Model Checking Real-Time Systems

Written Abstract for the Seminar “Recent Advances in Model Checking”

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Organizational information

This abstract is based on Chapter 29 of the Handbook of Model Checking [CHVB18]. Section 1 first introduces and motivates model checking applied to real-time systems, building on [CHVB18, Chapters 29.1 and 29.2]. Section 2 gives some formal definitions from [CHVB18, Chapters 29.2 and 29.7] about timed automata and related notions such as region equivalence and zones. Finally, Section 3 briefly introduces time-extended formalisms such as *Timed Temporal Logic* and *Timed Games* related to timed automata for the verification of real-time systems, building on [CHVB18, Chapters 29.6 and 29.9].

1 Introduction

Motivation In the early 1990s, Rajeev Alur and David Dill introduced a powerful and expressive formalism named *timed automata* which have proven very convenient for modeling and reasoning about real-time systems. It is nowadays a standard tool for the verification of real-time systems and has many applications in the industry, while also being a very active research area.

Outlook The most basic problem in model-checking is *reachability*, i.e. given a structure, one wants to determine whether a set of locations is reachable from an initial location. Since model-checking reasons on properties written in temporal logic, we need *timed temporal logic* that can express properties on timed automata. With such a formalism, one may eventually be interested in the controller synthesis problem, which can be expressed in the formalism of *timed games*. My presentation will therefore be articulated around these notions.

2 Timed Automata

Preliminaries In this chapter, time values are equated with non-negative real numbers of $\mathbb{R}_{\geq 0}$. A *time sequence* is a finite or infinite non-decreasing sequence of time values. A *timed word* over $\Sigma \times \mathbb{R}_{\geq 0}$ is a word over the alphabet Σ sequentially paired with a time sequence.

Let C be a finite set of variables called *clocks*. A *valuation* over C is a mapping $v\colon C \to \mathbb{R}_{\geq 0}$. The set of valuations over C is denoted $\mathbb{R}_{\geq 0}^C$ and $\mathbf{0}_C$ denoted the valuation assigning 0 to every clock of C.

For any valuation v and any time value t, the valuation $v + t$ denotes the valuation obtained by shifting all values of v by t. For any subset r of C, $v[r]$ is the valuation obtained by resetting all clocks of r in v.

A *constraint* φ over C is recursively defined by the following grammar:

$$\varphi ::= x \odot k \mid \varphi \wedge \varphi$$

where $x \in C$, $k \in \mathbb{Z}$ and $\odot \in \{<, \leq, =, \geq, >\}$. The set of constraints over C is denoted $\Phi(C)$. We say that a valuation v over C satisfies $x \odot k$ when $v(x) \odot k$, and when v satisfies a constraint φ, we write $v \models \varphi$. The set of valuations satisfying a constraint φ is denoted $[\![\varphi]\!]_C$.

Timed Automata A timed automaton is basically a finite automaton with (real-time) constraints on the states. The following formal definition is a reformulation of [CHVB18, Chapter 29.2, Definition 1].

Definition 1. A *Timed Automaton* (TA) $\mathcal{A}$ is the data $(L, l_0, C, \Sigma, I, E)$ where L is a finite set of *locations* with initial location $l_0 \in L$, C is a finite set of *clocks*, Σ is a finite set of *actions*, $I \colon L \to \Phi(C)$ is an *invariant mapping* and $E \subseteq L \times \Phi(C) \times \Sigma \times 2^C \times L$ is a set of edges.

Any edge $(\ell, \varphi, a, r, \ell') \in E$ is denoted $\ell \xrightarrow{\varphi,a,r} \ell'$ where φ is a *guard*, and r is a subset of clocks that are set to zero after taking the transition.

An example of TA is given in Fig. 1.

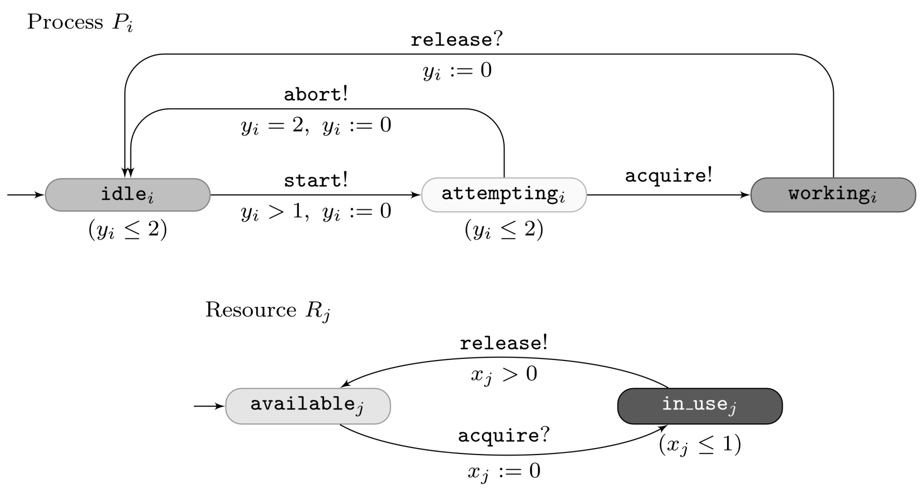


Figure 1: Two TA modeling processes which can use resources.
Figure taken from [CHVB18, Chapter 29.2]

Semantics The *operational semantics* of a TA $\mathcal{A} = (L, \ell_0, C, \Sigma, I, E)$ is the infinite-state timed transition system $[\![A]\!] = (S, s_0, \mathbb{R}_{\geq 0} \times \Sigma, T)$, where

$$S := \{(\ell, v) \in L \times \mathbb{R}_{\geq 0}, v \models I(\ell)\}, \quad s_0 := (\ell_0, \mathbf{0}_C),$$

$$T := \{(\ell, v) \xrightarrow{d,a} (\ell', (v+d)[r]) \mid d \in \mathbb{R}_{\geq 0}, \forall d' \in [0, d], v + d' \models I(\ell) \wedge \exists \ell \xrightarrow{\varphi,a,r} \ell' \in E, v + d \models \varphi\}$$

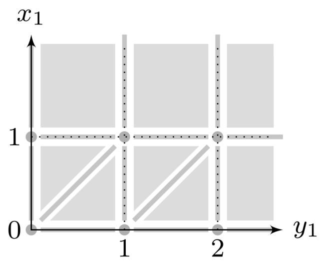
Parallel Composition It is possible to compose several TA in parallel. Informally, parallel composition roughly consists in pairing the automata and taking the conjunction of their invariants, distinguishing two types of transitions, namely the synchronous[1] and asynchronous transitions. The point is that systems can be defined in a compositional way.

Region Equivalence Informally, two valuations are region equivalent if a TA cannot differentiate between them and we write[2] this relation $\cong_M$. This notion is extended to states of the TA by defining $(\ell, v) \cong_M (\ell', v')$ iff $v \cong_M v'$ and $\ell = \ell'$. The equivalence class of (ℓ, v) is denoted $[\ell, v]_{\cong_M}$. As an

[1] Synchronous transitions are given by a *synchronization function*.

[2] $M := (M_x)_{x \in C}$ where M_x is the maximal constant clock x is compared to in the TA.

example, we give a representation of clock regions for the parallel composition of P_1 and R_1 in Fig. 2.



P_1	R_1	f_1
`start!`	–	`start`
`abort!`	–	`abort`
`acquire!`	`acquire?`	`acquire`
`release!`	`release?`	`release`

Figure 2: Clock regions for $(P_1||R_1)_{f_1}$ where f_1 is given by its table of values.
Figure taken from [CHVB18, Chapter 29.3]

From this notion, we can define the *region automaton* $\mathcal{R}_{\cong_M}(\mathcal{A}) = (S, s_0, \Sigma, T)$ which basically consists in quotienting $\mathcal{A}$ w.r.t. $\cong_M$:

$$S = (L \times \mathbb{R}_{\geq 0})_{/\cong_M} \quad s_0 = [\ell_0, \mathbf{0}_C]_{\cong_M} \quad T = \{[\ell, v]_{\cong_M} \xrightarrow{a} [\ell', v']_{\cong_M} \mid \exists d, (\ell, v) \xrightarrow{d,a} (\ell', v')\}$$

The region automaton $\mathcal{R}_{\cong_M}(\mathcal{A})$ is a finite automaton whose size is exponential compared with the size of $\mathcal{A}$. It can be used to check, reachability properties and "one of the most fundamental theorems" [CHVB18, Chapter 29.2, Theorem 1] states that the reachability problem (or equivalently emptiness) in TA is PSPACE-complete [AD94, Section 4.5].

Zones The number of states obtained from the region partitionning is unfortunately extremely large, so the notion of *zones* was introduced as a coarser representation of the state space. *Zones* are exactly the sets $[\![\varphi]\!]_C$ of valuations satisfying any guard $\varphi \in \Phi_d(C)$[3]. Similarly to the region automaton, we can define the *zone automaton*, except that the zone automaton is infinite so the zones need to be *normalized*, which basically consists in quotienting each zone w.r.t. the equivalence relation $\cong_M$[4].

Extensions Many extensions of TA were proposed in the literature, such as allowing *diagonal constraints*, *updatable* TA[5] or adding *urgency* requirements. However, it has been shown that such extensions are no more expressive than the original class of TA. Furthermore, most attempted extensions generally lead to undecidability of the reachability problem, e.g. by simply adding constraints of the form $\lambda x + \mu y \leq k$ or $x := x + k$, or more complex classes such as *hybrid* automata.

3 Model checking timed systems

Timed Temporal Logic We extend the usual theory of LTL presented in [CHVB18, Chapter 2] to timed systems with MTL (*Metric Temporal Logic*). Given a set of atomic propositions P, an interval I and a *time-constrained until* operator $\mathbf{U}_I$, the formulas of MTL are defined as follows:

$$\varphi ::= p \in P \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \mathbf{U}_I \varphi$$

Similarly, the *constrained always* operator $\Box_I$ and *constrained eventually* operator $\Diamond_I$ with $\mathbf{U}_I$ can be defined in this time-constrained setting. Two different semantics are usually adopted for MTL,

[3] $\Phi_d(C)$ is the set of *diagonal constraints* over C, i.e. constraints of the form $x - y \odot k$.

[4] This quotient does not produce a zone in general, other operators are therefore used in practice, such as the *extrapolation* of a *shortest-path-closed* DBM (*difference-bound matrix*).

[5] Clocks can be updated to any value instead of being reset to 0.

namely the *continuous* and *pointwise* semantics. Informally, the pointwise semantics allows one to assert formulas only at discrete set of points, while the continuous semantics allows one to assert formulas at arbitrary time points.

References

[AD94] Rajeev Alur and David L. Dill. A theory of timed automata. *Theoretical Computer Science*, 126:183–235, 1994.

[CHVB18] Edmund M. Clarke, Thomas A. Henzinger, Helmut Veith, and Roderick Bloem. *Handbook of Model Checking.* Springer Publishing Company, Incorporated, 1st edition, 2018.