# Contact informations

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Title: *Parameter control in the presence of uncertainties*

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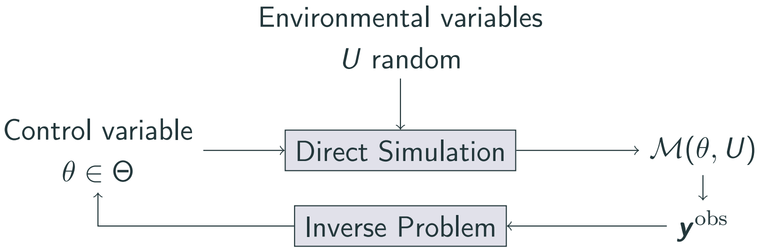
# Research topics

- Optimisation under Uncertainties
  - Notions of robustness in a calibration context
  - Regret-based family of estimators
- Adaptive Strategies using GP for the estimation
  - GP formulation of the regret
  - SUR (Stepwise Uncertainty Reduction) strategies of design enrichment
  - AK-MCS (Adaptive Kriging Monte Carlo Sampling) for batch selection of points

# Computer code and inverse problem

Input
- $\theta$: Control parameter
- $u$: Environmental variables, realisations of r.v. $U$

Output
- $\mathcal{M}(\theta, u)$: Quantity to be compared to observations



# Regret-based formulations

- $(\theta, u) \longmapsto J(\theta, u)$ objective function, *strictly positive*
- Best performance given $u$: $J^* : u \mapsto \min_{\theta \in \Theta} J(\theta, u)$

$\rightarrow$ Find $\hat{\theta}$ such that $J(\hat{\theta}, U)$ "close to" $J^*(U) = J(\theta^*(U), U)$ with high enough probability

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- We define the regret as $r(\theta, u) = J(\theta, u) - J^*(u)$ or $r(\theta, u) = \frac{J(\theta,u)}{J^*(u)}$
- $\Gamma_\alpha(\theta) = \mathbb{P}_U\left[r(\theta, U) \leq \alpha\right]$: Probability that the regret is below a level $\alpha$

**Regret-based family of estimators (Trappler et al., 2020)**

$$\left\{\hat{\theta} \mid \hat{\theta} = \operatorname*{arg\,max}_{\theta \in \Theta} \Gamma_\alpha(\theta), \alpha > 1\right\}$$

# Adaptive strategies for the estimation of $\Gamma_\alpha$

Let $Z \sim \mathrm{GP}(m_Z, \sigma_Z^2)$ be a GP constructed on $\Theta \times \mathbb{U}$ based on $J$.
We use $Z$ to estimate $\Gamma_\alpha$

- Reduction of the *augmented* IMSE of $\Delta = Z - \alpha Z^*$
- Improve estimation of the set $\{\Delta \leq 0\}$, using probability of coverage

- Sampling in the margin of uncertainty of $\{\Delta \leq 0\}$, and transformations to get a batch of points (AK-MCS)

# Adaptive strategies for the estimation of the quantile of the regret

Under certain conditions, the ratio $Z/Z^*$ is approximately log-normally distributed. Let us define $\Xi = \log Z/Z^*$

- Reduction of the augmented IMSE of $\Xi$
- Sampling in regions of interest, and transformations (QeAK-MCS (Razaaly, 2019))

# Conclusion

## Wrapping up

- Problem of a *good* definition of robustness
- Tuning $\alpha$ or $\eta$ reflects risk-seeking or risk-adverse strategies
- Strategies rely heavily on surrogate models, to embed aleatoric uncertainties directly in the modelling

## Perspectives

- Cost of computer evaluations $\rightarrow$ limited number of runs?
- In low dimension, CROCO very well-behaved.
- Dimensionality of the input space $\rightarrow$ reduction of the input space?

# References


Razaaly, N. (2019). *Rare Event Estimation and Robust Optimization Methods with Application to ORC Turbine Cascade*. These de doctorat, Université Paris-Saclay (ComUE).

Trappler, V., Arnaud, É., Vidard, A., and Debreu, L. (2020). Robust calibration of numerical models based on relative regret. *Journal of Computational Physics*, page 109952.

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The minimizer can be seen as a random variable:

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$\longrightarrow$ estimate its density (how often is the value $\theta$ a minimizer)

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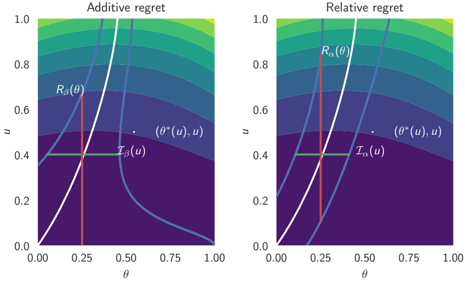
How to take into account values not optimal, but not too far either $\longrightarrow$ relaxation of the equality with $\alpha > 1$:

$$\Gamma_\alpha(\theta) = \mathbb{P}_U\left[J(\theta, U) \leq \alpha J^*(U)\right]$$

# Why the relative regret ?




- Relative regret
  - $\alpha$-acceptability regions large for flat and bad situations ($J^*(u)$ large)
  - Conversely, puts high confidence when $J^*(u)$ is small
  - No units $\to$ ratio of costs

# Notions of regret

Let $J^*(u) = \min_{\theta \in \Theta} J(\theta, u)$ and $\theta^*(u) = \arg\min_{\theta \in \Theta} J(\theta, u)$. The regret $r$:

$$r(\theta, u) = J(\theta, u) - J^*(u) = -\log\left(\frac{e^{-J(\theta,u)}}{\max_\theta\{e^{-J(\theta,u)}\}}\right) \quad (1)$$

$$= -\log\left(\frac{\mathcal{L}(\theta, u)}{\max_{\theta \in \Theta} \mathcal{L}(\theta, u)}\right) \quad (2)$$

$\rightarrow$ linked to misspecified LRT: maximize the probability of keeping $\mathcal{H}_0$: $\theta$ valid instead of $\arg\max \mathcal{L}$.

# PEI criterion

$Y \sim \mathrm{GP}(m_Y(\cdot), C_Y(\cdot, \cdot))$ on $\Theta \times \mathbb{U}$

$$\mathrm{PEI}(\theta, u) = \mathbb{E}_Y \left[ \left[ f_{\mathrm{min}}(u) - Y(\theta, u) \right]_+ \right] \tag{3}$$

where $f_{\mathrm{min}}(u) = \max\left\{\min_i J(\theta_i, u_i), \min_{\theta \in \Theta} m_Y(\theta, u)\right\}$