Cancer Biology 8347 - Cancer Systems Biology - Spring 2023

Problems on graph theory I

1. Average shortest path length and clustering coefficient are two measures of concentration in a graph. Often graphs that have high clustering coefficient will have low average shortest path length, but that is not always the case. We will look at two graphs on ten vertices with fifteen edges.

Note that you should work out answers as **exact fractions** and also state them as **decimals to 2 decimal places**. You should also **show your working** and **explain your reasoning**. All work should be done by hand (do not use computational tools!); the point is for you to get a feel for things by computing the numbers yourself.

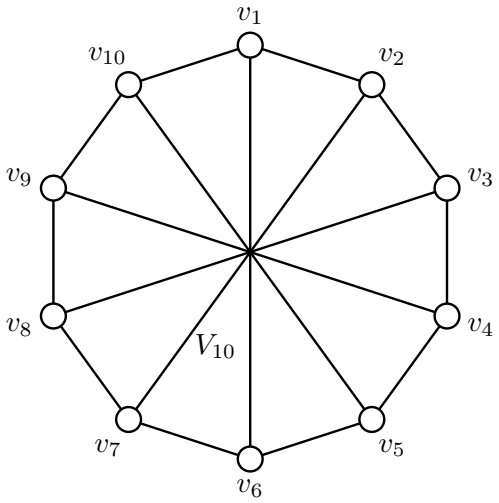
(a) Consider the graph shown at left below (this is known as the 10-*vertex Möbius ladder*, or V_{10}).

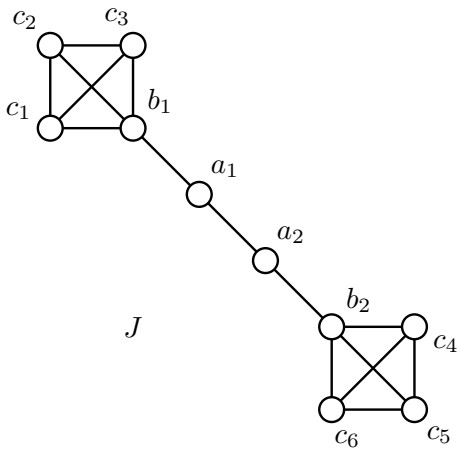
(i) Compute the clustering coefficient of the vertex v_1.

(ii) All vertices of V_{10} are *similar*, i.e., given any two vertices there is a symmetry that moves the first vertex to the second. Given this, what is the clustering coefficient of the whole graph?

(iii) What is the average length (number of edges) of a shortest path from v_1 to one of the other vertices? (Work out the length of a shortest path from v_1 to each of $v_2, v_3, \ldots, v_{10}$, and average these nine numbers.)

(iv) The average shortest path length for the whole graph turns out to be equivalent to the average over all vertices v_i of the average shortest path length from v_i to all other vertices. Given that all vertices of V_{10} are similar, what is its average shortest path length?





(b) Consider the graph shown at right above, which we will call J.

(i) Compute the clustering coefficients for the vertices a_1, b_1 and c_1.

(ii) Given that both vertices a_i are similar to a_1, both vertices b_i are similar to b_1, and all vertices c_i are similar to c_1, what is the clustering coefficient of the whole graph J?

(iii) Work out the average shortest path length from v to all other vertices for $v = a_1$, b_1 and c_1.

(iv) Given that both vertices a_i are similar to a_1, both vertices b_i are similar to b_1, and all vertices c_i are similar to c_1, what is the average shortest path length of the whole graph J?