Parameter control in the presence of uncertainties

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The 1D Shallow Water Equations

1D-SWE

$$\partial_t h + \partial_x(hu) = 0 \qquad \text{(Conservation)}$$

$$\partial_t(hu) + \partial_x\left(u^2 h + \frac{1}{2}gh^2\right) = -gh\partial_x Z - S \qquad \text{(Momentum)}$$

Quadratic Friction

$$S = -K\frac{|u|uh^{-\eta}}{h^2}, \quad \eta = 7/3 \tag{1}$$

Minimization of an objective function

We have $Y = \mathcal{H}W(K_{ref})$

$$\min_{K \in \mathcal{K}} j(K) = \frac{1}{2}||\mathcal{H}W(K) - Y||^2 \tag{2}$$

"Classical" optimization methods $\rightarrow$ Adjoint-based gradient
But what about uncertainties due to the environment ?

Introducing uncertainties

$\boldsymbol{X}_e$ random vector whose realizations $\boldsymbol{x}_e$ lies in $\mathbb{X}$

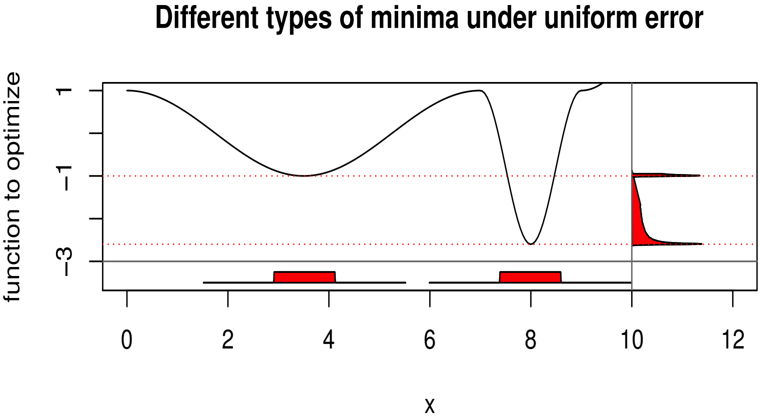
$$W(K) \quad \text{becomes} \quad W(\boldsymbol{x}_e, K) \tag{3}$$

and the (deterministic) quadratic error is now

$$j(\boldsymbol{x}_e, K) = \frac{1}{2}||\mathcal{H}W(\boldsymbol{x}_e, K) - Y||^2 \tag{4}$$

Influence of $\boldsymbol{X}_e$?
$\arg\min_K \mathbb{E}_{\boldsymbol{X}_e}[j(\boldsymbol{X}_e, K)]$?
$\arg\min_K \mathbb{V}\mathrm{ar}_{\boldsymbol{X}_e}[j(\boldsymbol{X}_e, K)]$?

Illustration of the robustness



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- Sensitivity analysis
  - Sobol' indices

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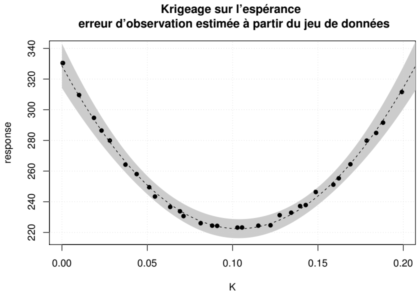

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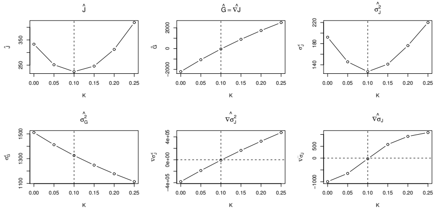

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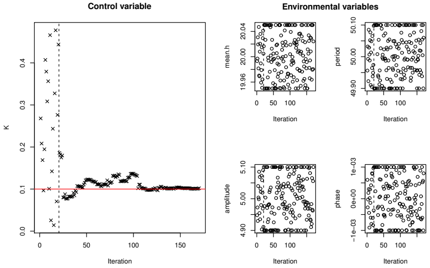

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  - Pareto front