Parameter control in the presence of uncertainties

Robust Estimation of Bottom friction

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Processus of modelling of physical systems

Uncertainties and errors are introduced at each stage of the modelling, by simplifications, parametrizations...

In the end, we have a set of parameters we want to calibrate, but how can we be sure that this calibration is acting upon the errors of the modelling, and does not compensate the effect of the natural variability of the physical system?

Outline

Introduction

Deterministic problem

Dealing with uncertainties

Robust minimization

Conclusion

Deterministic problem

Computer code and inverse problem

Input
- $\boldsymbol{k}$: Control parameter
- $\boldsymbol{u}$: Environmental variables (fixed and known)

Output
- $\mathcal{M}(\boldsymbol{k}, \boldsymbol{u})$: Quantity to be compared to observations

Environmental variables

$\boldsymbol{u} \in \mathcal{U}$ fixed

Control variable

$\boldsymbol{k} \in \mathcal{K}$

$$\boldsymbol{k} \in \mathcal{K} \longrightarrow \boxed{\text{Direct Simulation}} \longrightarrow \mathcal{M}(\boldsymbol{k}, \boldsymbol{u})$$

Computer code and inverse problem

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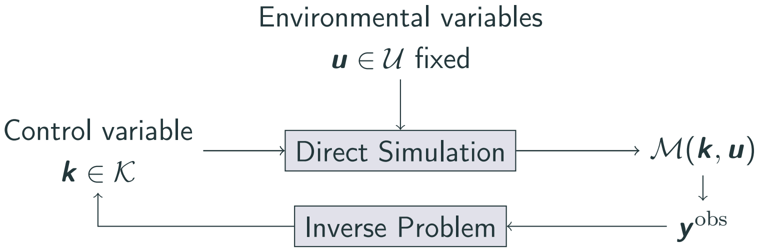
- $\boldsymbol{u}$: Environmental variables (fixed and known)

Output
- $\mathcal{M}(\boldsymbol{k}, \boldsymbol{u})$: Quantity to be compared to observations



Data assimilation framework

We have $\boldsymbol{y}^{\mathrm{obs}} = \mathcal{M}(\boldsymbol{k}_{\mathrm{ref}}, \boldsymbol{u}_{\mathrm{ref}})$ with $\boldsymbol{u}_{\mathrm{ref}} = \boldsymbol{u}$

$$\hat{\boldsymbol{k}} = \underset{\boldsymbol{k}\in\mathcal{K}}{\arg\min}\, J(\boldsymbol{k}) = \underset{\boldsymbol{k}\in\mathcal{K}}{\arg\min}\, \frac{1}{2}\|\mathcal{M}(\boldsymbol{k}, \boldsymbol{u}) - \boldsymbol{y}^{\mathrm{obs}}\|^2$$

→ Deterministic optimization problem

→ Possibly add regularization

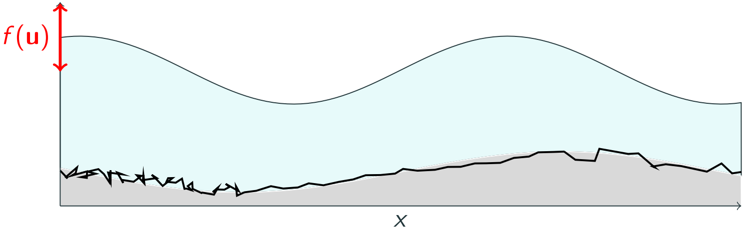
→ Classical methods: Adjoint gradient and Gradient-descent

BUT

- What if $\boldsymbol{u} \neq \boldsymbol{u}_{\mathrm{ref}}$?
- Does $\hat{\boldsymbol{k}}$ compensates the errors brought by this misspecification?

Context

- The friction $\boldsymbol{k}$ of the ocean bed has an influence on the water circulation
- Depends on the type and/or characteristic length of the asperities
- Subgrid phenomenon
- $\boldsymbol{u}$ parametrizes the BC



Dealing with uncertainties

Different types of uncertainties

Epistemic or aleatoric uncertainties? [WHR+03]

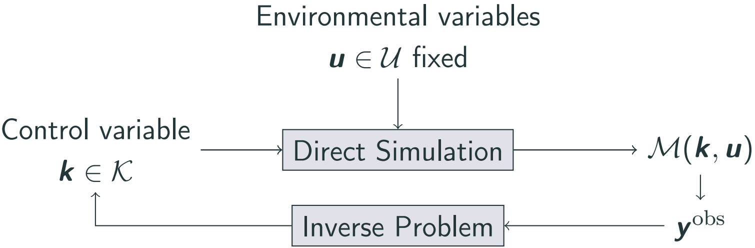
- Epistemic uncertainties: From a lack of knowledge, that can be reduced with more research/exploration
- Aleatoric uncertainties: From the inherent variability of the system studied, operating conditions

$\rightarrow$ But where to draw the line?

Our goal is to take into account the aleatoric uncertainties in the estimation of our parameter.

Aleatoric uncertainties

Instead of considering $\boldsymbol{u}$ fixed, we consider that $\boldsymbol{U}$ is a random variable (pdf $\pi(\boldsymbol{u})$), and the output of the model depends on its realization.



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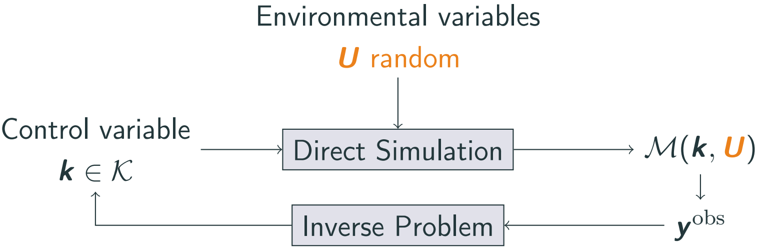


The cost function as a random variable

- Output of the computer code ($\boldsymbol{u}$ is an input):

$$\mathcal{M}(\boldsymbol{k}, \boldsymbol{u})$$

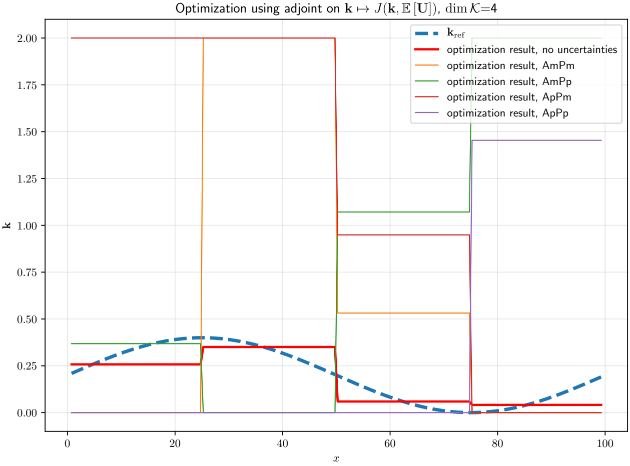
- The (deterministic) quadratic error is now

$$J(\boldsymbol{k}, \boldsymbol{u}) = \frac{1}{2}\|\mathcal{M}(\boldsymbol{k}, \boldsymbol{u}) - \boldsymbol{y}^{\mathrm{obs}}\|^2$$

"$\hat{\boldsymbol{k}} = \arg\min_{\boldsymbol{k}\in\mathcal{K}} J(\boldsymbol{k}, \boldsymbol{u})$" but what can we do about $\boldsymbol{u}$?

Toy Problem: Influence of misspecification of $\boldsymbol{u}_{\mathrm{ref}}$

Minimization performed on $\boldsymbol{k} \mapsto J(\boldsymbol{k}, \mathbb{E}[\boldsymbol{U}])$, for different $\boldsymbol{u}_{\mathrm{ref}}$:
Naïve approach



Robust Estimation of parameters

- Main objectives:
    I. Define criteria of robustness, based on $J(\boldsymbol{k}, \boldsymbol{u})$, that will depend on the final application
    II. For each criterion, be able to compute an estimate $\hat{\boldsymbol{k}}$ in a reasonable time
- Questions to be answered along the way:
    - Suitable prior distribution on $\boldsymbol{K} \sim \pi(\boldsymbol{k})$
    - Good exploration of $\mathcal{U}$, based on the density of $\boldsymbol{U}$ (Design of Experiment: LHS, Monte-Carlo, OA,...?)

Robust minimization

Criteria of robustness

Non-exhaustive list of “Robust” Objectives

- Worst case:

$$\min_{\boldsymbol{k}\in\mathcal{K}} \left\{ \max_{\boldsymbol{u}\in\mathcal{U}} J(\boldsymbol{k}, \boldsymbol{u}) \right\}$$

- M-robustness [LSN04]:

$$\min_{\boldsymbol{k}\in\mathcal{K}} \mathbb{E}_U \left[J(\boldsymbol{k}, \boldsymbol{U}) \right]$$

- V-robustness [LSN04]:

$$\min_{\boldsymbol{k}\in\mathcal{K}} \mathbb{V}\mathrm{ar}_U \left[J(\boldsymbol{k}, \boldsymbol{U}) \right]$$

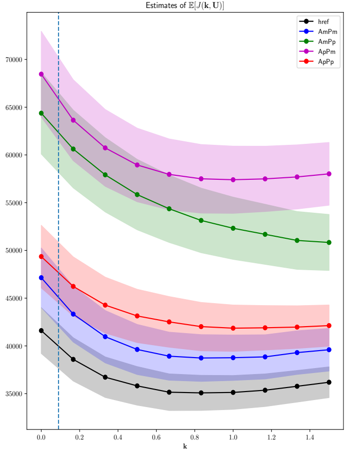
- Multiobjective [Bau12]:

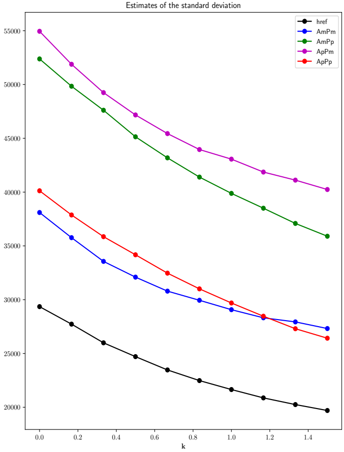
Pareto frontier

- Region of failure given by $J(\boldsymbol{k}, \boldsymbol{u}) > T$ [BGL+12]:

$$\max_{\boldsymbol{k}\in\mathcal{K}} R(\boldsymbol{k}) = \max_{\boldsymbol{k}\in\mathcal{K}} \mathbb{P}_U \left[J(\boldsymbol{k}, \boldsymbol{U}) \leq T \right]$$

Toy Problem: Minimization of mean value





$\longrightarrow$ Quite different from the value $\boldsymbol{k} \approx 0.1$ obtained by optimization knowing the true value of $\boldsymbol{u}_{\mathrm{ref}}$

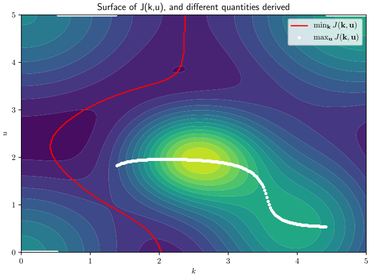
The minimizer as a random variable:

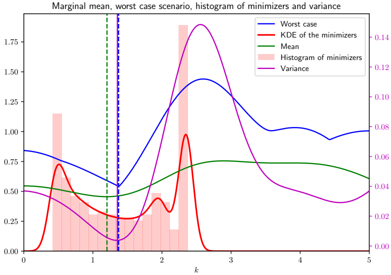
$$\boldsymbol{K}_{\arg\min} = \arg\min_{\boldsymbol{k}\in\mathcal{K}} J(\boldsymbol{k}, \boldsymbol{U})$$

$\longrightarrow$ estimate its density (how often is the value $\boldsymbol{k}$ a minimizer)

$$R(\boldsymbol{k}) = \mathbb{P}_U\left[\boldsymbol{k} = \arg\min_{\tilde{\boldsymbol{k}}} J(\tilde{\boldsymbol{k}}, \boldsymbol{U})\right]$$

$$= \mathbb{P}_U\left[J(\boldsymbol{k}, \boldsymbol{U}) \leq \min_{\tilde{\boldsymbol{k}}} J(\tilde{\boldsymbol{k}}, \boldsymbol{U})\right]$$





"Most Probable Estimate", and relaxation

The minimizer as a random variable:

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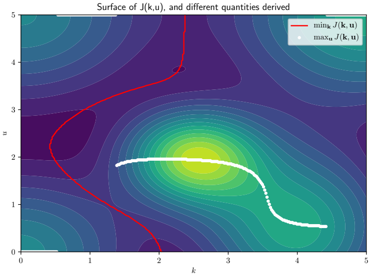
$\longrightarrow$ estimate its density (how often is the value $\boldsymbol{k}$ a minimizer)

$$R(\boldsymbol{k}) = \mathbb{P}_U\left[\boldsymbol{k} = \arg\min_{\tilde{\boldsymbol{k}}} J(\tilde{\boldsymbol{k}}, \boldsymbol{U})\right]$$

$$R_\alpha(\boldsymbol{k}) = \mathbb{P}_U\left[J(\boldsymbol{k}, \boldsymbol{U}) \leq \alpha \min_{\tilde{\boldsymbol{k}}} J(\tilde{\boldsymbol{k}}, \boldsymbol{U})\right]$$

$\longrightarrow$ Relaxation of the constraint with $\alpha \geq 1$



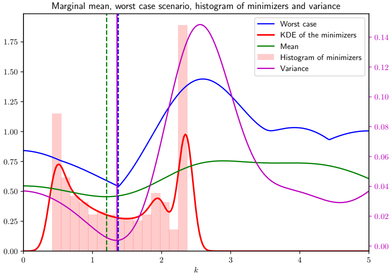
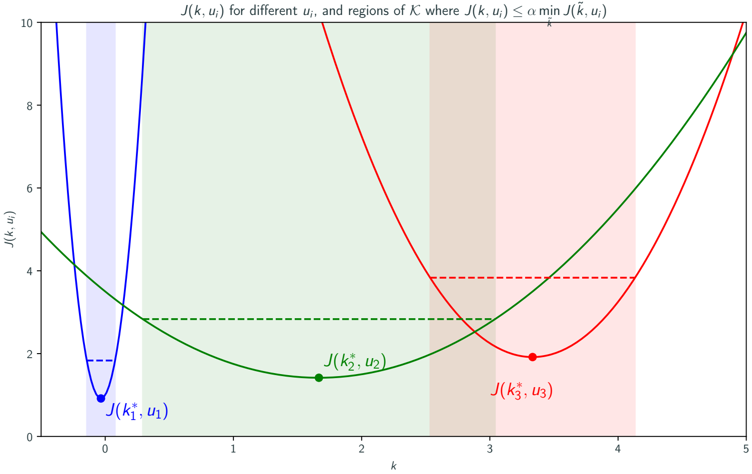
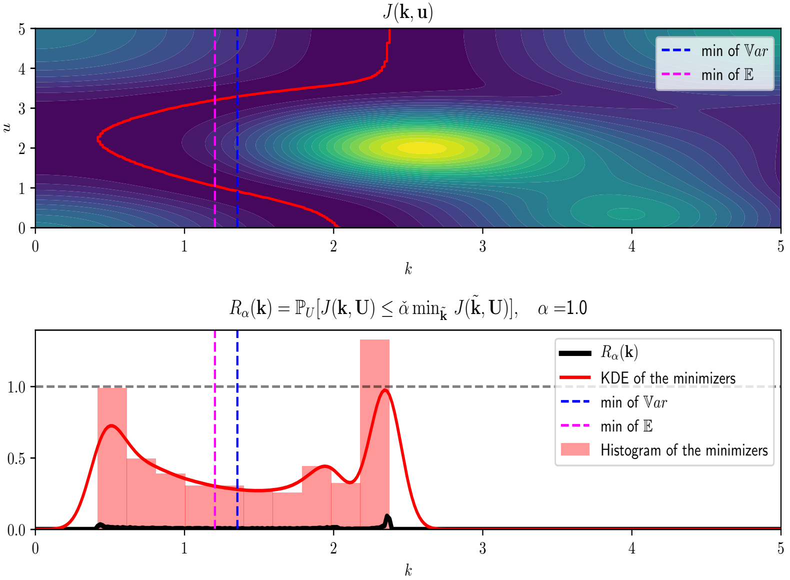


Illustration of the relaxation



MPE and relaxation



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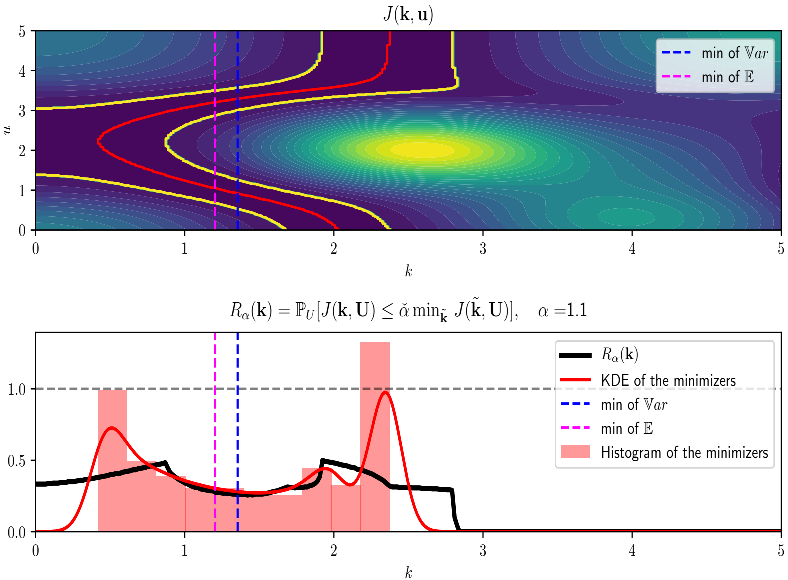

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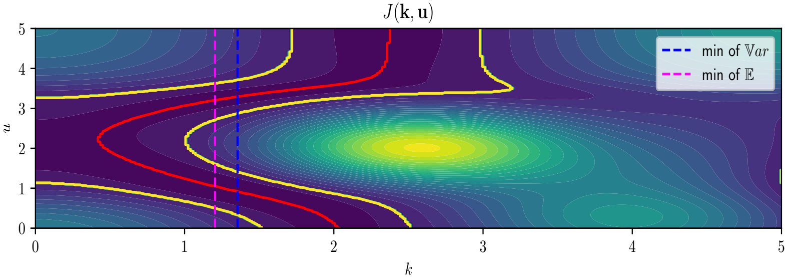

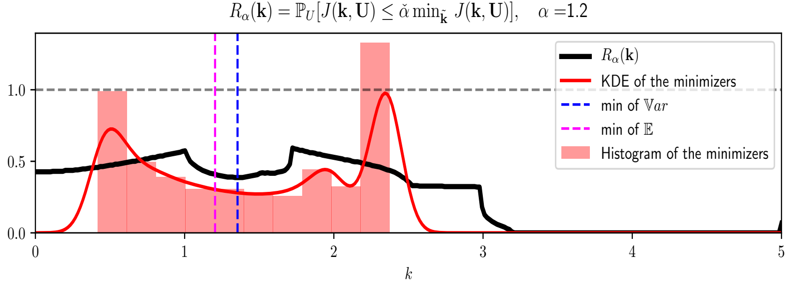

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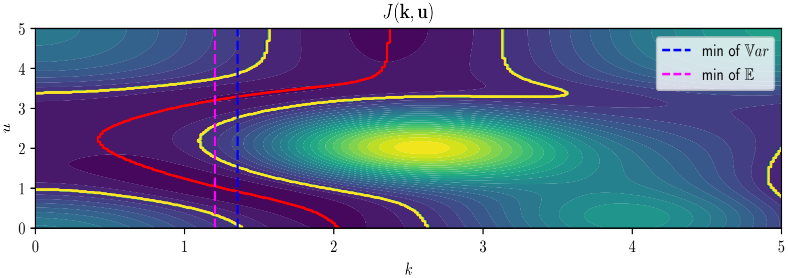

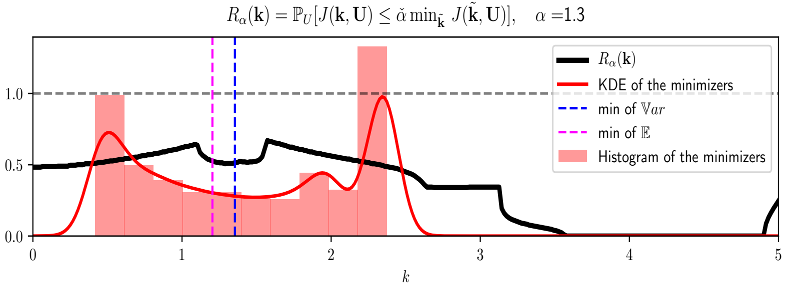

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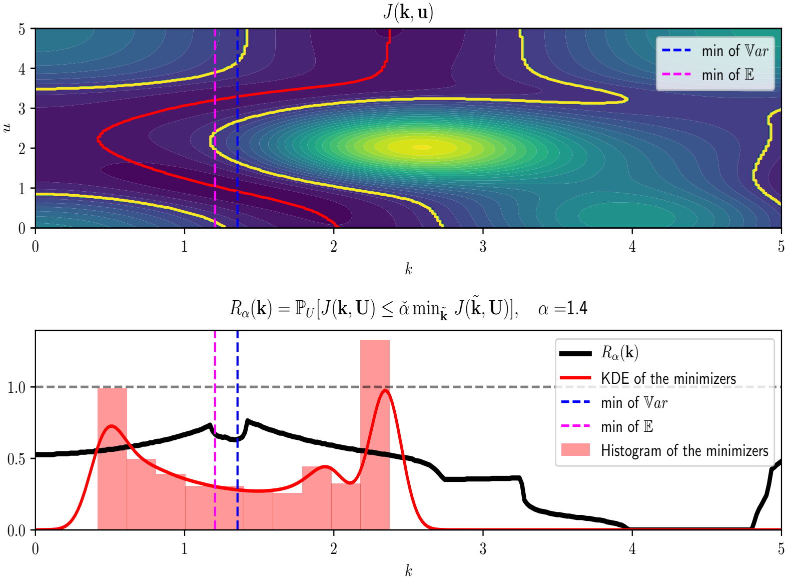

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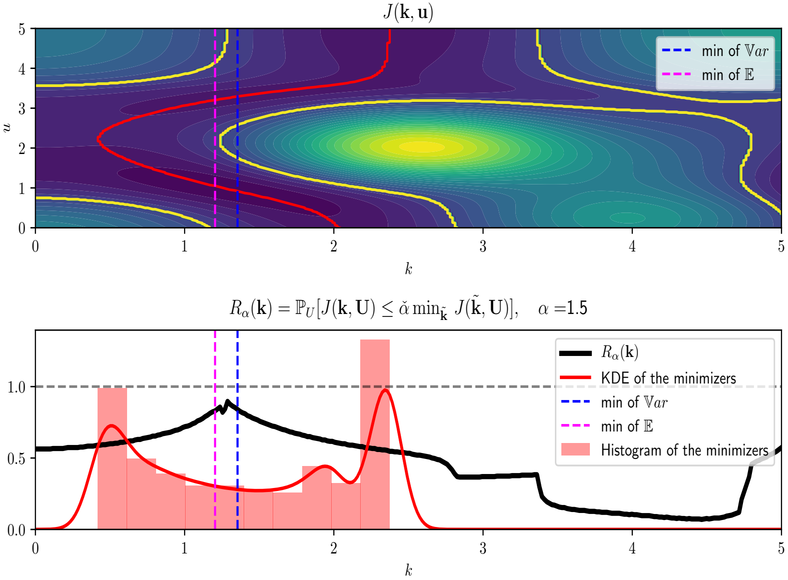

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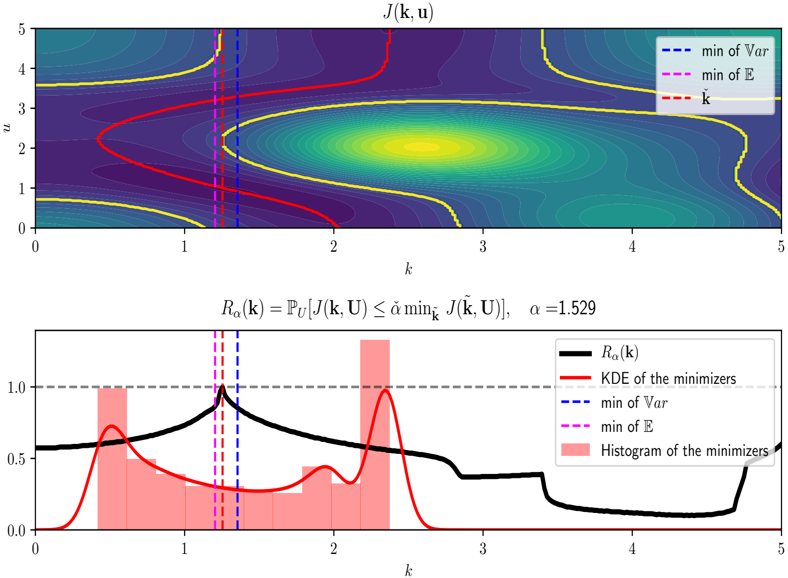
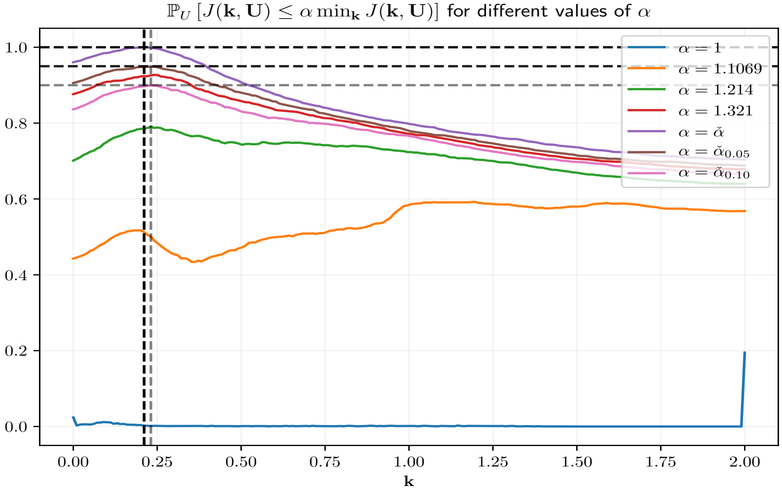


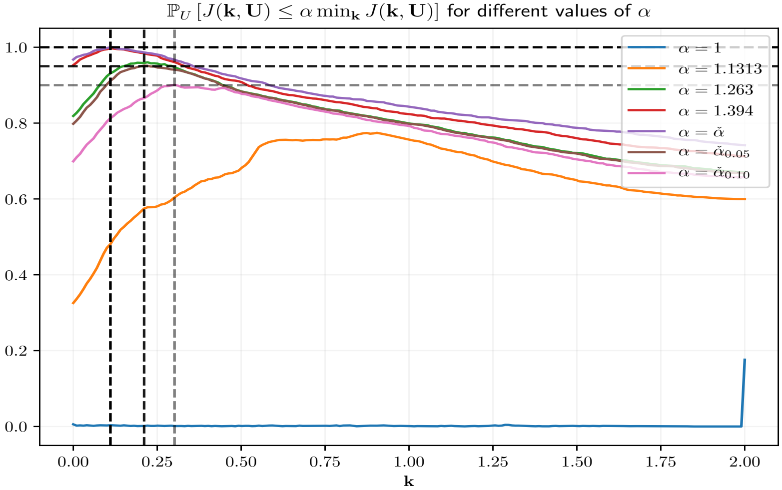
MPE and relaxation: SWE toy problem, $\dim \mathcal{K} = 1$, $\dim \mathcal{U} = 2$

Known $\boldsymbol{u}_{\mathrm{ref}} = \mathbb{E}[\boldsymbol{U}]$



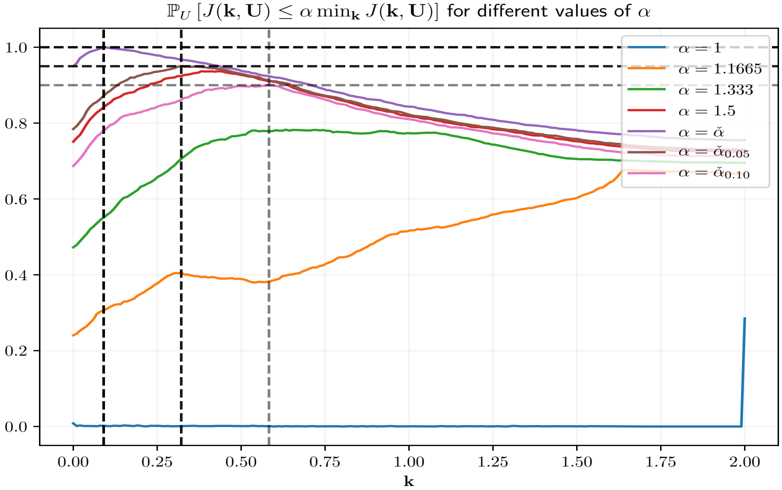
MPE and relaxation: SWE toy problem, $\dim \mathcal{K} = 1$, $\dim \mathcal{U} = 2$

$$\boldsymbol{u}_{\mathrm{ref}} = \mathbb{E}[\boldsymbol{U}] + (-0.2, -0.1)$$



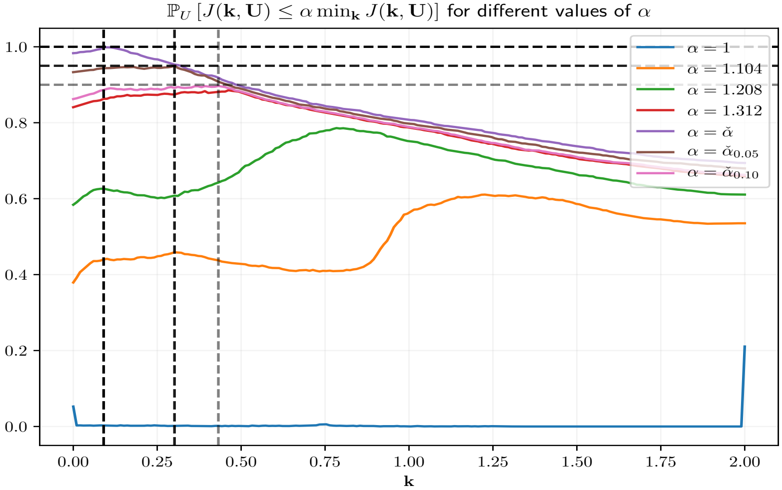
MPE and relaxation: SWE toy problem, $\dim \mathcal{K} = 1$, $\dim \mathcal{U} = 2$

$$\boldsymbol{u}_{\mathrm{ref}} = \mathbb{E}[\boldsymbol{U}] + (-0.1, +0.2)$$



MPE and relaxation: SWE toy problem, $\dim\mathcal{K}=1$, $\dim\mathcal{U}=2$

$$\boldsymbol{u}_{\mathrm{ref}} = \mathbb{E}[\boldsymbol{U}] + (+0.2, +0.1)$$



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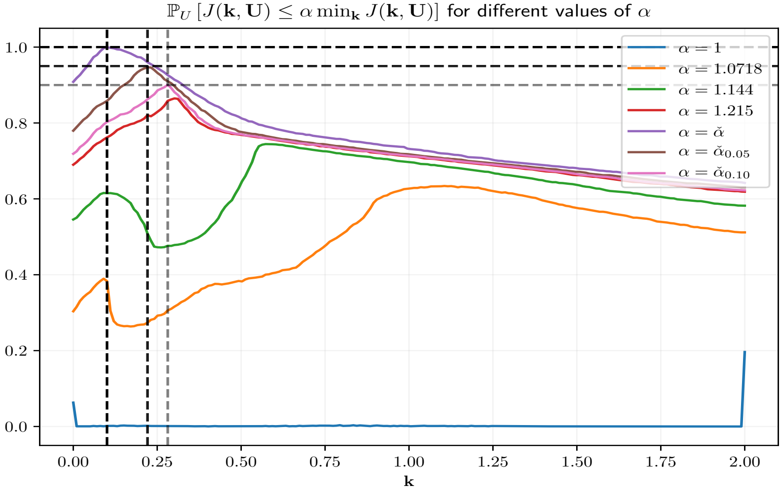

Conclusion

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Wrapping up

- Problem of a “good” definition of robustness
- Strategies rely heavily on surrogate models, to embed aleatoric uncertainties directly in the modelling

Perspective and future work

- Cost of computer evaluations $\rightarrow$ limited number of runs?
- Dimensionality of the input space $\rightarrow$ reduction of the input space?
- How to deal with uncontrollable errors $\rightarrow$ realism of the model?

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