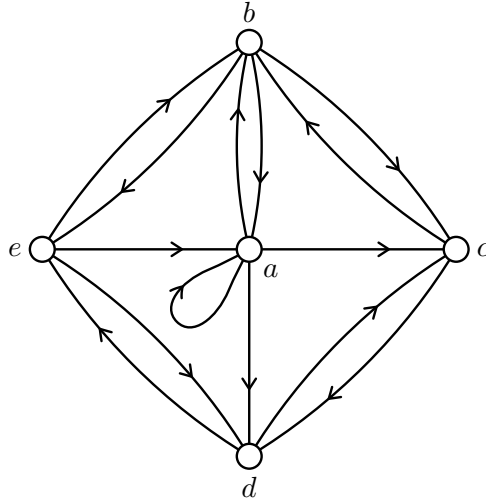


**Cancer Biology 8347 - Cancer Systems Biology - Spring 2023**  
**Problems on graph theory II**

2. Consider the boolean network shown below, where every vertex has a threshold of 2. In other words, for a vertex to be 'on' at time  $t + 1$ , it must have at least two incoming edges from vertices that are 'on' at time  $t$ .



For each of the following initial states (at time  $t = 0$ ), construct the states at times  $t = 0, 1, 2, 3, \dots$ . Show the state for each value of  $t$  by coloring in the vertices that are 'on' and leaving open the vertices that are 'off'. Label each state with ' $t = 0$ ', ' $t = 1$ ', and so on.

Continue until the states settle into an *attractor*. An attractor is a sequence of states (maybe just one state, or maybe several) that will repeat in a cyclic pattern. You can recognize that you have reached an attractor by the fact that you repeat a state  $S$  that you have seen before. Then the states following the first occurrence of  $S$  will be repeated over and over again.

When you recognize that you have reached an attractor, describe the cycle of states in the attractor.

**Note:** Copies of the network are provided on the following pages. Please use these to show your work.

- (a) Initial state at  $t = 0$  has vertices  $a, c$  and  $d$  'on', vertices  $b$  and  $e$  'off'.
- (b) Initial state at  $t = 0$  has vertices  $a, b$  and  $c$  'on', vertices  $d$  and  $e$  'off'.

