Credit Portfolio optimisation

Portfolio Optimisation Strategy

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Personal note

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1 INTRODUCTION

1.1 Portfolio Management

Portfolio Management (PM) has the mandate to manage the Wholesale Clients Loan Portfolio worldwide. It aims to optimise the portfolio by improving the risk/return profile. It is divided into a number of departments namely: Portfolio Strategy, Business Management, Portfolio Structuring & Execution and Credit Portfolio Management. Portfolio Strategy aligns a strategy that optimises the uses of the bank's capital by adjusting the Portfolio through optimising the risk/return profile. Business Management is responsible for reporting and analysis of the portfolio and performs research on portfolio parameters and development of the Loan Pricing Tool. Portfolio Structuring & Execution executes the disposal strategy aligned by Portfolio Strategy. Within the PM team, CPM officers handle all lending products for our wholesale clients. The team functions as a global centre of excellence, involved in structuring, advising, offering, executing and monitoring credits.

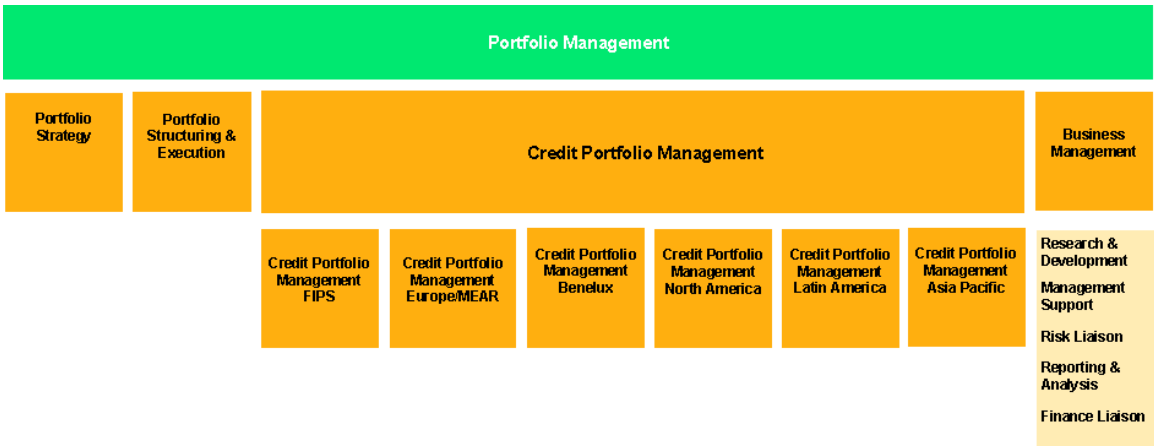

Figure i Organisation chart Portfolio Management

1.2 Models

ABN-AMRO bank makes use of more than one model to monitor the portfolio. Next to internal models Portfolio Management make uses of KMV portfolio manager. The underlying model has the property that it makes use of several parameters that are market controlled in contrary to the internal information the Loan Pricing Tool and the RAROC-engine base their calculations on. The models and their differences will be discussed in chapter 2.

1.3 Project description

The main object of this exercise is to construct a method to improve the existing portfolio. Selling credit risks is not the only method to reduce portfolio risk, also investing in parts of the portfolio that are underinvested can improve the portfolio. To identify potential investments and divestments with an optimal diversification effect, an efficient methodology should be constructed, which can be applied to the internal models used by ABN AMRO.

Selling and buying risk in such a way that the portfolio improves, while certain restrictions on capital allocation specified by the lender are met, is the proposed optimising strategy.

Primary goal is to analyse some approaches and construct a method that could be used to find an effective strategy that eventually can be implemented. The final product should be a method and eventually an application that gives an advice on selling and buying risk in given relationships, facilities or even industry-sectors, so that the portfolio improves.

2 MODELS FOR RISK AND PROFIT MEASUREMENTS

2.1 Introduction

Credit risk refers to the risk that counterparty does not fulfil its contractual obligation or the quality of an issuer deteriorates. In the models of ABN AMRO, credit risk is the risk a counterparty does not fulfil its obligation due to default.

To meet the objective of this research first in-depth knowledge of Credit Risk is required. Literature study, particularly in the LPT and RAROC engine, has been performed to get a better understanding in how Credit Risk is modelled within ABN AMRO.

2.2 Risk measurements used in the portfolio

2.3 Profit and performance measurements

Economic Profit/Loss

2.4 Models

2.5 Model differences

2.6 Overview model differences

3 MARKOWITZ'S MODERN PORTFOLIO THEORY

3.1 Introduction

A deepening inquiry into the theory concerning the Portfolio optimisation problem was done. In the following paragraphs highlights of some literature that could be useful in the future practice of this internship will be described.

3.2 Markowitz's Modern Portfolio Theory

Modern portfolio theory is constructed initially by Harry Markowitz and described in his book [**I**]. It is based on sophisticated investment decision approach that permits an investor to classify, estimate, and control both the kind and amount of expected risk and return. One of the essential parts of the modern portfolio theory is the quantification of the risk/return ratio.

Mean variance model

Modern Portfolio Theory is associated with mean variance return/risk analysis. A mean variance model minimises the portfolio risk for a given level of expected return. Markowitz identifies the variation or the standard deviation of the portfolio return as the portfolio risk. The mean-variance model has the following assumptions.

First of all, the model is based on a single period model of investment. This means that the investor allocates its wealth among different assets in the beginning and harvests the returns at the end. Three measures are necessary for using the mean-variance model. The standard deviation of the return of each asset i (denoted by σ_i), the expected return over a given time of period per asset and the correlation between each pair of assets are required. The variance or the standard deviation of an assets return over a given time of period is a standalone risk, also called undiversified risk. The general idea is to minimise the portfolio's standard deviation of returns for a given level of expected portfolio return, considering the correlation of each pair of individual asset. The portfolio risk, standard deviation of portfolio returns, is denoted by σ_p, and is derived by

$$\sigma_p = \sqrt{\left(\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij}\right)}$$

X_i represents the position of asset i in the portfolio.

$$\sum_{i=1}^{n} X_i = n$$

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

ρ_{ij} is the correlation between asset i and j.

Efficient frontier

Modern portfolio theory assumes that for a specified expected portfolio return, a rational investor would choose the portfolio with the smallest possible risk and visa versa. A portfolio is said to be *efficient* if there is no portfolio having the same standard deviation with a greater expected return and there is no portfolio having the same return with a lesser standard deviation. The *efficient frontier* is the collection of all these efficient portfolios. An example of the efficient frontier is displayed beneath.

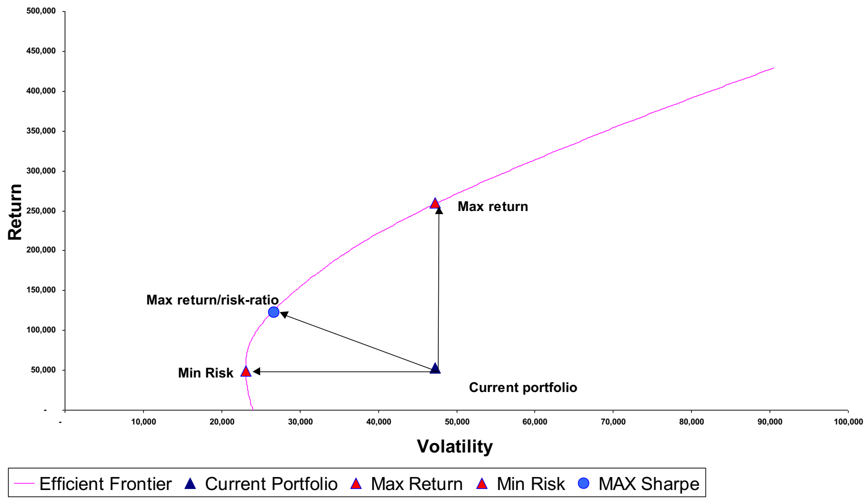

Figure ii: An example of the efficient frontier, with three optimisation directions (specified in paragraph 4.2)

The following mean variance model [**II**] is an example of a quadratic programming problem, which determines the efficient portfolio for a given level of expected return μ_p.

$$\text{Min} \quad \sqrt{\left(\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_{ij} \right)}$$

Subject to

$$\sum_{i=1}^{N} X_i = N \qquad (1)$$

$$\sum_{i=1}^{N} X_i \mu_i = \mu_p \qquad (2)$$

$$X_i \geq 0, \qquad for \quad i = 1,..,N \qquad (3)$$

Where μ_i is defined as the expected return on the facility i, and μ_p is the expected/desired return on the portfolio. The efficient frontier can be calculated by solving this problem for a number of portfolio returns.

Some additional restrictions could be applied to his model. An investor could constrain some asset's size with an upper- and lower bound. These constraints could be found necessary by the investor, as he must hold on to specific policy guidelines on investments. Also market specific information could make an investor to decide to limit some asset's size.

3.3 Optimise Credit Portfolio

In line with the modern portfolio theory the mean variance model can be used to achieve this goal. The difference in optimising credit portfolio with the previously discussed asset return portfolio is that the risk measures to be minimised are based on possible portfolio losses.

The goal is to minimise the risk on portfolio losses and increase the portfolio expected return.

4 OPTIMISING METHODOLOGIES FOR CREDIT PORTFOLIO

4.1 Introduction

4.2 KMV Portfolio Manager's optimisation methodology

4.3 Diversity factor

4.4 Proposed Credit Portfolio Optimisation

4.5 Investment constraints

5 IMPLEMENTATION OF OPTIMISATION METHOD

5.1 Introduction

As specified in the previous chapters, we would like to decrease the risk in terms of Economic Capital, while maintaining the same or increase Economic Profit. Since Economic Capital is hard to calculate analytically, we introduce to optimise the portfolio UL. This second order measure of the loss-distribution does not predict any losses suffered in the tail of the distribution, but we assume that when we decrease the UL, Economic Capital will decrease too.

5.2 Optimisation problem

In paragraph 4.4 the applicability of Modern Portfolio Theory on Credit Portfolio was discussed. In the same paragraph an optimisation problem was constructed to optimise Credit Portfolio using the mean variance analyse. In the following paragraph, the reduced gradient algorithm is discussed to solve the constructed problem.

5.3 Algorithm - gradient algorithm

A possible approach towards solving the problem can be done by using gradient-based methods. The method used for this problem is based on the so-called steepest decent algorithm. The algorithm iterates on the gradient of UL_p using position ULC_i for each facility. In each iteration the algorithm derives the gradient of each facility. Part of the exposure of the facility with the greatest gradient is transferred to the facility with the smallest gradient.

Theoretically the gradient of each facility is given by:

$$\frac{\partial UL_p}{\partial X_i} = \frac{\frac{1}{2}\times 2\times X_i UL_i^2 + \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} X_j UL_i UL_j \rho_{i,j}}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} X_i X_j UL_i UL_j \rho_{i,j}}} =$$

$$= \frac{X_i UL_i^2 + \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} X_j UL_i UL_j \rho_{i,j}}{UL_p} = \frac{X_i UL_i^2 + \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} X_j UL_i UL_j \rho_{i,j}}{UL_p} \times \frac{X_i}{X_i} =$$

$$= \frac{ULC_i}{X_i}$$

This gradient will only work for a portfolio with counterparties with the exact same characteristics. Such a portfolio is called a homogeneous portfolio. A homogenous portfolio consists of counterparties with the same exposure size, probability of default, et cetera. The risk contribution is then only influenced by correlation and not by exposure size. Using this gradient on the current Credit Portfolio will result in removing counterparties with the greatest risk contribution. The counterparty could easily be a diversifying one, and will only be removed because of its relative big stand alone risk. This is contrary to the goal of optimising the portfolio by diversification. It's obvious that a large exposure (with a big stand alone risk) could have a big contribution to portfolio risk, but it is not necessary true that a large exposure automatically contributes to a worse portfolio. The impact of changing the position of facility i should then not be based on the effect of a small change in X_i, but on the change in exposure or stand alone risk.

Because the portfolio is in fact heterogeneous, we should look into what effect an increase of stand-alone UL_i have on UL_p. Therefore the new gradient is based on the new portfolio weights UL_i

The new gradient is calculated by the following formula:

$$\textit{gradient of facility i :} \qquad \frac{\partial UL_p}{\partial UL_i} = \frac{\frac{1}{2} \times 2 \times UL_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} UL_j \rho_{i,j}}{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} UL_i UL_j \rho_{i,j}}} =$$

$$= \frac{\dfrac{UL_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} UL_j \rho_{i,j}}{UL_p} \times UL_i}{UL_i} =$$

$$= \frac{\dfrac{\sum_{j=1}^{n} UL_i UL_j \rho_{i,j}}{UL_p}}{UL_i} = \frac{ULC_i}{UL_i}$$

It so happened that the gradient is in fact the numerator of the first additional diversity factor. Optimising the portfolio with the first additional diversity factor will in fact give the same results as with the reduced gradient method. This is because of constancy of the factor 's denominator, which is the same for each facility.

By using stand alone risk (UL) as the portfolio weight (decision variable), we adjust the optimisation problem to the following one:

$$\min \quad \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} UL_i UL_j \rho_{L_i,L_j}}$$

Subject to

$$\sum_{i=1}^{n} UL_i^{new} \geq \sum_{i=1}^{n} UL_i^{Old} \qquad (1)$$

$$\sum_{i=1}^{n} EL_i^{new} = \sum_{i=1}^{n} EL_i^{Old} \qquad (2)$$

$$\sum_{i=1}^{n} Limit_i^{new} \geq \sum_{i=1}^{n} Limit_i^{Old} \qquad (3)$$

$$UL_i \geq 0 \qquad for \quad i = 1,..,n \qquad (4)$$

The objective function has been changed and UL_i is now considered to be the decision variable or the portfolio weight. (1) constrains the possibility that the portfolio risk decreases by removing stand alone risk. The idea is to reduce the portfolio risk by moving stand alone risk to diversifying parts of the portfolio. In (2) the same level of Expected Loss is demanded. This result from the mean variance theory discussed before. In (3) the portfolio limits after optimisation should at least be the same as the initial portfolio.

5.4 Implementation

Optimisation using a Gradient algorithm

6 RESULTS OPTIMISATION

6.1 Current portfolio

6.2 Portfolio after optimisation

After optimisation the sum of individual undiversified risk measure UL_i hardly changes. The limit amount and exposure at default increase substantially, while portfolio UL and EC decreases. The interesting part of the optimisation is the fact that the undiversified risk of the portfolio hardly changes (increase of 0.2%), while the risk after diversification decreases substantially (decrease of 22.0%). Instinctively one would say the diversifying optimisation method should have a positive impact on the portfolio EC. In this case the portfolio EC decreases with the almost the same percentage.

However the statistical measures UL_p and the risk contribution ULC_i do not have a clear relation with portfolio EC, it can be used to improve EC. By diversifying by means of these statistical measures, the portfolio improves. Comparing the base portfolio with the optimised portfolio, we see that sectors improve in terms of concentration. Only agriculture has an intolerable increase of concentration.

A remarkable note to make is that the application returns a portfolio with a great increase in exposure, while reducing the amount of Economic Capital. This result is due to the fact that the algorithm decreases exposure in the parts of the portfolio that on average have a high negative effect on Portfolio Economic Capital, while increasing exposure in parts of the portfolio that have a minimal and diversifying effect.

6.3 Conclusion

The algorithm used has decreased the portfolio Economic Capital, without decreasing the total exposure or the limit amount. As expected the gradient method identify parts of portfolio exposure that cause concentration or diversification effects. Because the first diversity factor is based on the gradient, this factor can be used to identify counterparties that diversify or cause concentration. Even more, if the gradient in the algorithm is replaced with the first diversity factor, it will give the exact same result.

The constructed application, algorithm and the first diversity factor are suitable to identify potential investments and divestments with an optimal diversification effect.

6.4 Investment portfolio

The application is also suitable for investment purposes only. The application and algorithm is then adjusted to one that only searches for parts of the portfolio that has an optimal diversification effect.

Selling parts of the portfolio is removed from the algorithm. Per iteration, the algorithm will invest a specified amount of exposure. In each iteration the algorithm searches for the counterparty with the best gradient to increase the exposure with the specified exposure amount.

6.5 Conclusion and Further Research

The constructed application has proven its worthiness in improving the portfolio in terms of Economic Capital. One of the areas where this application can be used is investment strategy.

Investment Strategy

The application is adjusted to one that, for a given total investment amount, gives an optimal mix of investments, which maximally diversifies the portfolio.

Single Investments

For single investments or disposals the diversity factor is more than sufficient in identifying (un-) diversifying counterparties.

Genetic Algorithm

The optimisation method used for the application is based on the reduced gradient method. Although the constructed algorithm gives a reasonable result, other algorithms and methods should be investigated. One alternate methodology that could be useful is a hybrid genetic-quantitative method. In "A hybrid genetic-quantitative method for Risk-Return optimisation of credit portfolio" [**III**] a credit portfolio is optimised by using a hybrid Genetic Algorithm. The paper describes this algorithm and concludes that it leads to better convergence than the gradient algorithm while not suffering from the local optima problem, and within a reasonable time.

CVaR

Theoretically CVaR is the most preferable risk to minimise for Portfolio Optimisation. CVaR is considered to be a coherent risk measure and can easily be optimised by linear programming techniques. Further research on this topic is necessary to prove its applicability in practice.

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