]
 2 hbarEQ1 = True
   System = PyOR.Numerical MR(Spin list,hbarEQ1)
 4 | Sx,Sy,Sz = System.SpinOperator()
 1 help(System.SpinOperator)
Help on method SpinOperator in module PythonOnResonance:
SpinOperator() method of PythonOnResonance.Numerical MR instance
   Generate spin operators for all spins: Sx, Sy and Sz
   INPUT
   nill
   OUTPUT
   Sx : array [Sx of spin 1, Sx of spin 2, Sx of spin 3, ...]
   Sy: array [Sy of spin 1, Sy of spin 2, Sy of spin 3, ...]
   Sz : array [Sz of spin 1, Sz of spin 2, Sz of spin 3, ...]
```

```
1 Sp,Sm = System.PMoperators(Sx,Sy)
 2 help(System.PMoperators)
Help on method PMoperators in module PythonOnResonance:
PMoperators(Sx, Sy) method of PythonOnResonance.Numerical MR instance
    Generate spin operators for all spins: Sp (Sx + j Sy) and Sm (Sx - j Sy)
    INPUT
    ----
    Sx, Sy
    OUTPUT
    Sp : array [Sp of spin 1, Sp of spin 2, Sp of spin 3, ...]
    Sm : array [Sm of spin 1, Sm of spin 2, Sm of spin 3, ...]
```

Example: NOE (Hilbert and Liouville space)

Define Spin System (Case: Two spin half)

Define the Path to PyOR source code, PythonOnResonance.py

Define Spin System (Two Spin Half)

```
pathSource = '/media/HD2/Vineeth/PostDoc_Simulations/Github/PyOR_V1/Source' 1 Spin_list
```

```
Spin_list = [1/2, 1/2]
```

Load Python packages

from IPython.display import display, HTML display(HTML("<style>.container { width:100% !important; }</style>")) import sys sys.path.append(pathSource) import PythonOnResonance as PyOR import time import numpy as np import matplotlib.pyplot as plt from matplotlib import rc matplotlib import rc matplotlib notebook import sympy as sp from sympy import *

Define the unit of Hamiltonian

hbarEQ1 is True, then unit of Hamiltonian is in angular frequency

```
1 hbarEQ1 = True
```

Generate the spin operators: Sx, Sy and Sz

```
System = PyOR.Numerical_MR(Spin_list,hbarEQ1)
Sx,Sy,Sz = System.SpinOperator()
```

Generate the spin operators: S+ and S-

```
1 Sp,Sm = System.PMoperators(Sx,Sy)
```

Generating Zeeman Hamiltonian (Lab and Rotating Frame)

```
# Gyromagnetic Ratio
  Gamma = [System.gammaH1,System.gammaH1]
  # B0 Field in Tesla, Static Magnetic field (B0) along Z
   B0 = 9.4
  # Rotating Frame Frequency
  OmegaRF = [-System.gammaH1*B0,-System.gammaH1*B0]
  # Offset Frequency in rotating frame (Hz)
8 | \text{Offset} = [10.0, 20.0]
9 # generate Larmor Frequencies
10 LarmorF = System.LarmorFrequency(Gamma, B0, Offset)
11 # Lab Frame Hamiltonian
12 Hz lab = System.Zeeman(LarmorF,Sz)
13 # Rotating Frame Hamiltonian
14 Hz = System.Zeeman RotFrame(LarmorF,Sz,OmegaRF)
```

Larmor Frequency in MHz: [-400.22802765 -400.22803765]

Zeeman Basis Kets

```
Kets = System.Basis Ket()
 2 Kets
['|1/2,1/2>|1/2,1/2>',
 '|1/2,1/2>|1/2,-1/2>',
 '|1/2,-1/2>|1/2,1/2>',
 ' | 1/2, -1/2> | 1/2, -1/2> ' ]
```

Zeeman Basis Bras

```
Bras = System.Basis Bra()
 2 Bras
['<1/2,1/2|<1/2,1/2|',
 '<1/2,1/2|<1/2,-1/2|',
 '<1/2,-1/2|<1/2,1/2|',
 '<1/2,-1/2|<1/2,-1/2|'|
```

Zeeman Basis states

```
Basis Zeeman state = System.ZBasis H(Hz lab)
```

Zeeman Hamiltonian in Liouville Space (Zeeman Basis)

```
1 Hz L = System.CommutationSuperoperator(Hz)
```

Matrix representation of Zeeman Basis states

```
1 Matrix(Basis Zeeman state[0])
1.0
                                                       13
0
```

Initialize Density Matrix

```
Thermal DensMatrix = False
   if Thermal DensMatrix:
       # Spin temperature of individual spins (initial) Kelvin
       Tin = [300.0, 300.0]
       # Spin temperature of individual spins (equlibrium) Kelvin
       Tfi = [300.0, 300.0]
8
                                                                       System.DensityMatrix Components Dictionary(Basis PMZ, dic PMZ, rho)
10
       # High Temperature
11
       HT approx = False
                                                                   Density Matrix = 1.0 \text{ Iz}1 + -1.0 \text{ Iz}2
12
13
       # Initial Density Matrix
14
        rho in = System.EqulibriumDensityMatrix Advance(LarmorF,Sz,Tin,HT approx)
15
       # Equlibrium Density Matrix
16
17
        rhoeq = System.EqulibriumDensityMatrix Advance(LarmorF,Sz,Tfi,HT approx)
18 else:
19
        rho in = np.sum(Sz,axis=0)
        rhoeq = np.sum(Sz,axis=0)
20
```

```
System.DensityMatrix Components Dictionary(Basis PMZ, dic PMZ, rho in)
```

```
Density Matrix = 1.0 \text{ Iz}1 + 1.0 \text{ Iz}2
```

Converting initial and equlibrium density matrix into Liouvillian

```
rho in L = System.Vector L(rho in)
2 rhoeq L = System.Vector L(rhoeq)
```

Pulse (Hilbert Space)

```
flip angle1 = 0.0 # Flip angle Spin 1
flip angle2 = 180.0 # Flip angle Spin 2
rho = System.Rotate H(rho in,flip angle1,Sy[0])
rho = System.Rotate H(rho,flip angle2,Sy[1])
```

Pulse (Liouville Space)

```
1 rho L = System.Rotate L(rho in L,flip angle1,Sy[0])
2 rho L = System.Rotate L(rho L,flip angle2,Sy[1])
```

Relaxation in Hilbert Space

```
R1 = None
R2 = None

# Correlation Time
tau = [10.0e-12]

# Dipolar coupling constant (Hz)
bIS = [30.0e3]

Rprocess = "Auto-correlated Dipolar Homonuclear"

System.Relaxation_Constants(R1,R2)
System.Relaxation_Parameters(LarmorF, OmegaRF, tau, bIS)
```

Relaxation Rate Spin 1

```
R1_rate = System.RelaxationRate_H(Sz[0],Sz[0],Rprocess,R1,R2,Sx,Sy,Sz,Sp,Sm)
Rcross_rate = System.RelaxationRate_H(Sz[1],Sz[0],Rprocess,R1,R2,Sx,Sy,Sz,Sp,Sm)
Longit_relaxa = R1_rate + Rcross_rate
R2_rate = System.RelaxationRate_H(Sp[0],Sp[0],Rprocess,R1,R2,Sx,Sy,Sz,Sp,Sm)

print("T1 = %.5f and T2 = %.5f" % ((1.0/R1_rate).real, (1.0/R2_rate).real))
print("R1 = %.5f and R2 = %.5f" % ((R1_rate).real, (R2_rate).real))
print("Cross Relaxation rate = %.5f and time = %.5f" % (Rcross_rate.real, 1.0/Rcross_rate.real))
print("Longitudinal Relaxation rate = %.5f and time = %.5f" % (Longit_relaxa.real, 1.0/Longit_relaxa.real))

T1 = 5.63856 and T2 = 5.63482
R1 = 0.17735 and R2 = 0.17747
Cross Relaxation rate = 0.08856 and time = 11.29210
Longitudinal Relaxation rate = 0.26591 and time = 3.76070
```

Evolution of Density Matrix in Hilbert Space

```
dt = 0.0001
2 AQ = 50.0
3 Npoints = int(AQ/dt)

method = "ODE Solver"
6 ode_solver = 'DOP853'
7 System.ODE_Method(ode_solver)

start_time = time.time()
10 t, rho_t = System.Evolution_H(rhoeq,rho,Sx,Sy,Sz,Sp,Sm,Hz+Hj,dt,Npoints,method,Rprocess)
11 end_time = time.time()
12 timetaken = end_time - start_time
13 print("Total time = %s seconds " % (timetaken))

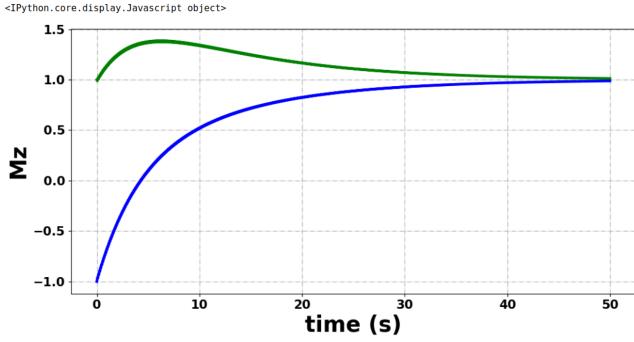
Tatal_time_22_42313264046002_casesde
1 System.PlottingMulti(1,[t,t],[signal_Z1,signal_Z2],"time (s)","Mz",["green","blue"])
```

Total time = 33.43313264846802 seconds

Expectation Value (Hilbert Space)

```
det_Z1 = Sz[0]
det_Z2 = Sz[1]

t, signal_Z1 = System.Expectation_H(rho_t,det_Z1,dt,Npoints)
t, signal_Z2 = System.Expectation_H(rho_t,det_Z2,dt,Npoints)
```



Relaxation in Liouville Space

```
1 R = None
2 Rprocess = "Auto-correlated Dipolar Homonuclear"
3 tau = [10.0e-12]
4 bIS = [30.0e3]
5 System.Relaxation_Parameters(LarmorF, OmegaRF, tau, bIS)
6 R_L = System.Relaxation_L(Rprocess,R,Sx,Sy,Sz,Sp,Sm)
```

Evolution of Density Matrix Liouville Space

```
method = "ODE Solver"
System.ODE_Method('DOP853')

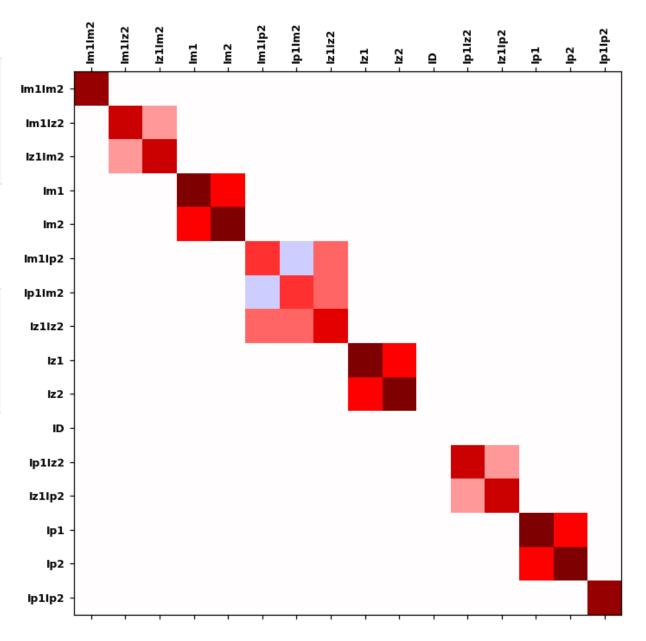
start_time = time.time()
t, rho_t = System.Evolution_L(rhoeq_L,rho_L,Sx,Sy,Hz_L,R_L,dt,Npoints,method)
end_time = time.time()
timetaken = end_time - start_time
print("Total time = %s seconds " % (timetaken))

(16, 500000)
Total time = 3.557140827178955 seconds
```

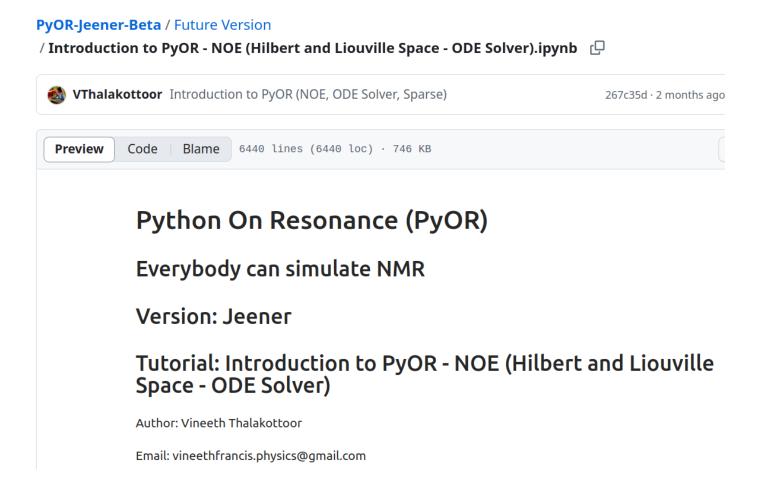
Expectation value (Liouville Space)

```
1 LEXP_Z1 = System.Detection_L(det_Z1)
2 LEXP_Z2 = System.Detection_L(det_Z2)

1 t, MZ_1 = System.Expectation_L(rho_t, LEXP_Z1, dt, Npoints)
2 t, MZ_2 = System.Expectation_L(rho_t, LEXP_Z2, dt, Npoints)
```



Look detailed notebook in Github



Example: Ethanolamine Spectra

Define Spin System (Four Spin Half)

```
1 Spin_list = [1/2, 1/2, 1/2, 1/2]
```

Define the unit of Hamiltonian

```
1 hbarEQ1 = True
```

Generate the spin operators: Sx, Sy, Sz, Sp and Sm

```
# Call Class "Numerical_MR" from PyOR package
System = PyOR.Numerical_MR(Spin_list,hbarEQ1)

Sx,Sy,Sz = System.SpinOperator()

Sp,Sm = System.PMoperators(Sx,Sy)
```

```
# Offset Frequency in rotating frame (Hz)
ref_freq = System.gammaH1*B0
Freq_1 = System.PPMtoHz(2.35,ref_freq)
Freq_2 = System.PPMtoHz(2.35,ref_freq)
Freq_3 = System.PPMtoHz(3.1,ref_freq)
Freq_4 = System.PPMtoHz(3.1,ref_freq)
Offset = [Freq_1,Freq_2,Freq_3,Freq_4]
```

```
# generate Larmor Frequencies
LarmorF = System.LarmorFrequency(Gamma,B0,Offset)

Larmor Frequency in MHz: [-500.14569182 -500.14569182 -500.14606693 -500.14606693]

# Lab Frame Hamiltonian
Hz_lab = System.Zeeman(LarmorF,Sz)

# Rotating Frame Hamiltonian
Hz = System.Zeeman_RotFrame(LarmorF,Sz,OmegaRF)
```

Generating Zeeman Hamiltonian (Lab and Rotating Frame)

```
# Gyromagnetic Ratio
Gamma = [System.gammaH1,System.gammaH1,System.gammaH1,System.gammaH1]

# B0 Field in Tesla, Static Magnetic field (B0) along Z
B0 = System.L500

# Rotating Frame Frequency
OmegaRF = [-System.gammaH1*B0,-System.gammaH1*B0,-System.gammaH1*B0]
```

Generating J Coupling Hamiltonian

```
Jlist = np.zeros((len(Spin list),len(Spin list)))
  Jlist[0][1] = 11.1
   Jlist[0][2] = 3.9
   Jlist[0][3] = 6.69
   Jlist[1][2] = 6.69
   Jlist[1][3] = 3.9
   Jlist[2][3] = 12.3
8
   Jcoupling Strong = True
10
   if Jcoupling Strong:
       Hj = System.Jcoupling(Jlist,Sx,Sy,Sz)
13
   else:
14
       Hj = System.Jcoupling Weak(Jlist,Sz)
```

Initialize Density Matrix

```
1 Thermal DensMatrix = False
 3 if Thermal DensMatrix:
       # Spin temperature of individual spins (initial) Kelvin
       Tin = [300.0, 300.0]
       # Spin temperature of individual spins (equlibrium) Kelvin
       Tfi = [300.0, 300.0]
       # High Temperature
11
       HT approx = False
12
13
       # Initial Density Matrix
14
       rho in = System.EqulibriumDensityMatrix Advance(LarmorF,Sz,Tin,HT approx)
15
       # Equlibrium Density Matrix
16
17
       rhoeq = System.EqulibriumDensityMatrix Advance(LarmorF,Sz,Tfi,HT approx)
18 else:
       rho in = np.sum(Sz,axis=0)
19
20
       rhoeq = np.sum(Sz,axis=0)
```

Pulse (Hilbert Space)

```
flip_angle = 90.0
rho = System.Rotate_H(rho_in,flip_angle,np.sum(Sy,axis=0))
```

Relaxation

```
R1 = 0.0
R2 = 0.0
Rprocess = "No Relaxation"
System.Relaxation_Constants(R1,R2)
```

Evolution of Density Matrix

```
Highest_Larmor_Frequency = Freq_4
fs = 4 * Highest_Larmor_Frequency
dt = 1.0/fs
AQ = 5.0
Npoints = int(AQ/dt)
print("Number of points in the simulation", Npoints)

"""
option for solver, "method": "Unitary Propagator" or "ODE Solver"
"""
method = "Unitary Propagator"

start_time = time.time()
t, rho_t = System.Evolution_H(rhoeq,rho,Sx,Sy,Sz,Sp,Sm,Hz+Hj,dt,Npoints,method,Rprocess)
end_time = time.time()
timetaken = end_time - start_time
print("Total time = %s seconds " % (timetaken))
```

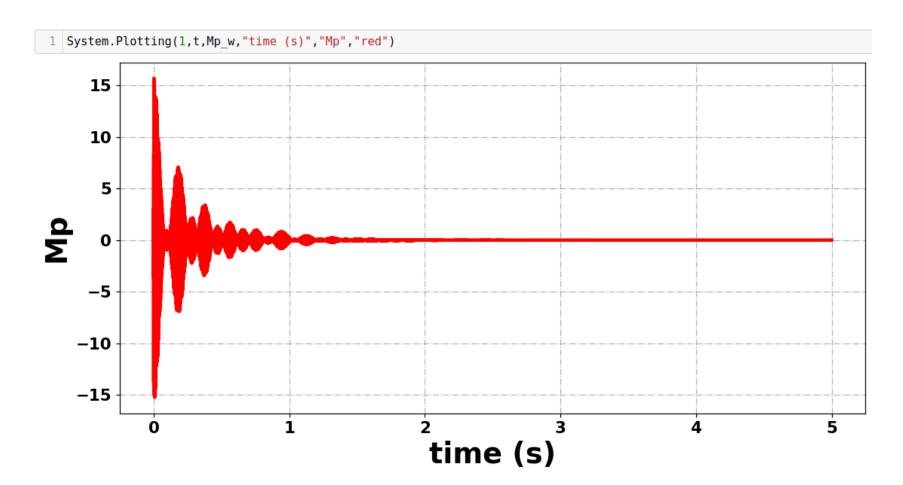
Number of points in the simulation 31008 Total time = 0.29144835472106934 seconds

Expectation value

```
1 t, Mp = System.Expectation_H(rho_t,np.sum(Sp,axis=0),dt,Npoints)
```

Windowing

```
1 Mp_w = System.WindowFunction(t,Mp,3.0)
```

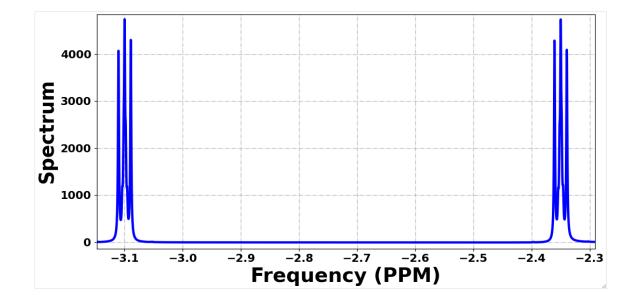


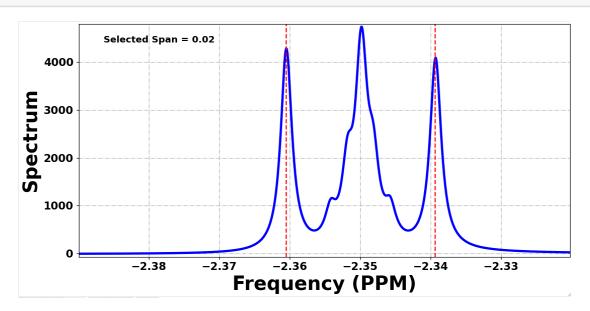
Fourier Transform

```
1 fs = 1.0/dt
2 freq, spectrum = System.FourierTransform(Mp_w,fs,5)
```

```
# Zero order phase correction
PH0 = 45.0
spectrum_PH0 = System.PhaseAdjust_PH0(spectrum,PH0)
# First order phase correction
pivot = -1550.8
slope = 1.1
spectrum_PH1 = System.PhaseAdjust_PH1(freq,spectrum_PH0,pivot,slope)

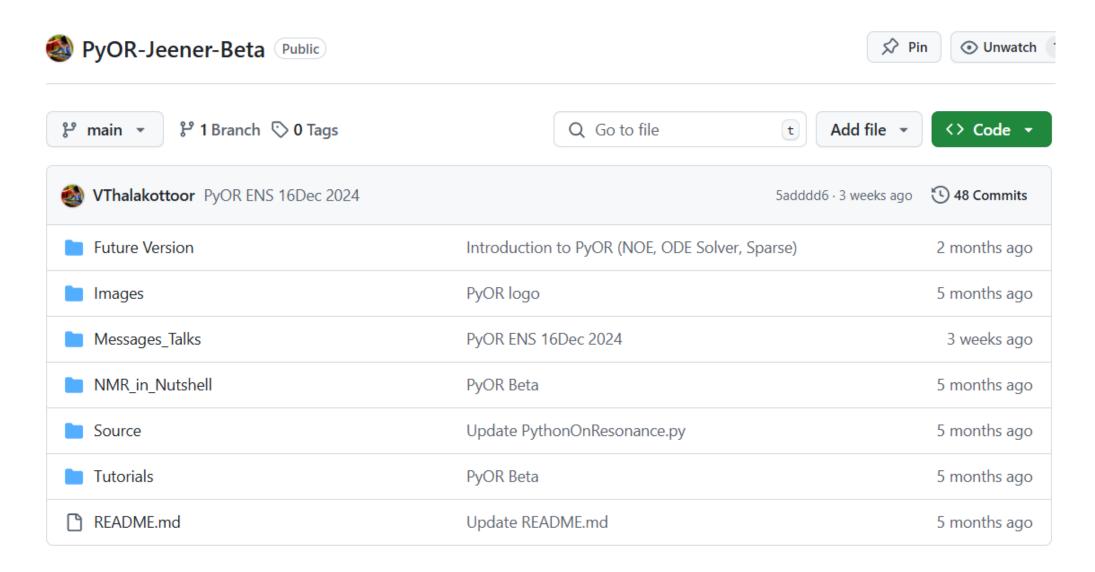
PPMscale = True
if PPMscale:
    fig,span_selector = System.Plotting_SpanSelector(2,System.HztoPPM(freq,ref_freq),spectrum_PH1,"Frequency (PPM)","Spectrum","blue")
else:
    fig,span_selector = System.Plotting_SpanSelector(2,freq,spectrum_PH1,"Frequency (Hz)","Spectrum","blue")
```





PyOR Jeener (beta version)

https://github.com/VThalakottoor/PyOR-Jeener-Beta



Future

- Shaped Pulse
- Solid State NMR, Quadrupolar NMR
- EPR
- Graphical User Interface (GUI)

Acknowledgement

Daniel Abergel and ANR (ANR-22-CE29-0006-01-DynNonlinPol)

PyOR is all yours

"Let what is created surpass the creator"

Thank You