

Python On Resonance (PyOR)

Everybody can simulate NMR

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Tutorial 14: Avoided Crossing Part 2

Example: Spin-Lock Induced Crossing (SLIC) - 2 spin half system

In previous tutorial we had plotted avoided crossing for two spin half system. In this tutorial we look into population "oscillation" between singlet and triplet states, when B1 amplitude (Spin Lock) equals J coupling (the SLIC condition).

Reference: PhD Thesis, Stephen J. DeVience, Harvard University, 2014.

Load Python packages and define path to the source file "PythonOnResonance.py"

```
In [1]: pathSource = '/media/HD2/Vineeth/PostDoc_Simulations/Github/PyOR_G/Source'
```

```
In [2]: from IPython.display import display, HTML
display(HTML("<style>.container { width:100% !important; }</style>"))
import sys
sys.path.append(pathSource)

import PythonOnResonance as PyOR

import time
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import rc
%matplotlib notebook
import sympy as sp
from sympy import *
```

Generating Spin System

```
In [3]: """
Define Spin quantum numbers of your spins in "Slist1".
Slist1[0] is spin of first particle and Slist1[1] is spin of second particle.
""";

Slist1 = [1/2, 1/2]
```

```
In [4]: """
Define Planck constant equals 1.
Because NMR spectroscopists are more interested to write Energy in frequency units.
```

```

if False then hbarEQ1 = hbar
""";

hbarEQ1 = True

```

```

In [5]: """
Generate Spin Operators
""";

System = PyOR.Numerical_MR(Slist1,hbarEQ1)

"""
Sx, Sy and Sz Operators
""";
Sx,Sy,Sz = System.SpinOperator()

"""
S+ and S- Operators
""";
Sp,Sm = System.PMoperators(Sx,Sy)

```

Zeeman Hamiltonian in Lab Frame

```

In [6]: """
Gyromagnetic Ratio
Gamma = [Gyromagnetic Ratio spin 1, Gyromagnetic Ratio spin 1, ...]
""";
Gamma = [System.gammaH1, System.gammaH1]

"""
Define the field of the spectrometer, B0 in Tesla.
""";
B0 = 4.7

"""
Define the chemical Shift of individual spins
Offset = [chemical Shift spin 1, chemical Shift spin 1, ..]
""";
Offset = [0,2.8] # Offset frequency in Hz
deltaV = Offset[1] - Offset[0] # Frequency difference between Spin 1 and 2
""";

Function "LarmorF" give the list Larmor frequencies of individual spins in lab frame
""";
LarmorF = System.LarmorFrequency(Gamma,B0,Offset)

Hz = System.Zeeman(LarmorF,Sz)

```

Larmor Frequency in MHz: [-200.11400882 -200.11401162]

Initialize Density Matrix

```

In [7]: """
We will generate Initial Density Matrix in two ways:
First we will generate a density matrix as we prefer say, Sz.
Second we will create density matrix at thermal equilibrium

First Case
""";

Thermal_DensMatrix = False

```

```

if Thermal_DensMatrix:
    Hz_EnUnit = System.Convert_FreqUnitsToEnergy(Hz)
    HT_approx = False # High Temperature Approximation is False
    T = 300 # Temperature in Kelvin
    rho_in = System.EquilibriumDensityMatrix(Hz_EnUnit,T,HT_approx)
    rhoeq = rho_in.copy()
else:
    rho_in = np.sum(Sz,axis=0) # Initial Density Matrix
    rhoeq = np.sum(Sz,axis=0) # Equilibrium Density Matrix
    print("Trace of density metrix = ", np.trace(rho_in))

```

Trace of density metrix = 0j

Zeeman Halitonian in Rotating Frame

```

In [8]: off = -2*np.pi*deltaV/2
OmegaRF = [-System.gammaH1*B0 + off, -System.gammaH1*B0 + off] # RF irradiation in the mid
Hxr = System.Zeeman_RotFrame(LarmorF, Sz, OmegaRF)

```

J Coupling Hamiltonian

```

In [9]: """
Define J couplings between individual spins
"""

Jlist = np.zeros((len(Slist1),len(Slist1)))
Jlist[0][1] = 17.4
Hj = System.Jcoupling(Jlist,Sx,Sy,Sz)

```

B1 Hamiltonian (Spin Lock)

```

In [10]: Omega1 = [Jlist[0][1],Jlist[0][1]] # SLIC condition: B1 amplitude equals J coupling between
Omega1Phase = [0,0]
Hrf = System.Zeeman_B1(Sx,Sy,Omega1,Omega1Phase)

```

Total Hamiltonian

```

In [11]: Hslic = Hxr + Hj + Hrf # Hamiltonina duirng Spin Lock

```

We will work with 4 Spin-Lock Eigenstates

1. Spin-Lock Eigenstates (when B1 amplitude of RF >> Chemical shift difference)

$$|\phi_+ \rangle = \frac{1}{2}(|\alpha\alpha \rangle + |\alpha\beta \rangle + |\beta\alpha \rangle + |\beta\beta \rangle) = \frac{1}{\sqrt{2}}|T_0 \rangle + \frac{1}{2}(|T_- \rangle + |T_+ \rangle)$$

$$|\phi_0 \rangle = \frac{1}{\sqrt{2}}(|\alpha\alpha \rangle - |\beta\beta \rangle) = \frac{1}{\sqrt{2}}(|T_- \rangle - |T_+ \rangle)$$

$$|\phi_S \rangle = \frac{1}{\sqrt{2}}(|\alpha\beta \rangle - |\beta\alpha \rangle) = |S_0 \rangle$$

$$|\phi_{-}\rangle = \frac{1}{2}(-|\alpha\alpha\rangle + |\alpha\beta\rangle + |\beta\alpha\rangle - |\beta\beta\rangle) = \frac{1}{\sqrt{2}}|T_0\rangle - \frac{1}{2}(|T_{-}\rangle + |T_{+}\rangle)$$

2. Zeeman Eigenstates

$$|\alpha\alpha\rangle, |\alpha\beta\rangle, |\beta\alpha\rangle, |\beta\beta\rangle$$

3. Singlet-Triplet States

$$|T_{+}\rangle, |T_0\rangle, |T_0\rangle, |S_0\rangle$$

In [12]:

```
"""
Zeeman eigen states
""";
B_Z = System.ZBasis_H(Hz)
Matrix(B_Z[0])
```

$$|1/2,1/2\rangle|1/2,1/2\rangle, |1/2,1/2\rangle|1/2,-1/2\rangle, |1/2,-1/2\rangle|1/2,1/2\rangle, |1/2,-1/2\rangle|1/2,-1/2\rangle$$

Out[12]:

$$\begin{bmatrix} 1.0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In [13]:

```
"""
Lets make Spin-Lock Eigenstates from Zeeman eigen states
""";
Phi_p = 0.5 * (B_Z[0] + B_Z[1] + B_Z[2] + B_Z[3]) # linear combination of triplet states
Phi_0 = (1/sqrt(2)) * (B_Z[0] - B_Z[3]) # linear combination of triplet states
Phi_S = (1/sqrt(2)) * (B_Z[1] - B_Z[2]) # Singlet State
Phi_m = 0.5 * (B_Z[1] + B_Z[2] - B_Z[0] - B_Z[3]) # linear combination of triplet states
Matrix(Phi_p)
```

Out[13]:

$$\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

In [14]:

```
"""
Let make a basis transformation operator, U
Inorder to transform all operators from Zeeman basis to Spin-Lock Eigenstates
""";
U = np.zeros((System.Vdim, System.Vdim))
U[:,0] = Phi_m.T
U[:,1] = Phi_0.T
U[:,2] = Phi_p.T
U[:,3] = Phi_S.T
Matrix(U)
```

Out[14]:

$$\begin{bmatrix} -0.5 & 0.707106781186548 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0.707106781186548 \\ 0.5 & 0 & 0.5 & -0.707106781186548 \\ -0.5 & -0.707106781186548 & 0.5 & 0 \end{bmatrix}$$

In [15]:

```
"""
```

```

Lets make the population operators from Spin-Lock Eigenstates
pop_P population of Phi_p # linear combination of triplet states
pop_0 population of Phi_0 # linear combination of triplet states
pop_S population of Phi_S # Singlet states
pop_M population of Phi_m # linear combination of triplet states
""";
pop_P = Phi_p @ Phi_p.T
pop_0 = Phi_0 @ Phi_0.T
pop_S = Phi_S @ Phi_S.T
pop_M = Phi_m @ Phi_m.T

```

```

In [16]: """
The population operators are in Zeeman basis
""";
Matrix(pop_P)

```

```

Out[16]: 
$$\begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$


```

```

In [17]: """
Convert the operators into Spin-Lock basis
""";
pop_P_SL = np.linalg.inv(U) @ pop_P @ U
pop_0_SL = np.linalg.inv(U) @ pop_0 @ U
pop_S_SL = np.linalg.inv(U) @ pop_S @ U
pop_M_SL = np.linalg.inv(U) @ pop_M @ U
Hslic_SL = np.linalg.inv(U) @ Hslic @ U # Changing the basis of Hamiltonian to Spin-Lock

```

```

In [18]: Matrix(pop_M)

```

```

Out[18]: 
$$\begin{bmatrix} 0.25 & -0.25 & -0.25 & 0.25 \\ -0.25 & 0.25 & 0.25 & -0.25 \\ -0.25 & 0.25 & 0.25 & -0.25 \\ 0.25 & -0.25 & -0.25 & 0.25 \end{bmatrix}$$


```

```

In [19]: Matrix(pop_0)

```

```

Out[19]: 
$$\begin{bmatrix} 0.5 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & 0.5 \end{bmatrix}$$


```

```

In [20]: Matrix(pop_P)

```

```

Out[20]: 
$$\begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$


```

```

In [21]: Matrix(pop_S) # Singlet

```

```
Out[21]: 
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & -0.5 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```

Pulse

```
In [22]: """
Rotate the magnetization about Y-axis, by an angle theta.
""";
pulse_angle = 90.0
rho = System.Rotate_H(rho_in,pulse_angle,np.sum(Sy,axis=0))
rho_SL = np.linalg.inv(U) @ rho @ U # Changing the basis of density matrix to Spin-Lock b
```

Relaxation Constant

```
In [23]: R1 = 1.0
R2 = 2.0
System.Relaxation_Constants(R1,R2)

Rprocess = "No Relaxation"
```

Evolution of Density Matrix under first Spin Lock

```
In [24]: dt = 1.0e-4
Spin_Lock_Time = 1
Npoints1 = int(Spin_Lock_Time/dt)
print("Number of points in the simulation", Npoints1)

"""
option for solver, "method": "Unitary Propagator" or "ODE Solver"
"""
method = "Unitary Propagator"

start_time = time.time()
t1, rho_t1 = System.Evolution_H(rhoeq,rho_SL,Sx,Sy,Sz,Sp,Sm,Hslic_SL,dt,Npoints1,method,Rp
end_time = time.time()
timetaken = end_time - start_time
print("Total time = %s seconds " % (timetaken))
```

Number of points in the simulation 10000
Total time = 0.05933833122253418 seconds

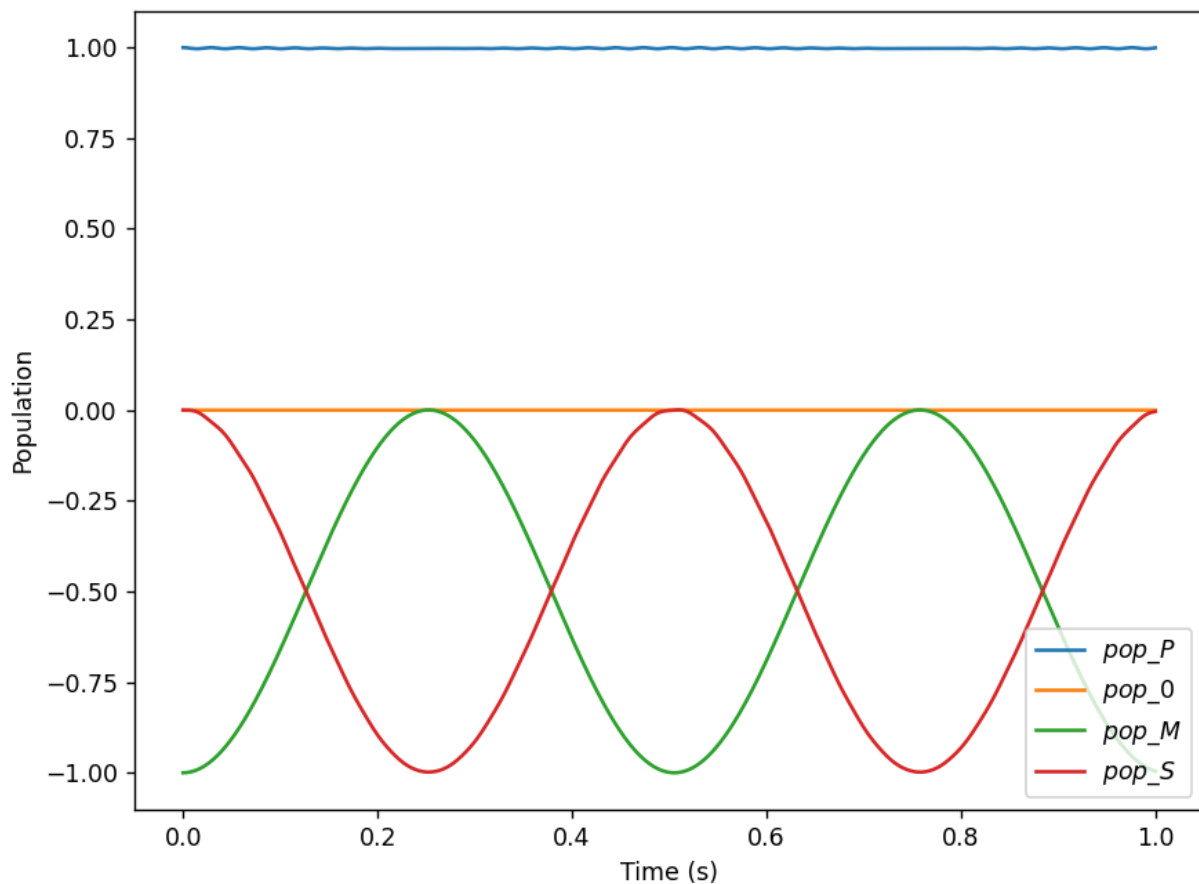
Expetation Value - Population

```
In [25]: start_time = time.time()
t, population_P = System.Expectation_H(rho_t1,pop_P_SL,dt,Npoints1)
t, population_0 = System.Expectation_H(rho_t1,pop_0_SL,dt,Npoints1)
t, population_M = System.Expectation_H(rho_t1,pop_M_SL,dt,Npoints1)
t, population_S = System.Expectation_H(rho_t1,pop_S_SL,dt,Npoints1)
end_time = time.time()
timetaken = end_time - start_time
print("Total time = %s seconds " % (timetaken))
```

Plotting the Population

In [26]:

```
plt.figure(1)
plt.plot(t, population_P, "-", label=r"$pop\_P$")
plt.plot(t, population_0, "-", label=r"$pop\_0$")
plt.plot(t, population_M, "-", label=r"$pop\_M$")
plt.plot(t, population_S, "-", label=r"$pop\_S$")
plt.xlabel("Time (s)")
plt.ylabel("Population")
plt.legend()
```



```
/opt/anaconda3/lib/python3.9/site-packages/numpy/core/_asarray.py:102: ComplexWarning: Casting complex values to real discards the imaginary part
  return array(a, dtype, copy=False, order=order)
/opt/anaconda3/lib/python3.9/site-packages/numpy/core/_asarray.py:102: ComplexWarning: Casting complex values to real discards the imaginary part
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/opt/anaconda3/lib/python3.9/site-packages/numpy/core/_asarray.py:102: ComplexWarning: Casting complex values to real discards the imaginary part
  return array(a, dtype, copy=False, order=order)
Out[26]: <matplotlib.legend.Legend at 0x7f07319ba610>
```

Conclusion:

So we can see the population between pop_M and pop_S oscillates. So if we wait $t_{SLIC,max} = \frac{0.707}{\Delta\nu}$, in this case is it 0.2525 seconds, $\Delta\nu$ is the chemical shift difference, we can populate the singlet state.

Plotting the Eigen Values

Any suggestion? write to me

If you see something is wrong please write to me, so that the PyOR can be error free.

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In []: