Predicting downturn LGD for a (confidential) portfolio

Bob Stroomer

1 July 2009

Preface

This thesis is part of my internship for the study Business Mathematics and Informatics at the VU University Amsterdam.

This is a thesis about LGD models. LGD stands for Loss Given Default and is an expectation for the losses a bank will experience on a debtor if the debtor defaults, especially in downturn situations. LGD is one of the parameters that could be used by banks to estimate the losses the bank will experience as a result of granting credit.

I wrote this thesis at the department Financial Services Risk Management (FSRM) of Ernst & Young Netherlands. Ernst & Young is an audit and advisory firm which has offices all over the world. The FSRM-department works on different kinds of projects that are related to managing risks within the financial industry. Many of the projects are related to developing, implementing or assessing risk-related models, including LGD-models, within financial institutions.

I would like to thank Rene Bekker from the VU University and Gerd-Jan van Wiggen and Vincent Stap from Ernst & Young FSRM for assisting me in writing this thesis. I would also like to thank Nico Warmer, partner at FSRM, for giving me the opportunity to write this thesis at Ernst & Young.

Summary

By providing loans, banks take the risk of losing money when the counter-party (debtor) is not able to fully repay the loan, in which case the debtor defaults. To stay liquid, banks hold some money aside to catch up losses that occur from defaulting debtors. The capital held aside is called Economic Capital. To determine the Economic Capital, banks predict the Expected Loss (EL) with the following parameters:

- Probability of Default (PD),

- Loss Given Default (LGD),

- Exposure At Default (EAD).

The Probability of Default is the probability that a debtor will default within one year, the Exposure at Default is the expected amount of capital a debtor owes the bank if he defaults. Finally, Loss Given Default is an the expected part of the loan that can't be recovered by the bank. For a single loan, LGD is generally defined as

$$LGD = \frac{\text{Loss amount}}{\text{Size of the loan at default}} \qquad (1)$$

and Expected Loss for an obligor is calculated with

$$EL = PD * EAD * LGD. \qquad (2)$$

The prediction of Economic Capital is subject to rules set by a banks' regulating institution. In the Netherlands, the rules are set by De Nederlandsche Bank (DNB). Worldwide, most of the local regulations are based on the Basel accord on banking supervision (Basel II accord). The Basel II accord is initiated by the regulators of the G-10 countries and provides guidelines for determining Economic Capital for different kinds of risk. Downturn LGD is one of the required parameters for calculating Economic Capital. It is the expected LGD given the fact that the economy is in a downturn situation.

In this thesis, four methods for predicting downturn LGD are described; three parametric models and one bootstrap method. The underlying idea of the parametric models is that the size of losses depend on the overall state of the economy. The health of a loan is determined by the overall state of the economy and an idiosyncratic factor that only influences a particular loan. If the health is below a pre-defined threshold, the loan defaults and the expected part of the loan that cannot be recovered is predicted.

The first model (Frye) assumes that losses are normally distributed. This model is used as a basis for many other approaches in determining downturn LGD. The second model (Pykhtin) assumes a lognormal distribution for the individual value of the loans. Unlike Frye, Pykhtin assumes that losses are influenced by the firms idiosyncratic factor. The third model (Tasche) assumes a beta-distribution for the losses. Tasche also provides a conservative (high) value for the variance of the losses. Finally, a bootstrap method is described, in which new losses are generated by taking draws from a pool of existing losses. Based on the new losses, a downturn LGD is determined.

The four models are applied to a confidential dataset

Contents

1 Introduction

By providing loans, financial institutions take the risk that the counterparty will be unable to repay the loan. If a counterparty is unable to fulfill its obligations, the bank has a loss. To catch up losses that result from defaulting debtors, banks put an amount of money aside. This is called regulatory capital. The amount of capital a bank has to put aside should be calculated according to rules and regulations set by the supervising authority. One of the parameters used in the calculations for regulatory capital is 'Loss Given Default' (LGD). LGD is a prediction for the fraction of the outstanding loan that can not be recovered by the bank.

In this paper, the role of LGD in the calculation of the regulatory capital is described. In the Netherlands, the guidelines of the Basel II accord are implemented in the regulations. A description of the Basel II accord and the role of LGD are given in Section 2 and Section 3 respectively. The accord focuses mainly on the LGD in a situation that is likely to happen only once in a thousand years. Such a situation is called a downturn situation.

In Section 4, four models are described that can be used for predicting LGD. Three of them are theoretical models that are described in many papers about LGD, the fourth is a statistical approach that uses bootstrapping.

Ernst & Young has provided a confidential dataset (confidential)....

2 Basel accords and capital requirements

In their daily activities, banks are exposed to certain kinds of risk. One of the risks is the possibility that debtors default and cannot pay back their loans. By managing such risks, financial institutions have to implement rules that are set by the regulating authority (supervisor). The Basel Committee is an international Committee of representatives of regulating authorities. The Basel Committee has set up a set of rules and regulations for financial institutions by means of the Basel accords. Worldwide, over 100 countries have implemented the guidelines of the Basel II accord in their laws. This chapter describes the principles of the Basel accords and the role of Loss Given Default (LGD) within these accords.

2.1 Basel Committee on Banking Supervision

The Basel Committee on Banking Supervision is an institution, created in 1974, which has the purpose to encourage convergence towards common approaches and standards. The committee is created by the governors of the central banks of the G-10 countries and is nowadays represented by senior representatives of the G-10 countries, Luxembourg and Spain. Usually, the members of the committee meet at the Bank for International Settlements in Basel.

The Basel Committee formulates broad supervisory standards and guidelines and recommends statements of best practice in banking supervision. The expectation is that a wide range of nations' authorities will implement them in their own national system.

The main achievements of the committee are the two Basel accords, which contain recommendations on banking laws and regulations. The Basel I accord was published in 1988 and is primarily focused on credit risk. In 2004, the Basel accord was succeeded by the Basel II accord, which focuses not only on credit risk, but also on operational risk and market risk. Originally, the Basel accord was implemented by the 12 members of the Basel Committee. Nowadays, over 100 countries have implemented the principles of the Basel accords in their laws.

2.2 Basel I accord

During the early 1980's, the members of the Basel Committee became concerned that the capital ratios of the main international banks were declining just as their risk exposure was increasing. In response to that trend, the

Basel Committee tried to achieve greater international convergence in the measurement of capital adequacy by setting up the Basel I accord [11].

The Basel I accord was originally created and adopted by the members of the Basel Committee in July 1988. The purpose of the accord is to set clear minimum capital standards for international operating banks and other financial institutions. The accord focuses mainly on credit risk, which is the possibility an obligor is not able to pay back a loan. To catch up losses as a result of credit risk, a bank has to keep an amount of capital aside. The Basel accord focuses on setting rules for determining the minimum capital a financial institution has to hold to catch up these losses.

The assets of a financial institution are grouped in five classes, each class with its own risk weight. The risk weight indicates the credit risk exposure of a loan. The risk weights of the classes are 0%, 10%, 20%, 50% and 100%. For example, loans to a financial institutions' home country sovereign have a risk weight of 0% and most corporate loans fall in the 100% category. The minimum capital a bank has to hold to catch up losses on loans is 8% of the risk weighted assests

$$K = 8\% \times \sum_{i=1}^{n} RW_i \times A_i, \tag{3}$$

where K is the capital requirement, n the number of assets, RW_i the risk weight of asset i and A_i the value of asset i. For example, suppose a bank has the following loans outstanding:

Counterparty	Risk-weight	Amount outstanding
Sovereign	0%	10.000.000
AA rated Bank	10%	5.000.000
A rated Bank	20%	1.000.000
Corporate	100%	2.500.000

Table 1: Example loan portfolio

Then, the minimum capital the bank has to hold to catch up losses on these loans is:

$$
\begin{aligned}
K \quad &= 8\% \quad \times \quad (\quad 0\% \quad \times \quad 10.000.000 \\
& \qquad\qquad\qquad + \quad 10\% \quad \times \quad 5.000.000 \\
& \qquad\qquad\qquad + \quad 20\% \quad \times \quad 1.000.000 \\
& \qquad\qquad\qquad + \quad 100\% \quad \times \quad 2.500.000 \quad) \\
\\
&= 8\% \quad \times \qquad 3.200.000 = 256.000.
\end{aligned}
\tag{4}
$$

The implementation of the accord started in 1989. In September 2003, all international operating banks within the G-10 countries were in compliance with the rules provided in the accord.

The weakness of the Basel I accord is the fact that it focuses almost exclusively on credit risk. Because other types of risk are not taken into account, there were concerns that the estimated risk weights did not properly reflect a financial institutions' actual risks. In 1996, the Committee issued additional amendments that incorporated market risk into the risk weights of asset classes. In 2004, the Basel I framework was succeeded by a new set of guidelines, the Basel II framework.

2.3 Basel II accord

In June 2004, the Basel Committee on banking supervision introduced the Basel II accord [12] as an improved version of the Basel I accord. Like the Basel I accord, the purpose of the Basel II accord is to provide an international set of rules and guidelines that can be used by banking regulators for creating rules about the amount of capital a bank has to put aside to catch up losses. The Basel II accord focuses not only on losses as a result of credit risk, but also on losses that result from market risk and operational risk.

Basel II replaces the outdated Basel I guidelines and is implemented in Europe by means of the CRD-guideline (Capital Requirements Directive). Almost all EU-member states have integrated the Basel II accord in their national laws. Worldwide, over 100 countries have implemented the principles of the accord.

The Basel II accord is based on three pillars. The first pillar is about the capital requirements. It sets some regulations to prevent the bank from suffering too much from losses due to credit risk, market risk and operational risk. The second pillar is about the supervisory review process. It gives guidelines about other kinds of risk. The third pillar tells something about market transparency. It includes regulations about the reporting to supervisory banks. The three pillars are discussed in more detail in the following subsections.

2.3.1 Pillar 1: Minimal capital requirements

Pillar 1 gives guidelines for the calculation of the minimal amount of capital a bank has to put aside, according to their regulator, for market risk, credit risk, and operational risk. Banks may choose from a couple of methods for calculating the minimal capital requirements. There is a simple method, called standardized approach, and there are more complex methods, based

on a banks internal ratings. For using an advanced method, a bank needs approval of the supervisory bank. In this pillar, financial institutions have to make predictions of the amount and severity of future losses. This is the pillar in which LGD is required.

Credit risk
Credit risk is the exposure of a bank to the possibility of losses due to debtors that are not able to pay back their loans. There are different types of debtors (sovereigns, banks and retail) and different types of loans (mortgages, personal loans, derivative transactions).

Operational risk
Operational risk covers the possibility of losses that occur because of inadequate or failing business processes. Fraudulent employees and crashing computer systems are typical examples of operational risk.

Market risk
Market risk is a firm's exposure to economic changes or events that have impact on large portions of the market. Examples are changes in equity (stock prices), interest rates, commodity prices and foreign exchange rates.

2.3.2 Pillar 2: The role of the supervisor

The second pillar is about the supervisory review process. This pillar gives some regulations about all risks that are not dealt with in pillar 1. The four principles of this pillar are:

Principle 1: Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels.

Principle 2: Supervisors should review and evaluate banks' internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital ratios. Supervisors should take appropriate supervisory action if they are not satisfied with the result of this process.

Principle 3: Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum.

Principle 4: Supervisors should seek to intervene at an early stage to prevent capital from falling below the minimum levels required to support the risk characteristics of a particular bank and should require rapid remedial action if capital is not maintained or restored.

2.3.3 Pillar 3: Market disclosure

Pillar 3 is the transparency pillar. It provides guidelines for market disclosure banks have to make. The purpose is to create a disclosed market in which counterparties of banks have a good picture of the overall risk position of the banks. The guidelines are related to:

- Application of the capital rules,

- the capital structure and amount of capital,

- the qualitative and quantitative managing of risks.

The provided guidelines give suggestions for disclosure to the market as well as reporting to the supervisors.

2.4 Determining capital requirements

To prevent a financial institution from suffering too much of losses related to defaulting debtors, it has to put capital aside. The amount of capital a financial institution has to put aside to catch up those losses is called the capital requirement. The losses that occur differ from year to year, depending on the number and severity of defaults. Figure 1 shows the variation of loss rates a bank could have over time. The loss rate is the average loss on a defaulted loan, expressed as a percentage of the outstanding amount.
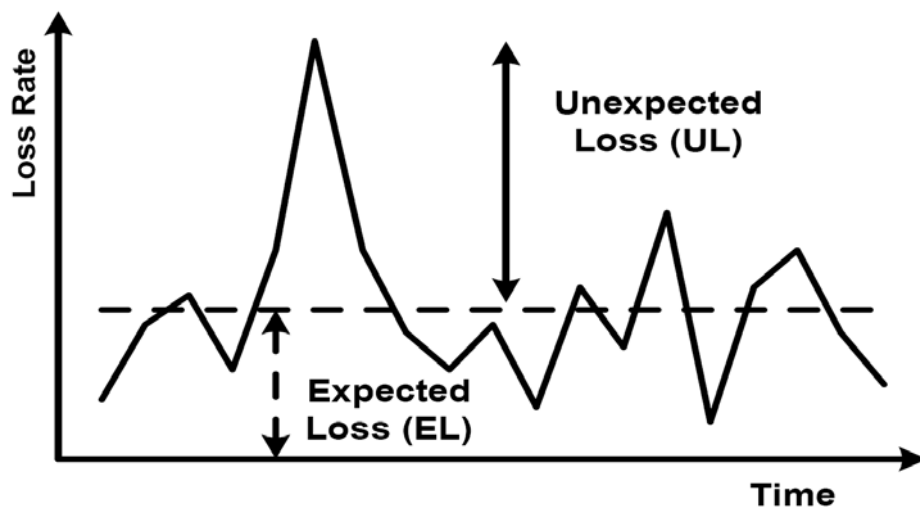

Figure 1: Losses over time (source: Bank for international settlements [13])

It is impossible to know in advance the losses that a financial institution will have in a year. However, a bank can make a prediction of the average level of credit losses that will occur. Those losses are known as expected losses (EL). In Figure 1, the dashed line represents the expected losses. Generally, expected losses are considered as a cost component of doing business. They are managed by means of provisioning and pricing of credit exposures.

If the actual losses on credit exposure exceed the expected losses, the financial institution needs to use (a part of) its own capital to catch them up. Such losses are shown in Figure 1 as the spikes above the expected losses. Unexpected losses do not happen every year, but can be potentially very large when they do occur. Unexpected losses are associated with being in the business. Financial institutions know that they occur, but do not know when and in what sizes. Interest rates and risk premia may absorb a part of the unexpected losses, but are most of the time insufficient to cover them all. Therefore, a financial institution has to hold capital.

In the worst case, a financial institution loses its entire portfolio of loans in a given year. That case is highly unlikely, but can occur in theory. Holding capital to catch up such a loss is economically very inefficient. Moreover, a financial institution aims to invest as much capital as possible in profitable investments. Capital that is held to catch up losses cannot be invested in profitable investments. Therefore, financial institutions try to minimize the capital they hold. However, if a financial institution has insufficient capital and profit to cover the losses on credit exposures, it is possible that it can't meet its own debt obligations. In that case, the financial institution could become insolvent. Financial institutions have to balance the risks and rewards of holding capital.

The approaches to determine the level of required capital focus on the frequency of financial institutions insolvencies arising from credit losses. By using a stochachstic credit portfolio model, it is possible to estimate a value for the amount of loss that will be exceeded with a small probability. This probability can be seen as the probability that a financial institution will become insolvent. The capital is set on a level for which the probability of getting insolvent is very small. In the Basel II accord, the confidence interval is set at 99.9%. In other words, the losses may only be larger than the required capital in 1 out of 1000 years.

Figure 2 shows a typical probability density function of a financial institutions' losses. Expected loss can be seen as the mean of the distribution. The figure shows that most of the losses occur around or below the value of the expected loss. The gray area is the possibility that a financial institution gets insolvent. The probability of getting insolvent is 100% - confidence level.

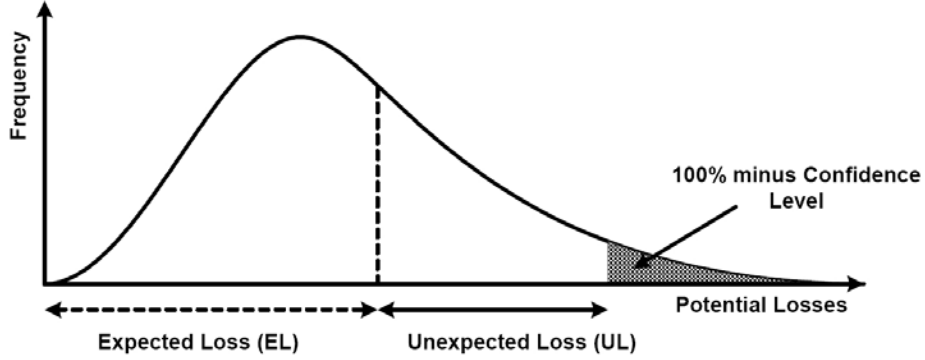To determine the capital requirement for credit risk under the Basel II

Figure 2: Loss distribution (source: Bank for international settlements [13])

accord, a financial institution can use three different methods; the Standardized Approach (SA), The Foundation Internal Ratings Based Approach (F-IRB) and the Advanced Internal Ratings Based Approach (A-IRB). The three approaches differ in level of difficulty and outcomes. The Standardized Approach is the easiest, but will give the highest capital requirements. The outcomes of the IRB approaches are based on the institutions' own models and historical data and are therefore more accurate for a specific financial institution.

2.4.1 Standardized Approach

The Standardized Approach is the easiest of the credit risk approaches. It works similar as the approach in the Basel I accord (see Section 2.2. For each debtor, the bank can look up a risk weight in a table, provided by the Basel Committee. The risk weight for an obligor depends on the type of loan and the credit profile of the obligor. It recognizes that different counterparts within the same category can have different risks weights. For example, a loan to an AA rated corporate has a lower risk weight than a loan to an A rated corporate. The risk-weighted assets (RWA) could then be calculated by the following formula:

$$\text{RWA} = \sum_{i=1}^{n} RW_i \times A_i. \tag{5}$$

Under the Standardized Approach, the minimum capital requirements for the bank is set at 8% of the risk-weighted assets, so the capital requirements (K) can be calculated with

8

$$K = \text{RWA} \times 0.08. \tag{6}$$

The risk weights are shown in Table 2. Some risky kinds of exposure have a risk weight of 150%, which means that holding capital equal to 8% of the exposure is insufficient.

Since this approach should suit for all financial institutions, the outcome is very conservative. If a financial institution has enough historical data about granted loans, it might better use one of the Internal Ratings Based (IRB) approaches.

Rating	Sovereign	Banks	Corporates
AAA to AA-	0%	20%	20%
A+ to A-	20%	50%	50%
BBB+ to BBB-	50%	50%	100%
BB+ to BB-	100%	100%	100%
B+ to B-	100%	100%	100%
Below B-	150%	150%	150%
Unrated	100%	50%	100%

Table 2: Risk weights under Basel II Standardized Approach

2.4.2 Internal Ratings Based Approaches

The losses caused by defaulting obligors fluctuate every year and depend on the number and severity of defaults. A financial institution never knows in advance the losses that will occur, but can forecast the expected losses based on three quantities:

- PD, Probability of Default, is the estimated portion of obligors that might default within a given time horizon.

- EAD, Exposure At Default, is the expected (discounted) amount of capital the financial institution still has to earn from an obligor, if the obligor defaults within a given time frame (mostly one year).

- LGD, Loss Given Default, is the expected part of the exposure that can not be recovered by the financial institution.

The Expected Loss ($E[L]$) for a bank is then calculated by

$$E[L] = PD \times LGD \times EAD. \tag{7}$$

9

PD and LGD are rates (percentages), EAD is a number (the amount of capital). The IRB approaches are built on those three quantities that are called the risk parameters.

A financial institution may only use the IRB approaches if it has permission of the supervising authority (in most cases, the central bank is the supervising authority). If a financial institution has satisfactory models and enough historical default data to make a prediction for the probability of default (PD), but has not sufficient data to make reliable predictions for LGD and EAD, the values for the latter two variables are provided by the supervising authority. In that case, the financial institution uses the Foundation-IRB approach. However, if a financial institution does have satisfactory models and enough historical data, it may use its own predictions for the values of LGD and EAD as well. In that case, the financial institution uses the Advanced-IRB approach.

2.4.3 Asymptotic Single Risk Factor model

In the Basel II accord, a theoretical foundation for the role of the systematic factor within the IRB approaches is included by means of the Asymptotic Single Risk Factor model. Under the Asymptotic Single Risk Factor (ASRF) model, the loss rate for a well diversified portfolio depends only on a single systematic risk factor which is standard normally distributed. The loss rate is independent of idiosyncratic factors, which only affect the losses on individual loans. Denote the systematic risk factor as X and the loss rate as L. The amount of capital a financial institution must hold can then be written as $E[L|X = \alpha]$. If α denotes the 99.9th percentile of the standard normal distribution, the amount of capital satisfies the 99.9% VaR (Value-at-Risk) target. In other words, there is a probability of 99.9% that the loss is less than or equal to $E[L|X = \alpha]$. The formula for $E[L|X = \alpha]$ is:

$$E[L|X = \alpha] = P(D = 1|X = \alpha) \times E[L|D = 1, X = \alpha] \qquad (8)$$

where D is an indicator function for the default event; $D = 1$ if an obligor defaults, else $D = 0$.

The first term of Formula (8), $P[D = 1|X = \alpha]$, is the conditional probability of default (CPD) given the 99.9th percentile of the systematic risk factor. The second term is the conditional loss given default (CLGD) and can be seen as downturn LGD. In other words, if α is the 99.9th percentile of the standard normal distribution, CPD and CLGD can be interpreted as the PD and LGD in a very bad year (the systematic factor is only worse in 0.1% of the years).

Financial institutions are not required to estimate the CPD for a borrower directly. To estimate the CPD for an exposure, a mapping function is provided by the Basel Committee that allows financial institutions to use their own estimates of expected Probability of Default. This function is the so-called Vasicek-distribution [19]:

$$CPD = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}}\right), \tag{9}$$

where $\Phi(x)$ is the standard normal distribution function and $\Phi^{-1}(p)$ is its inverse. Here, ρ is the asset-correlation parameter that determines the sensitivity of an assets' value to the systematic risk factor. The value of ρ is prescribed by the Basel Committee and depends on the default probability and the size of an exposure.

The combination of Formula (8) and Formula (9) gives the formula of the conditional expected loss rate for a unit of exposure:

$$E[L|X = \alpha] = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}}\right) \times CLGD \tag{10}$$

This equation forms the core of the IRB risk-weight functions prescribed by the Basel Committee. It gives an expectation of the losses in a downturn situation.

Under the IRB approaches in the Basel II accord, a bank calculates the capital requirements (K) to catch up unexpected losses with the following mapping function

$$\begin{aligned} K &= \left(\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}}\right) \times LGD - PD \times LGD\right) \\ &\times \left(\frac{1 + (M - 2.5) \times b(PD)}{1 - 1.5 \times b(PD)}\right) \end{aligned} \tag{11}$$

The first part of the equation is an indication for the unexpected losses on a unit of exposure and can be inferred from Equation 11. It is multiplied by the second part, a maturity adjustment with the parameters

$$\begin{aligned} b(PD) &= \text{Smoothed maturity function} &= (0.11852 - 0.05478 \times ln(PD))^2 \\ M &= \text{Effective maturity} &= \frac{\sum_{t=0}^{T} t \times CF_t}{CF_t} \end{aligned} \tag{12}$$

where T is the time until maturity of the loan and CF_t is the cash flow at time t.

2.4.4 Asset value correlation

The asset value correlation parameter, ρ, expresses the dependency of the asset value of one borrower to the asset value of another borrower. In other words, it determines the dependence of a borrowers' asset value on the systematic risk factor X. The larger the asset value correlation, the larger the probability a firm will default in years with low values of X.

The value of the asset correlation depends on the asset class of the borrower. Different kinds of assets/borrowers have different dependencies on the systematic factor. Figure 4 illustrates the role of the asset correlation for two kinds of loans with the same expected loss.
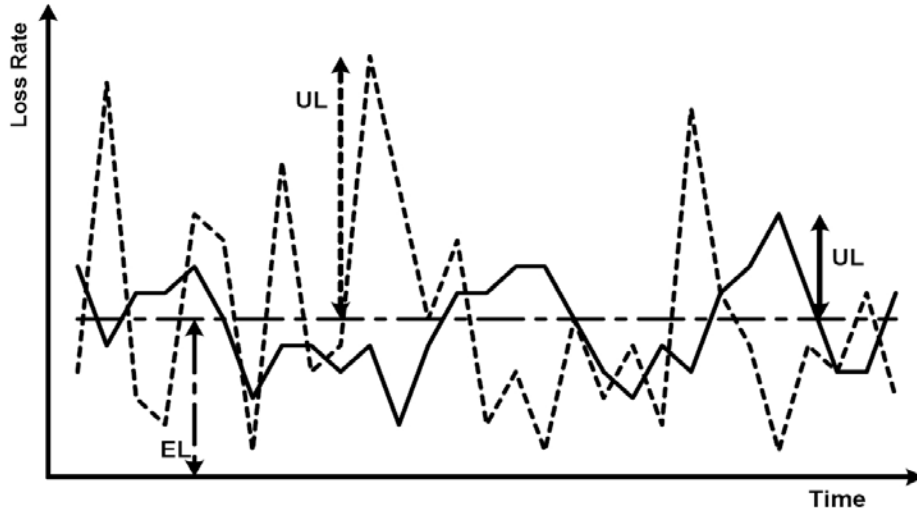

Figure 3: Losses over time for two kinds of loans with the same expected loss (source: Bank for international settlements [13])

The dashed line and the solid line represent the experienced losses within a timeframe for two loans with the same expected loss. The dashed line has a higher correlation among the individual loans. For the dashed line, the peaks are higher than for the solid line. If the systematic factor takes a low value, the negative effect will be larger for loans with a higher correlation with the systematic factor, so the total losses (expected losses plus unexpected losses) will be higher.

The value for the asset value correlation under the single risk factor approach for calculating the capital requirements is provided by the Basel Committee. It is derived from the analysis of datasets of the G-10 supervisors. The analysis of these datasets revealed two systematic dependencies:

- Asset correlations decrease with increasing PD's. For example, if a borrower has a high probability of default, the idiosyncratic risk components have a relatively large influence, so the dependence on the systematic factor is lower.

- Asset correlations increase with firm size. The larger a firm, the larger its dependency on the overall state of the economy. Smaller firms are more likely to default for individual reasons.

The function that is used to determine a firms' asset value correlation covers both of the above mentioned systematic dependencies:

$$
\begin{aligned}
\rho =\ & 0.12 \times (1 - e^{-50 \times PD})/(1 - e^{-50}) \\
+\ & 0.24 \times (1 - (1 - e^{-50 \times PD})/(1 - e^{-50})) \\
-\ & 0.04 \times (1 - (S - 5)/45).
\end{aligned} \tag{13}
$$

The function for the correlation has two limits for high and low PD's. The lower limit (high PD's) is set at 12%, the upper limit (low PD's) is 24%. The function itself is exponentially decreasing and looks as follows:
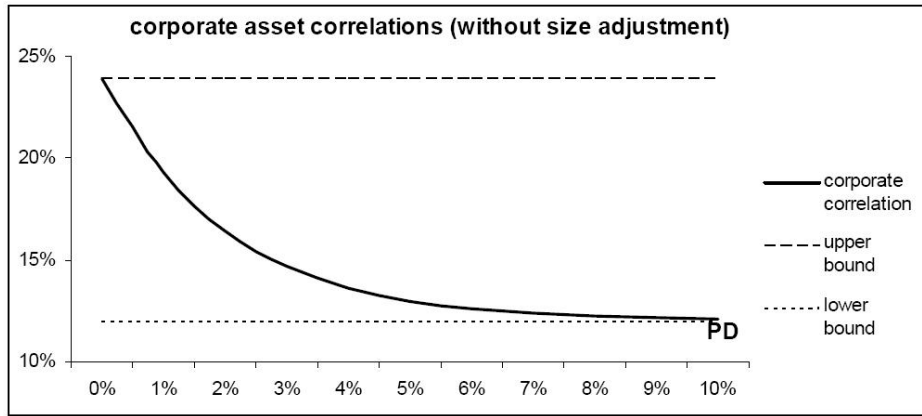

Figure 4: Asset correlation as function of PD (source: Bank for international settlements [13])

The last term in the equation, $0.04 \times (1 - (S - 5)/45)$, is an adjustment for the firm's size, S. It only affects borrowers with annual sales between \$5 million and \$50 million. For borrowers with annual sales above \$50 million, the size adjustment is 0, for borrowers with a firm size below \$5 million, the size adjustment is 0.04. In that case, the curve as shown in figure (above) shifts downward with 4%.

3 Loss Given Default

The Loss Given Default represents the expected percentage of loss a bank may suffer if an obligor defaults. It is also known as 1 - recovery rate. The recovery rate is the part of an exposure a bank can recover when an obligor defaults. In this section, some issues about measuring LGD are discussed.
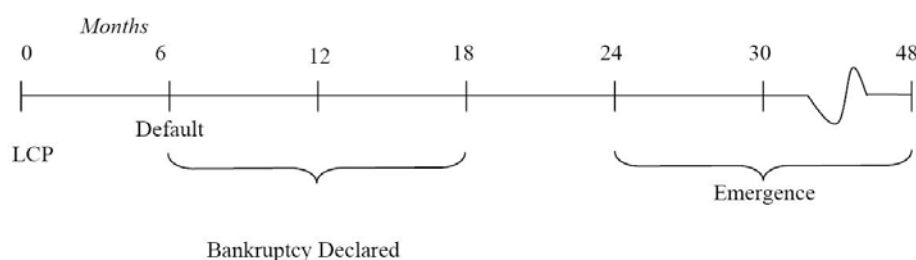

Figure 5: Timeline of a defaulted firm (source: Schuermann [17])

Once a firm is defaulted, the collection process starts. There are four key points within the collection process of a defaulted loan. First, there is the last cash payment (LCP). At that moment, the firm was still able to meet all its credit obligations. The next key moment is the declaration of default. At that moment, the firm is not able to meet all of its credit obligations and is considered default. For bonds, it is typically six months later, since most of them pay coupon twice a year. The third key moment is when bankruptcy is declared. Generally this moment occurs within one year after the default moment. It is possible that a firm is defaulted, but not yet declared bankrupt because of negotiations with its creditors. The last key moment is emergence from bankruptcy proceedings. Emergence typically occurs between two and four years after default. Cash flows can occur at any time after the last cash payment, but generally the largest part of the cash flows of a defaulted firm occurs during or within a short time after emergence.

3.1 Common characteristics of losses

The ability to determine LGD values based on a financial institutions' own portfolio and experienced losses is a good reason to use the Advanced IRB approach instead of the Foundation IRB or Standardized approach for determining the capital requirements under Basel II (see Subsections 2.4.1 and 2.4.2). The quality of the determined LGD depends on the information that is available to a financial institution.

14

Any financial institution that uses the Advanced-IRB approach needs to consider characteristics of its portfolio losses and recoveries. Studies and industry experiences have identified some common characteristics of losses. Supervisors need to be aware of those characteristics by assessing a financial institution's LGD-models. The main characteristics are:

- Most of the time, losses are either quite high (around 70-80%) or quite low (around 20-30%).

- The most important driver for a loan to determine if its losses are high or low is whether or not the loan is secured and its place in the obligors' capital structure.

- Losses are systematically higher in recessions.

- Industry of the obligor seems to matter. Asset-intensive industries (for example utilities) have lower losses than service sector firms.

- Size of the exposure does not matter.

In the following sections, some of the characteristics are discussed in more detail.

3.2 Bimodality

Generally, LGD distributions are bimodal. If you look at the distribution of experienced losses without regard to any factors or characteristics, there will be two peaks in it. Recovery is most of the times quite high or quite low. That makes sense, since losses tend to be quite small if the loan is secured (for example with collateral) and losses tend to be high for unsecured loans. This bimodality makes parametric modeling of losses difficult. If the mean of the distribution is somewhere in the range 40% - 60%, it is likely that only a small part of the losses is near the mean of the distribution, see Figure 6.

In Figure 6, the mean of the losses is slightly below 50%, but only a third of the experienced losses have an LGD between 35% and 65% Therefore, thinking in average LGD's can be misleading. The standard deviation and the shape of the distribution always need to be taken into account for determining the LGD-input for the calculation of capital requirements.
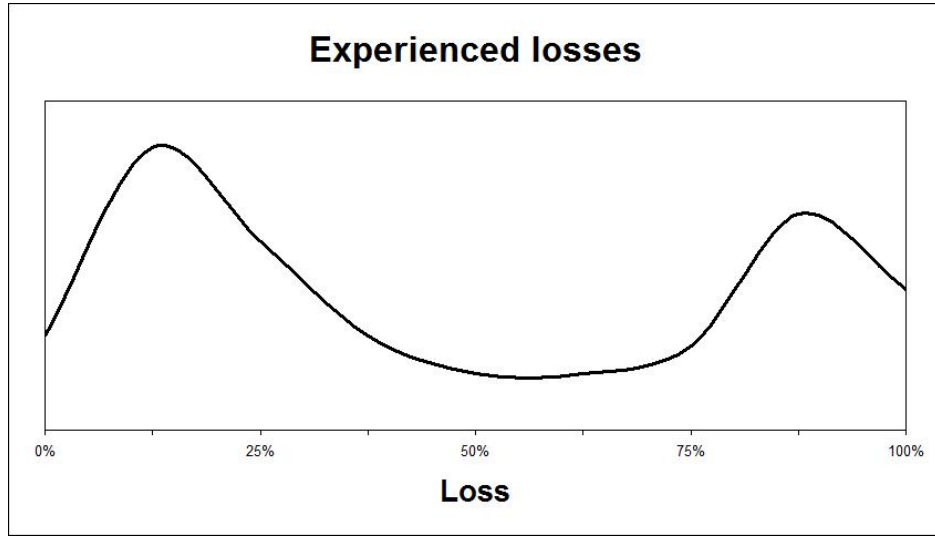
Figure 6: A typical probability distribution of losses on defaulted loans experienced by financial institutions

3.3 Capital structure

As explained in Section 3.2, losses are generally either quite high or quite low. The main driver that determines the size of the loss is the position of the loan in the obligor's capital structure. The capital structure is shown in Figure 7.

The capital structure determines which creditors are more senior than others. If a firm defaults, first the payments to the most senior debtors are made. Claims related to the default process and loans incurred after default are always more senior than all other claims. This system of seniority of claims is determined in law by means of the absolute priority rule (APR). The absolute priority rule is defined by Eberhart, Moore and Roenfeldt [3] as

> " The absolute priority rule states that a bankrupt firm's value is to be distributed to suppliers of capital such that senior creditors are fully satisfied before any distributions are made to more junior creditors, and junior creditors are paid in full before common shareholders. "

In practice, the APR is often violated. Several studies have shown that in 65%-80% of bankruptcies, shareholders receive payments while debtholders (who are more senior) are not fully paid off. The reason for this is the speed of resolution. Senior creditors agree on not being fully paid to resolve the bankruptcy faster.
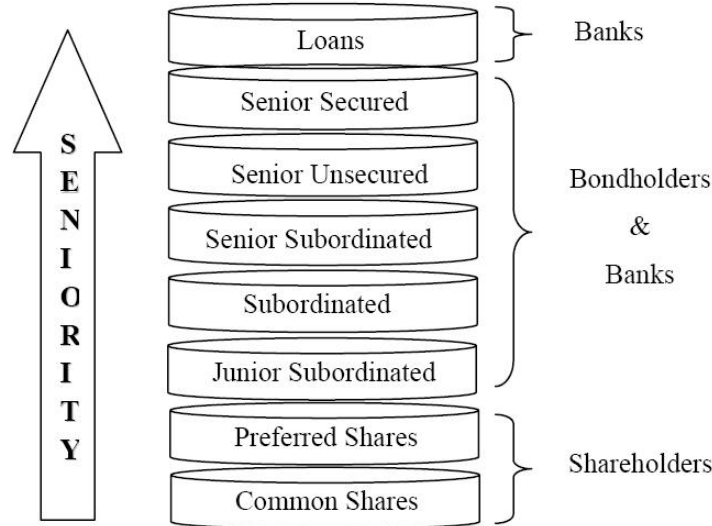
16

Figure 7: Capital Structure (source: Schuermann [17])

3.4 Downturn LGD

Because losses are systematically higher in recessions, financial institutions have to determine a downturn LGD. This downturn LGD should reflect the LGD in downturn conditions, since the number of defaults will be higher during that times. On the long term, an LGD should be at least 999 out of 1000 times smaller than or equal to the value of the downturn LGD. By using the long-run average LGD, a bank may get into trouble in periods of downturn, since especially in downturns the default rate as well as the Loss Given Default tend to be higher. Since there is no default data available to empirically determine downturn LGD, mathematical models are used to come to estimates or a very conservative value is chosen.

3.5 Default definition

A debt instrument (loan) can only experience losses if the obligor defaults. However, there is no clear definition of default. The Basel Committee has therefore included the following definition of default within the Basel II accord [12]:

" *A default is considered to have occurred with regard to a particular*

obligor when one or more of the following events has taken place."

- *It is determined that the obligor is unlikely to pay its debt obligations (principal, interest, or fees) in full.*

- *A credit loss event associated with any obligation of the obligor, such as charge-off, specific provision, or distressed restructuring involving the forgiveness or postponement of principal, interest, or fees.*

- *The obligor is past due more than 90 days on any credit obligation.*

- *The obligor has filed for bankruptcy or similar protection from creditors."*

The losses experienced in a default event strongly depend on the interpretation of default. For example, consider a firm that is past due more than 90 days on a credit obligation, but is able to pay all debts afterwards. According to the definition, the firm is defaulted. However, no losses occurred. In such cases, a financial institution can choose whether or not to see it as a defaulted loan.

3.6   Measuring LGD

LGD is defined as the ratio of losses to the exposure at default. To measure the LGD, the losses must be measured. In the collection process, the following three types of losses are included in the total loss on a loan:

- The loss of principal (missed income).

- The loss of carrying a non-performing loan (missed interest).

- The costs made for collecting the defaulted loan (workout expenses).

There are broadly three ways to measure the loss of a defaulted loan:

- Market LGD
  Market LGD can be measured for bonds and loans that are traded on the market. The actual prices at default are based on 'par' (100 cents per dollar) and can be translated in recovery (or loss). Market LGD is a good measurement for LGD, since the prices are observed soon. Moreover, the prices are based on market transactions, so there are no big valuation issues. The price at default is the investors' expected recovery, so all types of losses are included.

- Workout LGD
  Workout LGD is calculated by summation of the discounted expected cashflows of a defaulted loan. The tricky part is to determine the timing of the cashflows and the correct discount factors. The restructuring of a defaulted loan can lead into the issuance of risky and less risky assets. The discount factors for the payments are determined by the riskiness of the asset and the timing of the payment. In fact, after default, the financial institution is investor in a defaulted firm and has to value it accordingly.

- Implied market LGD
  Implied market LGD is measured by looking at the credit spread of risky, non-defaulted bonds that are currently traded. The spread above risk-free (government-) bonds is an indicator for the risk premium demanded by investors. This spread reflects expected losses (including PD and LGD), but also liquidity premiums. Studies have shown that the implied market LGD is higher than the actual loss rates.

3.7 LGD in our datasets' industry

confidential

4 Models

In this chapter, four models for predicting downturn LGD are discussed. ...(confidential)...

The first model is from John Frye [6], [7] and is referred to in many papers about LGD. Frye assumes that losses are normally distributed and partly based on the overall state of the economy.

The second model is Michael Pykhtins model [14]. It is based on Frye's model, but assumes that losses are lognormally distributed and also depend on a firm-specific variable that determines whether a firm defaults or not. In that way Pykhtin assumes that losses are larger for firms that are likely to default in the model.

The third model is proposed by Dirk Tasche [18]. Like the models of Frye and Pykhtin, Tasche assumes that losses depend on the overall state of the economy. The difference with Pykhtin is that Tasche assumes a beta-distribution for the losses and uses a conservative value for the variance instead of estimating the variance based on historical data.

The fourth model is based on a complete different approach, which is of a statistical nature. It is a bootstrap method that determines downturn LGD by bootstrapping losses from historical data. This method is described by Mathias Schmit [16]. The advantage of this method is that no input parameters have to be estimated. The disadvantage is that it performs relatively poor on small datasets.

4.1 Frye

In this subsection, a model will be used that is nearly identical to the credit capital model that Frye proposed [6]. The credit capital model calculates LGD by using an equation that determines collateral. Here, the losses are directly modeled, since the model will be applied to a set of losses without details about the collateral.

This model is quite similar to the models suggested by Gordy and Finger [8]. These models are driven by only one systematic risk factor, not by multiple correlated parameters. This simplification lacks a great deal of detail, but is appropriate for studying general influences, such as an economic downturn. The model presented here differs from Gordy and Finger by letting the default event as well as the recovery depend on the systematic factor and a firm-specific variable.

The systematic factor X is the main variable. It can be seen as the state of the economy. It affects the fortune of every firm and the amount of each recovery, so it influences the probability of default (PD_i) as well as the Loss Given Default (LGD_i) of firm i. The lower the value for X, the worse the economic situation. For low values of X, the model assumes a higher than average rate of defaults (Default Frequency, DF) and higher losses on the defaulted loans. X moves the *expected* rates of default and loss. On an individual level, X affects some losses more than others. So with a very high value of X (a very healthy economic situation), there are still defaults and high losses can still occur.

4.1.1 The model

For a given firm i in this model, its economic situation is affected by two factors. First, there is the systematic factor X. The impact of X on firm i is similar to its impact on all other firms. The other factor, Y_i, is the idiosyncratic factor. It can be seen as the individual fortune of firm i and affects only firm i's assets. The systematic factor X and the idiosyncratic factor Y_i determine the firm i's asset value A_i according to the formula:

$$A_i = \rho X + \sqrt{1 - \rho^2} Y_i. \tag{14}$$

The risk factors X and Y_i are assumed to be independent, standard normally distributed, which implies that A_i is also normally distributed. The parameter ρ in Equation (14) has a value between 0 and 1 and determines the dependency of firm i 's asset value on the systematic risk factor. If ρ has a high value, the value of every firm strongly depends on the general situation of the economy. If X also has a high value, there will be significantly less

defaults. On the other hand, if ρ is very small, the value of a firm is mainly driven by its individual fortune, the idiosyncratic factor Y_i. In that situation, the value of a firm is moving almost independently of the general economic situation.

In the model, a firm defaults if its asset value A_i falls below a threshold. The level of the threshold should be chosen to produce the long-term probability of default. Let D_i be a variable that symbolizes the default event of firm i and let PD_i be the probability of default, then

$$D_i = \begin{cases} 1 & \text{if } A_i < \Phi^{-1}(PD_i). \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

This model assumes the portfolio is quite large and fully diversified. The law of large numbers implies that the observed default frequency, DF_i, given a value for X, approximates its conditionally expected rate:

$$\begin{aligned} DF_i &= P\left(A_i < \Phi^{-1}(PD_i)|X = x\right) \\ &= P\left(\rho x + \sqrt{1-\rho^2}Y_i < \Phi^{-1}(PD_i)\right) \\ &= P\left(Y_i < \tfrac{\Phi^{-1}(PD_i)-\rho x}{\sqrt{1-\rho^2}}\right) = \Phi\left(\tfrac{\Phi^{-1}(PD_i)-\rho x}{\sqrt{1-\rho^2}}\right). \end{aligned} \tag{16}$$

To model the loss on a defaulted loan L_i, the systematic factor X is also used in a similar way as in modeling the default event. In addition to the systematic factor, there also is an idiosyncratic factor for each firm, Z_i. Also Z_i is assumed to be standard normally distributed, independent of X and Y_i. The loss is modeled as follows:

$$L_i = \mu + \sigma q X + \sigma \sqrt{1-q^2} Z_i \tag{17}$$

In this formula, μ can be interpreted as a firms' expected loss and σ as the unexpected loss. The effect of σ on the total loss is influenced by the systematic factor and the firms' idiosyncratic factor.

Since Z_i and X are independent, standard normally distributed, L_i is normally distributed with mean μ and variance σ^2. In this formula, q has a similar role as ρ in Equation (14); q determines the dependence of the loss on the systematic factor X. μ and σ are the mean and standard deviation of the recovery.

4.1.2 Applying the model to our data

confidential

4.2 Pykhtin

The model that is introduced by Michael Pykhtin [14] is quite similar to the credit capital model of Frye. As in Frye's model, it is assumed that returns depend on a systematic factor as well as an idiosyncratic factor. Like Frye, the systematic factor does not only influence the losses, but affects also a borrower's asset return. The difference with Frye is that this model assumes that losses are also affected by the firms idiosyncratic factor that affects the firms asset value. The idea behind this is that the loss suffered with a defaulting obligor decreases as the firms asset value decreases. A firm that is in trouble tends to spend less attention to the collateral of a loan, so the collateral value of a firm that is far below a 'default-threshold' is lower than the collateral value of a firm that is around that threshold.

Moreover, this model uses a lognormal distribution for the recoveries instead of the normal distribution that Frye proposed. The recovery on a loss is equal to $1 - LGD$. A lognormal distribution is used to ensure there are no negative recoveries, so the losses cannot exceed 100%.

4.2.1 The model

The model assumes that the asset return of firm i can be written as:

$$A_i = \rho X + \sqrt{1 - \rho^2} Y_i, \tag{18}$$

where X is the systematic factor and Y_i is the idiosyncratic factor of firm i. ρ is the dependency on the systematic risk factor and has a value between 0 and 1. X affects the asset value of all borrowers and can be interpreted as the general economic situation. Y_i affects only the asset value of borrower i and can be seen as the individual fortune of borrower i. X and Y_i are both independent, standard normally distributed.

The firm defaults if A_i falls below some threshold. The value of the threshold should be chosen in a way that the default probability is equal to the long term default rate. Let D_i indicate whether borrower i defaults, then D_i can be written as

$$D_i = \begin{cases} 1 & \text{if } A_i < \Phi^{-1}(PD_i), \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

If borrower i defaults, the amount of loss is determined by the value of the collateral. The loss in case of default (L_i) can be written as:

$$L_i = \max\{1 - R_i, 0\}, \tag{20}$$

where R_i is the recovery on loan i. If the recovery exceeds 1, the loss will be 0, so there are no negative losses. The difference between Frye's model and this model is the way the distributional properties are defined. Frye assumes the losses (and hence the recoveries) are normally distributed, this model uses a lognormal distribution for R_i:

$$R_i = \exp\left[\mu + \sigma B_i\right], \tag{21}$$

The random variable B_i is the standardized collateral return and is standard normally distributed. Frye argued that losses are related to the state of the economy. If the economy is in a bad situation, losses tend to be higher. Unlike Frye, Pykhtin argues that collateral value also depends on the asset value of a borrower. A borrower in financial distress might not invest in collateral maintenance, so the losses tend to be higher. B_i can be written as

$$B_i = \beta X + \gamma Y_i + \sqrt{1 - \beta^2 - \gamma^2} Z_i \tag{22}$$

where Z_i is an idiosyncratic factor that influences only the recovery on loan i. Z_i is standard normally distributed, independent of X and Y_i. This equation is only meaningful if β and γ are non-negative, satisfying $\beta^2 + \gamma^2 \leq 1$.

4.2.2 Applying the model to our data

confidential

4.3 Tasche

The single risk factor model that is introduced by Tasche is conceptually different from the models of Frye and Pykhtin. The outcomes of the models of Frye and Pykhtin are potential losses that are defined, but not observed. The approach of Tasche uses observed losses. In the models of Frye and Pykhtin, available losses are used to come to estimates for distributional parameters. In the model of Tasche, the losses themselves are used to as an empirical distirbution. Moreover, in the models of Frye and Pykhtin an estimation of the correlation parameter for both the default event and the LGD is required.

By modeling LGD via Tasches single risk factor model, the problems of Fryes and Pykhtins model are avoided. The model does not require correlation parameters, but can use LGD volatilities as input. The volatilities can be derived from the dataset, or can be fixed by supervisory authorities. Another difference between Tasches approach and the models of Frye and Pykhtin is the (assumed) distribution of the losses. Frye assumes the losses

are normally distributed around the expected loss and Pykhtin assumes log-normally distributed recoveries. Tasche uses a beta distribution to model the losses.

4.3.1 The model

The idea of the single risk factor approach is to describe the loss suffered with an obligor by a single variable. In Frye and Pykhtin, the loss is described by separate random variables for the default event and the amount of loss. Generally, the cumulative distribution function (cdf) for the loss suffered with an obligor is

$$F_L(t) = P(L \leq t) = 1 - p + pF_D(t), \tag{23}$$

where $p = P(L > 0)$ is the obligor's probability of default and $F_D(t) = P(L \leq t | L > 0) = p^{-1}P(0 < L \leq t)$ is the cumulative distribution function of the observed losses. Denote with $\Phi(x)$ the standard normal distribution function. Then, in a Basel II framework, L_i can be represented as

$$L_i = F_L^{-1}(\Phi(\sqrt{\rho}X + \sqrt{1 - \rho}Y_i)) \tag{24}$$

with independent, standard normally distributed random variables X and Y_i, asset correlation $\rho \in [0, 1]$ and the generalized inverse or quantile function F_L^{-1} of F_L. In general, the quantile function G^{-1} of a distribution function G is defined as

$$G^{-1}(z) = \min\{t : G(t) \geq z\}. \tag{25}$$

Applying Definition (25) to F_L^* yields the representation

$$F_L^{-1}(z) = \begin{cases} 0 & \text{if } z \leq 1 - p; \\ F_D^*(\frac{z-1+p}{p}) & \text{if } z > 1 - p. \end{cases} \tag{26}$$

The quantile function of the obligor's loss distribution F_D^{-1} again has to be determined according to Equation (25). Together, Equations (24) and (26) imply

$$L = \begin{cases} 0 & \text{if } \sqrt{\rho}X + \sqrt{1 - \rho}\xi \leq \Phi^{-1}(1 - p); \\ F_D^*(\frac{\Phi(\sqrt{\rho}X + \sqrt{1-\rho}\xi)-1+p}{p}) & \text{otherwise.} \end{cases} \tag{27}$$

Equation (27) can be equivalently written as the product of the indicator function of the default event $\sqrt{\rho}X + \sqrt{1 - \rho}Y_i > \Phi^{-1}(1 - p)$ and the factor $F_D^*(\frac{\Phi(\sqrt{\rho}X + \sqrt{1-\rho}\xi)-1+p}{p})$. The second factor can - similarly to Pykhtin's model - be interpreted as loss in case of default.

4.3.2 Applying the model to the data

confidential

4.4 Bootstrap method

The final method differs completely from the previous three. The previous three models generate portfolio's of losses according to a distribution. The realized losses in the dataset are used to estimate the parameters of those distributions. In the bootstrap method, portfolio's of losses are generated by resampling from the original dataset, so no distributional assumptions need to be made.

Moreover, some additional (correlation-)parameters have to be estimated for the previous models. In the bootstrap method, no distributional assumptions are made and no additional parameters have to be estimated.

The first step is taking a random portfolio of n defaults for a randomly chosen year. Drawing a year can be seen as determining the systematic factor for the loss portfolio. Each year has the same probability to be drawn. In the dataset, defaults are available for the time-window 1999-2007, so each year has a probability of 1/9 to be drawn. This portfolio is called p.

The next step is to generate a portfolio of i losses. This is done by drawing i times a loss out of portfolio p. The LGD for the generated portfolio is the average of all drawn losses. This process is repeated a large number of times. The downturn LGD will be the value at the 99.9%-quantile of the generated LGD's.

4.4.1 Applying the model to the data

confidential

5 Data

confidential

6 Results

confidential

7   Conclusion

confidential

8 References

References

[1] Edward L. Altmann. Default recovery rates and lgd in credit risk modeling and practice: An updated reveiw of the literature and empirical evidence. 2006.

[2] Michael Barco. Going downturn. 2007.

[3] Allan C. Eberhart, William T. Moore, and Rodney L. Roenfeldt. Security pricing and deviations from the absolute priority rule in bankruptcy proceedings. 1990. 3.3

[4] Andrea Resti Edward Altmann and Andrea Sironi. Analyzing and explaining default recovery rates. 2001.

[5] Andrea Resti Edward Altmann and Andrea Sironi. Recovery risks. 2005.

[6] John Frye. Collateral damage. 2000. 4, 4.1

[7] John Frye. Depressing recoveries. 2000. 4

[8] Michael Gordy. A risk-factor model foundation for ratings-based bank capital rules. 2002. 4.1

[9] Joocheol Kim and KiHuyng Kim. Loss given default modeling under the asymptotic single risk factor assumption. 2006.

[10] Amit Kulkarni. Monte carlo simulation of economic capital requirement and default protection premium.

[11] Basel Commitee on banking supervision. Basel accord on banking supervision. 2004. 2.2

[12] Basel Commitee on banking supervision. International convergence of capital measurement and capital standards (basel ii accord). 2004. 2.3, 3.5

[13] An Explanatory Note on the Basel II IRB Risk Weight Functions. 1, 2, 3, 4

[14] Michael Pykhtin. Unexpected recovery risk. 2003. 4, 4.2

[15] Min Qi and Xiaolong Yang. Loss given default of high loan-to-value residential mortgages. 2007.

[16] Til Schuermann. What do we know about loss given default? 2004. 5, 7

[17] Dirk Tasche. The single risk factor approach to capital charges in case of correlated loss given default rates. 2004. 4

[18] Oskar Vasicek. Loan portfolio value. 2002. 2.4.3

[19] RMA Capital working group. Downturn lgds for basel ii. 2005.