Analyze of Schwarz algorithms for idealized ocean-atmosphere coupling

Tutor : Éric Blayo et Florian Lemarié

Sophie THERY

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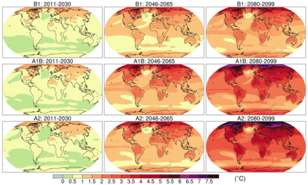
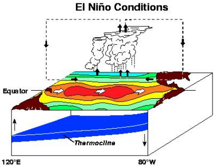
General framework: ocean-atmosphere coupling

Physical phenomena governed by the ocean atmosphere interactions:

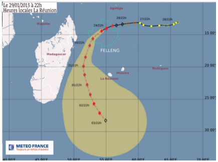
- climate
- El Niño
- tropical cyclones
- ...



climate modeling



seasonal forecasts

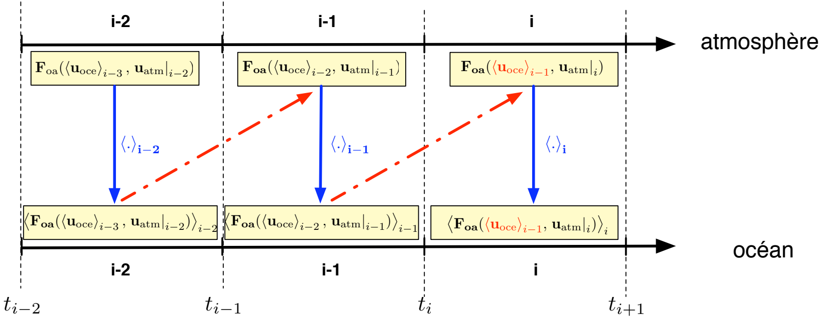


short term predictions

Motivations

The current methods of oceans-atmospheres coupling are unsatisfactory :

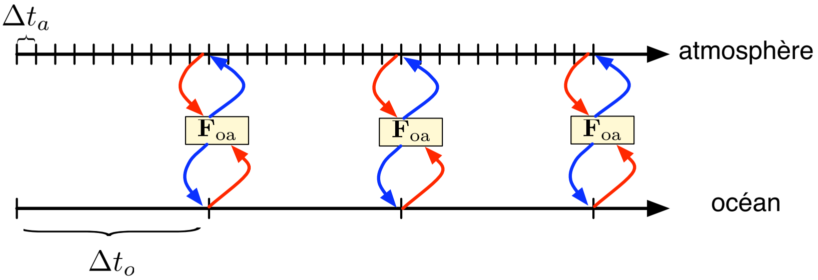
- Asynchrone coupling
  - Balance of the flows average on every window of time
  - problem of synchronisation



Motivations

The current methods of oceans-atmospheres coupling are unsatisfactory :

- Asynchrone coupling
- Synchrone coupling
  - lot of communication ⇒ inefficient implementation
  - problems of physical validity



Proposed solutions: Schwarz algorithms

Proof of concept : Simulation of the tropical cyclone Erica (2003) :

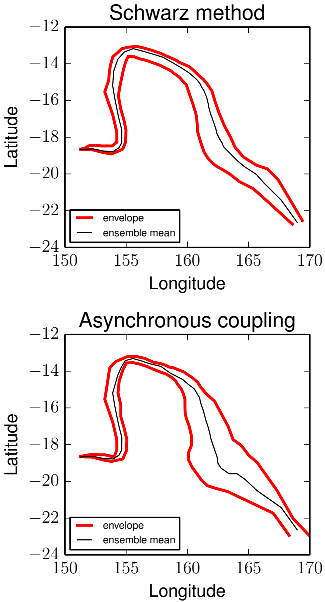
- ROMS : Oceanic model

  (Shchepetkin-McWilliams, 2005)

- WRF : Atmospheric model

  (Skamarock-Klemp, 2007)

Impact



$\Rightarrow$ Reduction of the uncertainty on the trajectory and the intensity of the cyclone.

Lemarié et al. 2014

Aim of the internship: Study of the convergence of the Schwarz algorithms on simplified ocean-atmosphere coupling

Ocean atmosphere coupling model

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- Navier-Stokes equations

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- Dominant physics : vertical axis (1d)

Ocean atmosphere coupling model

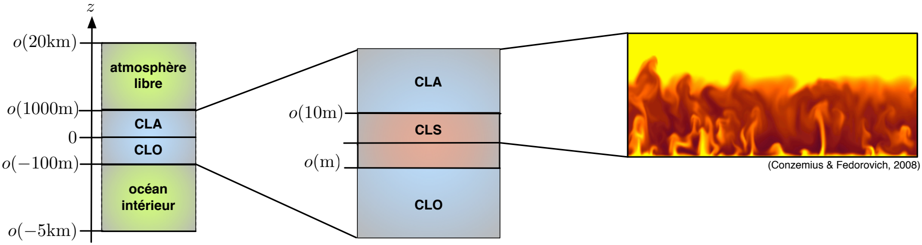
- Navier-Stokes equations
- Dominant physics : vertical axis (1d)
- Dominant terms

$$\begin{cases} \partial_t u - fv + \partial_z(wu) = \nu \partial^2_{zz} u \\ \partial_t v + fu + \partial_z(wv) = \nu \partial^2_{zz} v \end{cases}$$

  - (u, v, w) : speed
  - z : altitude
  - ν : mollecular diffusion
  - f : Coriolis frequency

Simplified ocean atmosphere coupling model

Small-scale turbulence parametrisation



(Conzemius & Fedorovich, 2008)

Simplified ocean atmosphere coupling model

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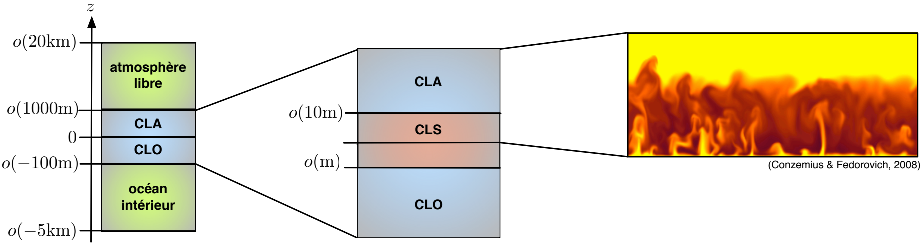


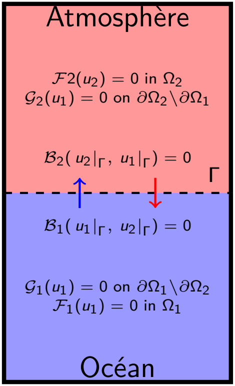
(Conzemius & Fedorovich, 2008)

⇒ *Reynolds average*

$$\begin{cases} \partial_t u - fv - \partial_z(D(z)\partial_z u) = F^u \\ \partial_t v + fu - \partial_z(D(z)\partial_z v) = F^v \end{cases}$$

with $D(z) = az + b > 0$,

Our model of study



Sur $\Omega_2 \times [0, T]$:

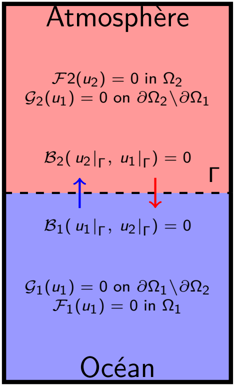
$$\begin{cases} \partial_t u_2 - f v_2 - \partial_z(D_2(z)\partial_z u_2) = F_2^u \\ \partial_t v_2 + f u_2 - \partial_z(D_2(z)\partial_z v_2) = F_2^v \end{cases}$$

Sur $\Omega_1 \times [0, T]$:

$$\begin{cases} \partial_t u_1 - f v_1 - \partial_z(D_1(z)\partial_z u_1) = F_1^u \\ \partial_t v_1 + f u_1 - \partial_z(D_1(z)\partial_z v_1) = F_1^v \end{cases}$$

$+$ initial conditions
$+$ outside conditions
$+$ interfaces conditions

Our model of study



Sur $\Omega_2 \times [0, T]$:

$$\begin{cases} \partial_t u_2 - fv_2 - \partial_z(D_2(z)\partial_z u_2) = F_2^u \\ \partial_t v_2 + fu_2 - \partial_z(D_2(z)\partial_z v_2) = F_2^v \end{cases}$$

Sur $\Omega_1 \times [0, T]$:

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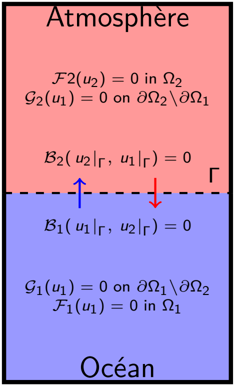
+ initial conditions
+ outside conditions
+ interfaces conditions

$\Rightarrow$ complexe values $U = u + iv$

Ocean-atmosphere coupling model



$$
\left\{
\begin{array}{l}
\text{Sur } \Omega_1 \times [0, T] : \\
\partial_t U_1 + ifU_1 - \partial_z(D_1(z)\partial_z U_1) = \tilde{F}_1 \\
\text{Sur } \Omega_2 \times [0, T] : \\
\partial_t U_2 + ifU_2 - \partial_z(D_2(z)\partial_z U_2) = \tilde{F}_2 \\
+ \text{ Conditions initiales} \\
+ \text{ Conditions au limites extérieurs} \\
+ \text{ Conditions d'interfaces}
\end{array}
\right.
$$

Théorie de couche limite :

$$D_j(z) = a_j z + b_j > 0$$

Schwarz algorithms

Algorithm 1 Schwarz algorithms

Require: u_2^0 sur Γ

$n = 0$

while non convergence ou $n < n_{max}$ **do**

solve

$$\begin{cases} \mathcal{L}_1 u_1^n = f_1 & \text{sur } \Omega_1, \\ \mathcal{G}_1 u_1^n = g_1 & \text{sur } \partial\Omega_1^{ext}, \\ \mathcal{B}_1 u_1^n = \mathcal{B}_1 u_2^{n-1} & \text{sur } \Gamma. \end{cases}$$

then solve

$$\begin{cases} \mathcal{L}_2 u_2^n = f_2 & \text{sur } \Omega_2, \\ \mathcal{G}_2 u_2^n = g_2 & \text{sur } \partial\Omega_2^{ext}, \\ \mathcal{B}_2 u_2^n = \mathcal{B}_2 u_1^n & \text{sur } \Gamma. \end{cases}$$

end while

State of the art

		Coefficients de diffusion **constants**	Coefficients de diffusion **affines**
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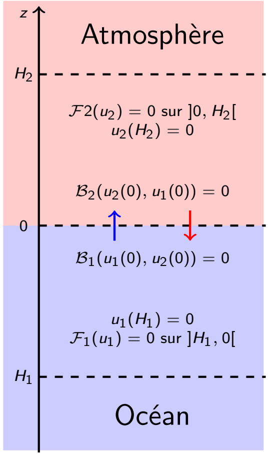
Aim of the internship: Study of the convergence of the Schwarz algorithms on simplified ocean-atmosphere coupling

We focus on two questions

- How the coriolis effect impact the convergence of the algorithm ?
- How "freeze" the diffusion coefficient by a constant impact the convergence ?

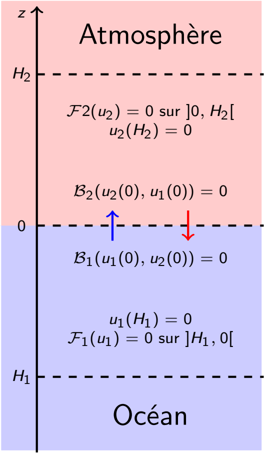
System verified by the errors



- On each area:

$$\begin{cases} -\partial_z(D_j(z)\partial_z u_j^n(z)) = F_j^u & \text{on } \Omega_j \\ u_j^n(H_j) = G_j & \\ \mathcal{B}_j u_j^n(0) = \mathcal{B}_j u_k^{n-1}(0) & \end{cases}$$

System verified by the errors



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- System verified by the errors :
  $e_j^n(z) = u_j^n(z) - u_j^*(z)$

$$\begin{cases} -\partial_z(D_j(z)\partial_z e_j^n(z)) = & 0 \quad \text{sur } \Omega_j \\ e_j^n1(H_j) = & 0 \\ \mathcal{B}_j e_j^n(0) = & \mathcal{B}_j e_k^{n-1}(0) \end{cases}$$

Convergence factor

Definition of the convergence factor:

$$\rho = \frac{||e_j^n||}{||e_j^{n-1}||}$$

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Study of the convergence :

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Method for the stationary case :

- Solve probleme without interface conditions $\Rightarrow e_j^n(z) = C_j^n \, \tilde{e}_j(z)$

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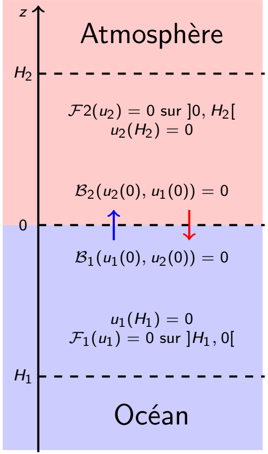
Method for the stationary case :

- Solve probleme without interface conditions $\Rightarrow e_j^n(z) = C_j^n \; \tilde{e}_j(z)$
- Convergence factor: $\rho = \left| \frac{C_j^n}{C_j^{n-1}} \right|$

  Where C_j^n is determined by the interface conditions.

Dirichlet-Neumann conditions



Dirichlet-Neumann interface conditions : exemple without Coriolis effect

$$\begin{cases} -\partial_z(D_j(z)\partial_z e_1^n(z)) &= 0 \\ & \text{sur }]H_1, 0[\\ e_1^n(H_1) &= 0 \\ {\color{red} e_1^n(0)} &= {\color{red} e_2^{n-1}(0)} \end{cases}$$

$$\begin{cases} -\partial_z(D_j(z)\partial_z e_2^n(z)) &= 0 \\ & \text{sur }]0, H_2[\\ e_2^n(H_2) &= 0 \\ {\color{purple} D_2(0)\partial_z e_2^n(0)} &= {\color{purple} D_1(0)\partial_z e_1^n(0)} \end{cases}$$

Convergence factor in stationary case without Coriolis effect, with Dirichlet-Neumann conditions:

$$\rho_{0,DN} = \frac{\int_{\Omega_2} (D_2(z))^{-1} dz}{\int_{\Omega_1} (D_1(z))^{-1} dz}$$

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- depends on D_j on all the area.
- $D_j(z)$ is not necessary constant.
- particular case :

$D_j(z)$ =constant

$$\rho_{0,DN}^{cst} = \frac{H_2 D_1}{H_1 D_2}$$

$D_j(z) = a_j z + b_j$:

$$\rho_{0,DN}^{var} = \frac{a_1}{a_2} \frac{\ln(1 + H_2 \frac{a_2}{b_2})}{\ln(1 + H_1 \frac{a_1}{b_1})}$$

Study of the convergence with Coriolis effect

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With Coriolis effect

System to solve :

$$\left\{ \begin{array}{rll} ife_1^n(z) - \partial_z(D_1(z)\partial_z e_1^n(z)) & = 0 & \text{sur }]H_1, 0[\\ e_1^n(H_1) & = 0 & \\ e_1^n(0) & = e_2^{n-1}(0) & \end{array} \right.$$

$$\left\{ \begin{array}{rll} ife_2^n(z) - \partial_z(D_2(z)\partial_z e_2^n(z)) & = 0 & \text{sur }]0, H_2[\\ e_2^n(H_2) & = 0 & \\ D_2(0)\partial_z e_2^n(0) & = D_1(0)\partial_z e_1^n(0) & \end{array} \right.$$

- with D_j =constant $\Rightarrow$ well known solutions
- with $D_j = a_j z + b_j > 0$

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- with D_j =constant $\Rightarrow$ well known solutions
- with $D_j = a_j z + b_j > 0$

Bessel's equation

$$z\partial_{zz}^2 u + (2\alpha - 2\beta\nu + 1)z\partial_z u + (\beta^2\gamma^2 z^{2\beta} + \alpha(\alpha - 2\beta\nu))u = 0$$

Convergence factor in stationary case without Coriolis effect, with Dirichlet-Neumann conditions:

- D_j =constant :

$$\rho_{DN}^{cst} = \sqrt{\frac{D_1}{D_2}} \left| \frac{\tanh(H_2\lambda_2)}{\tanh(H_1\lambda_1)} \right|$$

with $\lambda_j = \sqrt{i\frac{f}{D_j}}$ and $f \neq 0$.

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- $D_j = a_j z + b_j$:

$$\rho_{DN}^{var} = \sqrt{\frac{D_1(0)}{D_2(0)}} \left| \frac{I_0(\mu_2(H_2))K_0(\mu_2(0)) - K_0(\mu_2(H_2))I_0(\mu_2(0))}{I_0(\mu_1(H_1))K_0(\mu_1(0)) - K_0(\mu_1(H_1))I_0(\mu_1(0))} \right| \times \left| \frac{I_0(\mu_1(H_1))K_1(\mu_1(0)) + K_0(\mu_1(H_1))I_1(\mu_1(0))}{I_0(\mu_2(H_2))K_1(\mu_2(0)) + K_0(\mu_2(H_2))I_1(\mu_2(0))} \right|$$

with $\mu_j(z) = 2\sqrt{\frac{if}{a_j}\left(z + \frac{b_j}{a_j}\right)}$

Summary of the stationary case

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Plan

Study of the convergence in the instationary case

$$\left\{\begin{array}{rll}\partial_t e_j^n(t,z) + ife_j^n(t,z) - \partial_z(D(z)\partial_z e_j^n(t,z)) & = 0 & \text{sur }]0,T[\times\Omega_j \\ e_j^n(t,H_j) & = 0 & \text{sur }]0,T[\\ \mathcal{B}e_j^n(t,0) & = \mathcal{B}e_k^n(0) & \text{sur }]0,T[\\ e_j^n(0,z) & = 0 & \text{sur } \Omega_j\end{array}\right.$$

Study of the convergence in the instationary case

$$\begin{cases} \partial_t e_j^n(t,z) + ife_j^n(t,z) - \partial_z(D(z)\partial_z e_j^n(t,z)) & = 0 & \text{sur }]0,T[\times\Omega_j \\ e_j^n(t,H_j) & = 0 & \text{sur }]0,T[\\ \mathcal{B}e_j^n(t,0) & = \mathcal{B}e_k^n(0) & \text{sur }]0,T[\\ e_j^n(0,z) & = 0 & \text{sur } \Omega_j \end{cases}$$

Fourier transform on the time

$\hat{u}(\omega,z) = \displaystyle\int_{-\infty}^{\infty} u(t,z)e^{-i\omega t}dt$, with $\omega \in \mathbb{R}$

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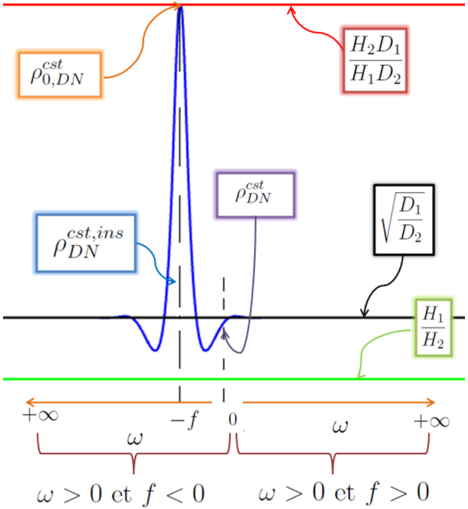
$\hat{u}(\omega,z) = \int_{-\infty}^{\infty} u(t,z)e^{-i\omega t}dt$, with $\omega \in \mathbb{R}$

$$\left\{\begin{array}{rll} i(f+\omega)\hat{e}_j^n(\omega,z) - \partial_z(D(z)\partial_z \hat{e}_j^n(\omega,z)) & = 0 & \text{sur } \Omega_j \\ \hat{e}_j^n(\omega,H_j) & = 0 & \\ \mathcal{B}\hat{e}_j^n(\omega,0) & = \mathcal{B}_j\hat{e}_k^n(\omega,0) & \end{array}\right.$$

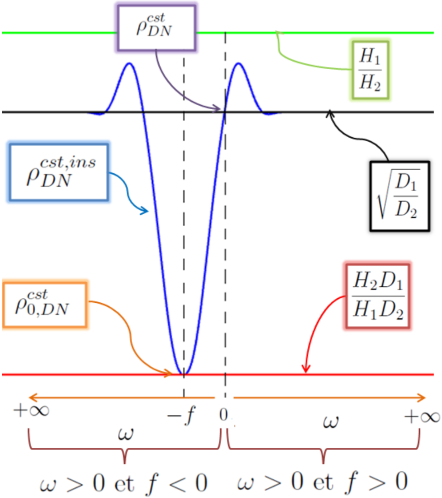
$\Rightarrow$ Stationary case equations with $f \to f+\omega$

Convergence factor in instationary case with Coriolis effect, with Dirichlet-Neumann conditions :



If
$\rho_{0,DN}^{var} \geq \sqrt{\frac{b_1}{b_2}}$

Facteur de convergence pour le cas instationnaire, des coefficients affines et conditions de Dirichlet-Neumann



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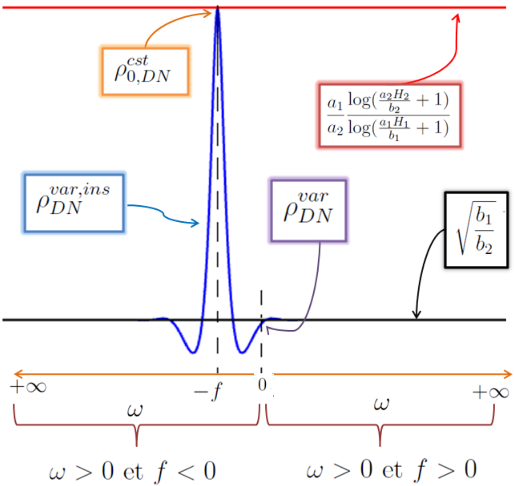

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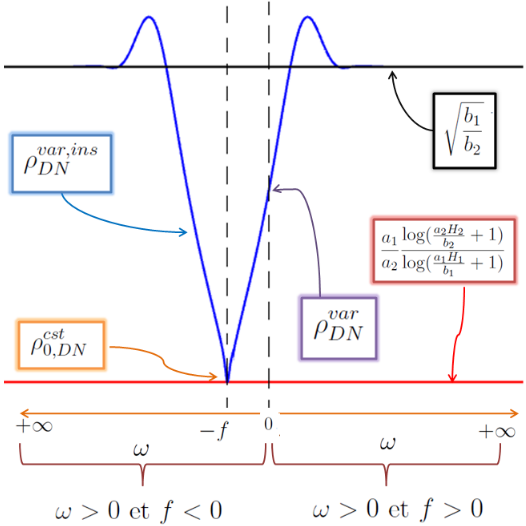

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Instatio-nnaire	Sans effet de Coriolis	Martin, V. *Schwarz Waveform Relaxation Methods for oceanographic equations* (2003) Bennequin D, Gander M.J, Halpern L : *Optimized Schwarz Waveform Relaxation Methods for Convection Reaction Diffusion Problems* (2004)	Lem...rié, F :*Toward an Optimized ...lobal-in-Time Schwarz Algorithm for Diffusion Equations with Discontinuous and Spatially Variable Coefficient.* (2013)
	Avec effet de **Coriolis**	Gander M.L *Optimized schwarz methods for model problems with continuously variablecoefficients* (2016)	

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- How can we optimize the convergence ?

Numerical results

- No disymetries between $f < 0$ et $f > 0$.
- When $\omega << |f|$ or $\omega >> |f|$ theoretical results and numerical results are consistant.
- When $\omega \approx |f|$: dispersion of the numerical results.

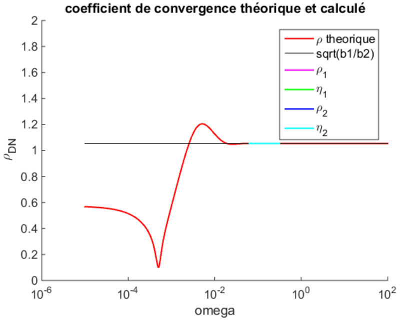
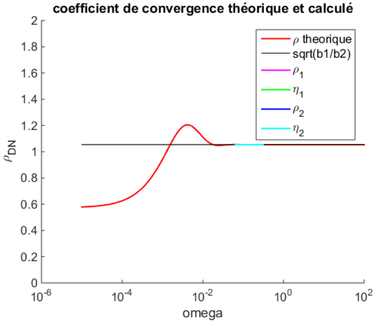


Figure: Theoretical and numerical convergence factor, $f < 0$



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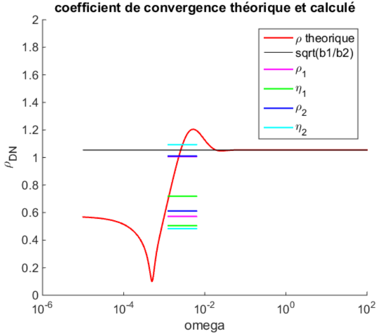

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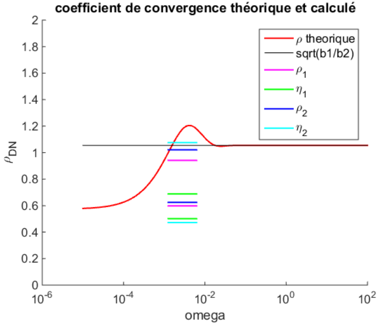

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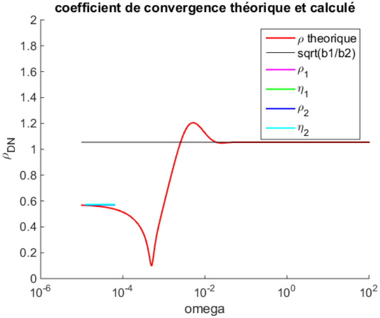

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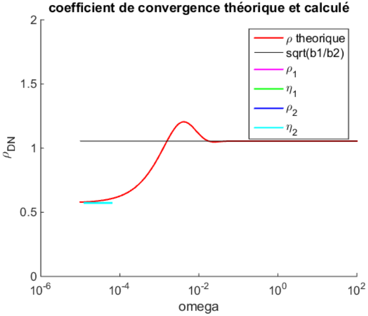

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- Easy to determinate in the stationary case.
- In the instationary case, a numerical method can calculate "good" Robin coefficients.
- Could we be help by the optimal coefficients from the stationary case to find better coefficient in the instationary case ?

During the Phd

- More realistic models.
  $\Rightarrow$ more complicated interface conditions.
- Link with Charles's thesis.
- Realistic applications in collaboration with climatologists

Thanks