Parameter estimation in the presence of uncertainties

Robust estimation of bottom friction

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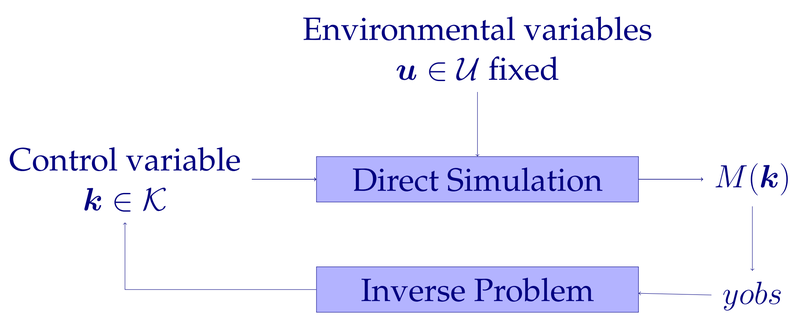
Abstract

Many physical phenomena are modelled numerically in order to better understand and/or to predict their behaviour. However, some complex and small scale phenomena can not be fully represented in the models. The introduction of ad-hoc correcting terms is usually the solution to represent these unresolved processes, but those need to be properly estimated. A good example of this type of problem is the estimation of bottom friction parameters of the ocean floor. This task is further complicated by the presence of uncertainties in certain other characteristics linking the bottom and the surface (eg boundary conditions). Classical methods of parameter estimation usually imply the minimisation of an objective function, that measures the error between some observations and the results obtained by a numerical model. The optimum is directly dependent on the fixed nominal value given to the uncertain parameter ; and therefore may not be relevant in other conditions. Strategies taking into account those uncertainties will be presented and applied on an academic model of a coastal area, in order to find an optimal value in a robust sense.

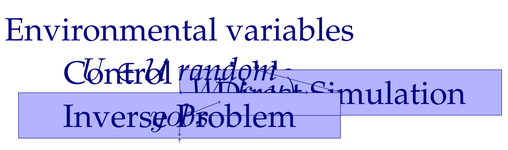
Introduction

- Numerical models of physical systems cannot represent fully the reality
  $\rightarrow$ Subgrid phenomena need to be parametrized
- How to choose the parameter value in a robust way ?

Classical deterministic inverse problem



Introducing randomness



Main Objectives

1. Define a relevant definition of robustness
2. Nullam at mi nisl. Vestibulum est purus, ultricies cursus volutpat sit amet, vestibulum eu.
3. Praesent tortor libero, vulputate quis elementum a, iaculis.
4. Phasellus a quam mauris, non varius mauris. Fusce tristique, enim tempor varius porta, elit purus commodo velit, pretium mattis ligula nisl nec ante.
5. Ut adipiscing accumsan sapien, sit amet pretium.
6. Estibulum est purus, ultricies cursus volutpat
7. Nullam at mi nisl. Vestibulum est purus, ultricies cursus volutpat sit amet, vestibulum eu.
8. Praesent tortor libero, vulputate quis elementum a, iaculis.

Case

- (Deterministic) Computer code: Maps bottom friction to the observation of the sea surface height

$$\begin{aligned} M : \mathcal{K} \times \mathcal{U} &\longrightarrow \mathcal{Y} \\ (\boldsymbol{k}, \boldsymbol{u}) &\longmapsto M(\boldsymbol{k}, \boldsymbol{u}) \end{aligned}$$

- Uncertainty quantification (UQ): Model the uncertainties with a random variable

$$\boldsymbol{U} \quad \text{has density} \quad \boldsymbol{u} \mapsto p_U(\boldsymbol{u})$$

- Compare observations with output of the model, using a precision matrix Σ^{-1}:

$$\begin{aligned} j : \mathcal{K} \times \mathcal{U} &\longrightarrow \mathcal{Y} \\ (\boldsymbol{k}, \boldsymbol{u}) &\longmapsto j(\boldsymbol{k}, \boldsymbol{u}) = \|M(\boldsymbol{k}, \boldsymbol{u}) - \boldsymbol{y}^{\text{obs}}\|^2_{\Sigma^{-1}} \end{aligned}$$

Two alternatives:

- Consider the random variable indexed by $\boldsymbol{k} \in \mathcal{K}$:

$$j(\boldsymbol{k}, \boldsymbol{U}) \quad \text{random variable with sample space } \mathbb{R}^+$$

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Bayesian framework

Results

Donec faucibus purus at tortor egestas eu fermentum dolor facilisis. Maecenas tempor dui eu neque fringilla rutrum. Mauris *lobortis* nisl accumsan. Aenean vitae risus ante. Phasellus imperdiet, tortor vitae congue bibendum, felis enim sagittis lorem, et volutpat ante orci sagittis mi. Morbi rutrum laoreet semper. Morbi accumsan enim nec tortor consectetur non commodo nisi sollicitudin. Proin sollicitudin. Pellentesque eget orci eros. Fusce ultricies, tellus et pellentesque fringilla, ante massa luctus libero, quis tristique purus urna nec nibh.

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 1: Table caption

Nulla ut porttitor enim. Suspendisse venenatis dui eget eros gravida tempor. Mauris feugiat elit et augue placerat ultrices. Morbi accumsan enim nec tortor consectetur non commodo. Pellentesque condimentum dui. Etiam sagittis purus non tellus tempor volutpat. Donec et dui non massa tristique adipiscing. Quisque vestibulum eros eu. Phasellus imperdiet, tortor vitae congue bibendum, felis enim sagittis lorem, et volutpat ante orci sagittis mi. Morbi rutrum laoreet semper. Morbi accumsan enim nec tortor consectetur non commodo nisi sollicitudin.

Figure 1: Figure caption

In hac habitasse platea dictumst. Etiam placerat, risus ac. Adipiscing lectus in magna blandit:

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 2: Table caption

Vivamus sed nibh ac metus tristique tristique a vitae ante. Sed lobortis mi ut arcu fringilla et adipiscing ligula rutrum. Aenean turpis velit, placerat eget tincidunt nec, ornare in nisl. In placerat.

Figure 2: Figure caption

Conclusions

- Pellentesque eget orci eros. Fusce ultricies, tellus et pellentesque fringilla, ante massa luctus libero, quis tristique purus urna nec nibh. Phasellus fermentum rutrum elementum. Nam quis justo lectus.
- Vestibulum sem ante, hendrerit a gravida ac, blandit quis magna.
- Donec sem metus, facilisis at condimentum eget, vehicula ut massa. Morbi consequat, diam sed convallis tincidunt, arcu nunc.
- Nunc at convallis urna. isus ante. Pellentesque condimentum dui. Etiam sagittis purus non tellus tempor volutpat. Donec et dui non massa tristique adipiscing.

Forthcoming Research

Vivamus molestie, risus tempor vehicula mattis, libero arcu volutpat purus, sed blandit sem nibh eget turpis. Maecenas rutrum dui blandit lorem vulputate gravida. Praesent venenatis mi vel lorem tempor at varius diam sagittis. Nam eu leo id turpis interdum luctus a sed augue. Nam tellus.

Acknowledgements

Etiam fermentum, arcu ut gravida fringilla, dolor arcu laoreet justo, ut imperdiet urna arcu a arcu. Donec nec ante a dui tempus consectetur. Cras nisi turpis, dapibus sit amet mattis sed, laoreet.