Zad. 3. De Casteljau metoda za efikasno računanje Bezierovog splajna. Pokažite da je metoda ispravna, tj. da za proizvoljni $u \in [0,1]$ i

$$r_i = (1-u)p_i + up_{i+1}, \quad i = 0,1,2$$
$$s_i = (1-u)r_i + ur_{i+1}, \quad i = 0,1$$
$$t_0 = (1-u)s_0 + us_1$$

vrijedi $f(u) = t_0$.

Dokaz:

$$f(u) = b_0(u)p_0 + b_1(u)p_1 + b_2(u)p_2 + b_3(u)p_3$$

$$b_0(u) = (1-u)^3$$
$$b_1(u) = 3u(1-u)^2$$
$$b_2(u) = 3u^2(1-u)$$
$$b_3(u) = u^3$$

$$f(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3$$

$$t_0 = (1-u)s_0 + us_1 = (1-u)\big((1-u)r_0 + ur_1\big) + u\big((1-u)r_1 + ur_2\big) =$$

$$= (1-u)^2 r_0 + (1-u)ur_1 + u(1-u)r_1 + u^2 r_2 =$$

$$= (1-u)^2 r_0 + 2u(1-u)r_1 + u^2 r_2 =$$

$$= (1-u)^2\big((1-u)p_0 + up_1\big) + 2u(1-u)\big((1-u)p_1 + up_2\big) + u^2\big((1-u)p_2 + up_3\big) =$$

$$= (1-u)^3 p_0 + u(1-u)^2 p_1 + 2u(1-u)^2 p_1 + 2u^2(1-u)p_2 + u^2(1-u)p_2 + u^3 p_3 =$$

$$= (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3.$$

$$= f(u). \quad \blacksquare$$