The Walker:

$\frac{dx}{dt} = v$, $v =$ constant (speed), find $x(t) = ?$

$$dx = v\,dt \Rightarrow \int_0^t dx = \int_0^t v\,dt$$

$$x(t)\Big|_0^t = vt\Big|_0^t \Rightarrow x(t) - x(0) = vt$$

$$x(t) = vt + x_0$$

x_0 → starting (initial position)

$x(t)$ ↑ position at time t

v ↓ speed

t → time

project 2:

$$\frac{dy}{dt} = ay + b \quad ; \quad a = \text{const}, \; b = \text{cost}, \; y(0) = y_0$$

meth 1:

set $ay + b = z \Rightarrow a\,dy = dz \Rightarrow dy = \frac{1}{a} dz$

$ay(t=0) + b = z(t=0) \Rightarrow ay_0 + b = z_0 \; ! \; \exists C$

$$\frac{\frac{1}{a} dz}{dt} = z \Rightarrow \frac{dz}{dt} = a\,dz \Rightarrow z(t) = z_0 e^{at}$$

$$ay(t) + b = (ay_0 + b)e^{at} \Rightarrow ay(t) = ay_0 e^{at} + be^{at} - b$$

$$\Rightarrow y(t) = y_0 e^{at} - \frac{b}{a}(1 - e^{at})$$

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meth 2:

$$\frac{dy}{ay+b} = dt \Rightarrow \frac{1}{a}\frac{a\,dy}{ay+b} = dt \Rightarrow \frac{a\,dy}{ay+b} = a\,t$$

$$\ln(ay+b)\Big|_0^t = at\Big|_0^t \Rightarrow \ln(ay+b) - \ln(ay_0+b) = at$$

$$\ln\left(\frac{ay(t)+b}{ay_0+b}\right) = at \Rightarrow ay(t) + b = (ay_0+b) = e^{at}$$

$$\Rightarrow y(t) = y_0 e^{at} - \frac{b}{a}(1 - e^{at})$$

Meth 3:

$$\int_0^t a(s)\,ds = \int_0^t a\,ds = as\Big|_0^t = \boxed{at}$$

$$\int_s^t a(w)\,dw = \int_s^t a\,dw = aw\Big|_s^t = \boxed{at-as}$$

$$y(t) = y_0 \boxed{e^{at}} + \int_0^t \boxed{e^{at-as}}\, b\,ds$$

$$= y_0 \cancel{a} e^{at} + e^{at} b \underline{\int_0^t e^{-as}\,ds}$$

$$= y_0 e^{at} + e^{at} b \left(\frac{1}{-a} e^{-as}\Big|_0^t\right)$$

$$= y_0 e^{at} + b e^{at}\left(-\frac{1}{a} e^{-at} + \frac{1}{a}\right)$$

$$= y_0 e^{at} - \frac{b}{a}\left(1 - e^{at}\right)$$

① $\frac{dy}{dt} = ay \longrightarrow y(t) = y_0 e^{at}, \quad y_0 = y(t=0)$

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② $\frac{dy}{dt} = \underbrace{ay}_{\text{Growth}} + \underbrace{e^{st}}_{\text{time-dependant source}}, \quad y(t=0) = y_0, \quad y(t) = ?$

$$y(t) = y_n(t) + y_p(t) = C_1 e^{at} + C_2 e^{st}$$

$$\frac{dy_p}{dt} = a y_p + e^{st} : \quad s C_2 e^{st} = a C_2 e^{st} + C_2 e^{st}$$

$$e^{st} \neq 0 \Rightarrow s C_2 = a C_2 + C_2 \Rightarrow C_2 = \frac{1}{s-a}$$

$$y(t) = C_1 e^{at} + \frac{1}{s-a} e^{st}$$

$$y(0) = y_0 = C_1 + \frac{1}{s-a} \Rightarrow C_1 = y_0 - \frac{1}{s-a}$$

$$y(t) = \frac{e^{st} - e^{at}}{s-a} + y_0 e^{at} = y_p(t) + y_n(t)$$

$$\text{if } s = a \Rightarrow y(t) = t e^{at} + y_0 e^{at}$$

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③ $\frac{dy}{dt} = ay + \cos \omega t, \quad y(0) = y_0, \quad y(t) = ?$

$$y(t) = y_n(t) + y_p(t)$$

$$y_p(t) = N \cos \omega t + M \sin \omega t$$

$$\frac{dy}{dt} = y - y^3 = y(1-y^2) = f(y)$$

fixed points : $\frac{dy}{dt} = 0$

$y^* : 0 , +1 , -1$

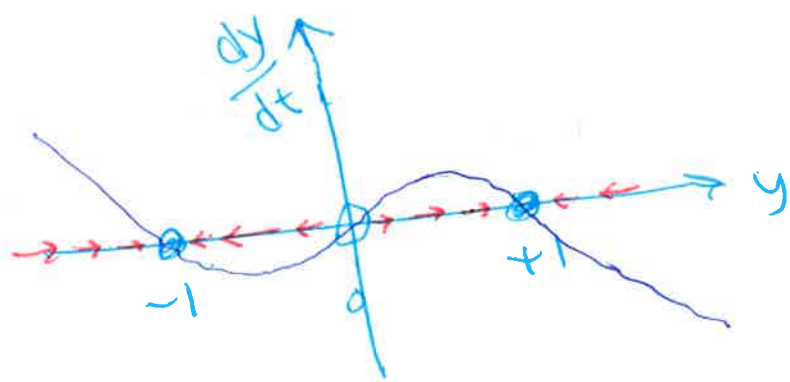
stability analysis : $f'(y^*) = ?$

$$f'(y) = \frac{df}{dy} = 1 - 3y^2$$

$y^* = 0 , \; f'(0) = 1$ unstable

$y^* = +1 , \; f'(+1) = -2$ stable

$y^* = -1 , \; f'(-1) = -2$ stable



$$\frac{dx}{dt} = x(3-x-2y) = 3x - x^2 - 2xy = f(x,y)$$

$$\frac{dy}{dt} = y(3-2x-y) = 3y - y^2 - 2xy = g(x,y)$$

Fixed points: $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$

(x^*, y^*) : $(0,0)$, $(0,3)$, $(3,0)$, $(1,1)$

Jacobian matrix: stability analysis:

$$J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}$$

$$= \begin{bmatrix} 3-2x-2y & -2x \\ -2y & 3-2y-2x \end{bmatrix}$$

$J_{(0,0)} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ unstable $(\lambda_{1,2} > 0)$ eigenvalues

$J(0,3) = \begin{bmatrix} -3 & 0 \\ -6 & -3 \end{bmatrix}$ stable $(\lambda_{1,2} < 0)$

$J(3,0) = \begin{bmatrix} -3 & -6 \\ 0 & -3 \end{bmatrix}$ stable $(\lambda_{1,2} < 0)$

$J(1,1) = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$ saddle point: unstable

$\lambda_1 * \lambda_2 < 0$