Robust estimation of parameters in the presence of random input

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Abstract. Classical methods of parameter estimation usually imply the minimisation of an objective function, that measures the error between some observations and the results obtained by a numerical model. In the presence of random inputs, the optimum is directly dependent on the fixed nominal value given to the uncertain parameter ; and therefore may not be relevant in other conditions.

In this paper, we are going to present strategies taking into account those uncertainties and apply them on an academic model of a coastal area, in order to find an optimal value in a robust sense.

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1 Introduction

1.1 Motivation

Das and Lardner (1991, 1992)

1.2 State of the art

Kuczera et al. (2010)

2 Deterministic setting and limitations

2.1 Context and notations

We assume that we have access to a computer code $\mathcal{M}$, simulating a physical phenomenon over a specific time window. This numerical model takes two types of inputs: a control variable, named $\mathbf{k} \in \mathcal{K} \subset \mathbb{R}^p$, and an environmental variable, named $\mathbf{u} \in \mathcal{U} \subset \mathbb{R}^l$. The control variable represents the quantities that are assumed to be constant, and comes from a lack of knowledge of the real physical settings. In the scope of our application, this parameter is the ocean bed friction induced by its rugosity. The environmental variables are the operating conditions, such as the boundary conditions. The output of the computer code is a physical quantity $\mathcal{M}(\mathbf{k},\mathbf{u}) \in \mathcal{Y}$ (such as the sea surface height) that will be compared to an observation $\mathbf{y}^o$. In order to use this model,

2.2 Parameter calibration in a deterministic setting

We assume that we dispose of an observation $\mathbf{y}^o$, that has been generated with a couple of unknowns $\mathbf{k}^t$ and $\mathbf{u}^t$ (where $(\cdot)^t$ stands for "truth"): $\mathbf{y}^o = \mathcal{M}(\mathbf{k}^t,\mathbf{u}^t)$. We also assume that we have background values for $\mathbf{k}$ and $\mathbf{u}$: $\mathbf{k}^b$ and $\mathbf{u}^b$. Calibration in this setting boils down to the minimisation of the cost function J defined by

$$\begin{aligned} J(\mathbf{k}) &= \frac{1}{2}(\mathcal{M}(\mathbf{k},\mathbf{u}^b) - \mathbf{y}^o)^T\mathbf{R}^{-1}(\mathcal{M}(\mathbf{k},\mathbf{u}^b) - \mathbf{y}^o) \\ &\quad + \frac{1}{2}(\mathbf{k}-\mathbf{k}^b)^T\mathbf{B}^{-1}(\mathbf{k}-\mathbf{k}^b) \\ &= \frac{1}{2}\|\mathcal{M}(\mathbf{k},\mathbf{u}^b) - \mathbf{y}^o\|^2_{\mathbf{R}^{-1}} + \frac{1}{2}\|\mathbf{k}-\mathbf{k}^b\|^2_{\mathbf{B}^{-1}} \end{aligned} \tag{1}$$

and after minimization, we get $\hat{\mathbf{k}}$ such that

$$J(\hat{\mathbf{k}}) = \min_{\mathbf{k}\in\mathcal{K}} J(\mathbf{k}) \tag{2}$$

Minimizing J will indeed compensate the error introduced by the parametrization $\mathbf{k}$, but will also compensate another misspecification: choosing $\mathbf{u}^b$ instead of $\mathbf{u}^t$ in the computer code. In the end, the value $\hat{\mathbf{k}}$ obtained will be optimal for this configuration. However, if the operating conditions do change, we do not have any information on the performance of $\hat{\mathbf{k}}$ in another setting. The solution to jointly estimate $\mathbf{k}$ and $\mathbf{u}$.

3 Different natures of parameters and uncertainties

As put in Walker et al. (2003), different types of parameters can be established

- Exact and fixed parameters, that are "universal" mathematical constant, or physical constants that are considered well known, such as the acceleration of gravity g.
- A priori chosen parameters, which are chosen and fixed to a certain value considered invariant
- Calibrated parameters, which are unknown and needs to be calibrated.

3.1 Probabilistic point of view

We model the environmental variables as a random variable: $\mathbf{U}$ whose sample space is $\mathcal{U}$. The probability measure of $\mathbf{U}$ is $\mathbb{P}_U$, and its density, if it exists, is p_U.

3.2 The cost function as a random variable

In this case, the cost function defined in (1) is no longer a function of solely $\mathbf{k}$, but of $\mathbf{U}$ as well:

$$J(\mathbf{k},\mathbf{U}) = \frac{1}{2}\|\mathcal{M}(\mathbf{k},\mathbf{U}) - \mathbf{y}^o\|^2_{\mathbf{R}^{-1}} + \frac{1}{2}\|\mathbf{k}-\mathbf{k}^b\|^2_{\mathbf{B}^{-1}} \tag{3}$$

3.3 Definitions of robustness

Robustness does have a lot of meanings, especially in the probabilities and statistics community. Indeed, a definition of the robustness of an estimator is a measure of the sensibility of said estimator to outliers (Huber, 2011). In other cases, robustness in a Bayesian framework refers to the sentivity of a wrong specification of the priors (Berger et al., 1994). In this paper, we are going to focus on two different aspects:

- Optimisation of the mean response (M-robustness in Lehman et al. (2004))

$$\text{minimize : } \mathbb{E}_U\left[J(\mathbf{k},\mathbf{U})\right] = \int_{\mathcal{U}} J(\mathbf{k},\mathbf{u})\,\mathrm{d}\mathbb{P}_U(\mathbf{u}) \tag{4}$$

$$= \int_{\mathcal{U}} J(\mathbf{k},\mathbf{u})p_U(\mathbf{u})\,\mathrm{d}\mathbf{u} \tag{5}$$

- Study of the distribution of the minimizers, assuming the inverse problem is well-posed to ensure the existence and the unicity of the $\arg\min$

$$\mathbf{K}_{\text{opt}} = \underset{\mathbf{k}\in\mathcal{K}}{\arg\min}\, J(\mathbf{k},\mathbf{U}) \tag{6}$$

It is worth noting that this distribution can be rewritten closer to a reliability formulation:

$$p_{K_{\text{opt}}}(\mathbf{k}) \propto \mathbb{P}_U\left[J(\mathbf{k},\mathbf{U}) \leq \min_{\tilde{\mathbf{k}}\in\mathcal{K}} J(\tilde{\mathbf{k}},\mathbf{U})\right] \tag{7}$$

4 Methods and applications

4.1 Optimisation of the mean response

Janusevskis and Le Riche (2010), Miranda et al. (2016)

5 Conclusions

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Code availability. TEXT

Data availability. TEXT

Code and data availability. TEXT

Sample availability. TEXT

Appendix A

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Author contributions. TEXT

Competing interests. TEXT

Disclaimer. TEXT

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