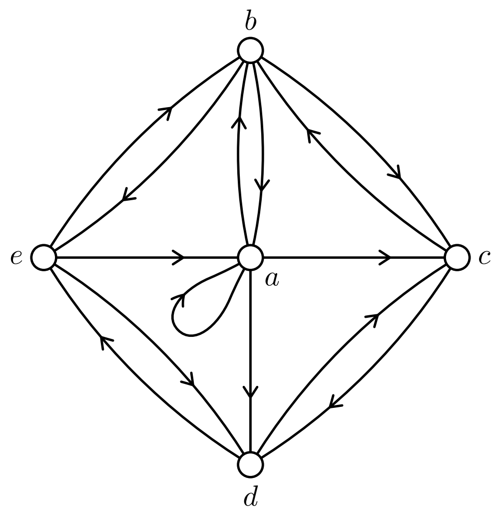
Cancer Biology 8347 - Cancer Systems Biology - Spring 2023

Solutions to problems on graph theory II

2. Consider the boolean network shown below, where every vertex has a threshold of 2. In other words, for a vertex to be 'on' at time $t+1$, it must have at least two incoming edges from vertices that are 'on' at time t.



For each of the following initial states (at time $t = 0$), construct the states at times $t = 0, 1, 2, 3, \ldots$. Show the state for each value of t by coloring in the vertices that are 'on' and leaving open the vertices that are 'off'. Label each state with '$t = 0$', '$t = 1$', and so on.

Continue until the states settle into an *attractor*. An attractor is a sequence of states (maybe just one state, or maybe several) that will repeat in a cyclic pattern. You can recognize that you have reached an attractor by the fact that you repeat a state S that you have seen before. Then the states following the first occurrence of S will be repeated over and over again.

When you recognize that you have reached an attractor, describe the cycle of states in the attractor.

Note: Copies of the network are provided on the following pages. Please use these to show your work.

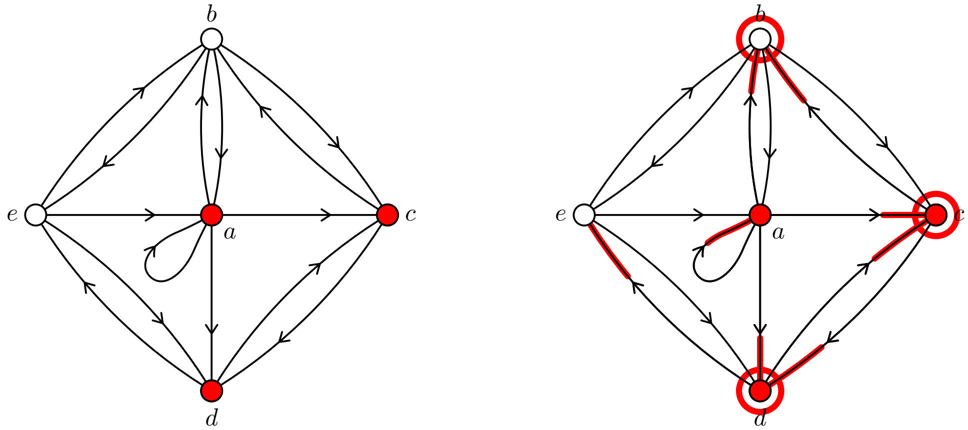
(a) Initial state at $t = 0$ has vertices a, c and d 'on', vertices b and e 'off'.

(b) Initial state at $t = 0$ has vertices a, b and c 'on', vertices d and e 'off'.

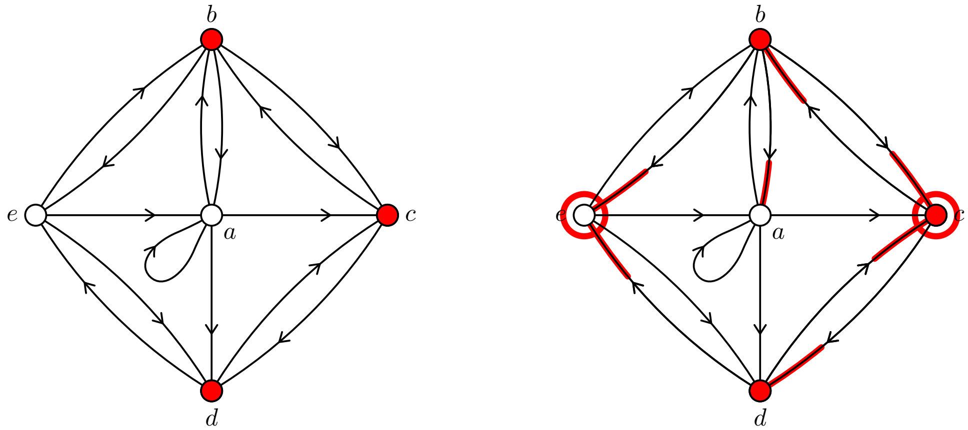
Solution to 2. At each step we show the state, and then the information leading to the next state: we highlight the head ends of edges whose tails are 'on', and circle any vertex that meets the threshold to be turned on at the next step.

(a) Starting with vertices a, c, d turned on:

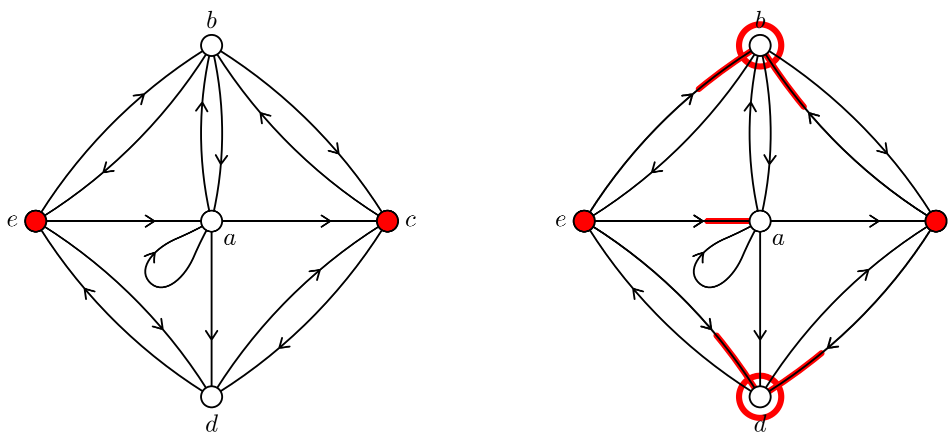
$t = 0$



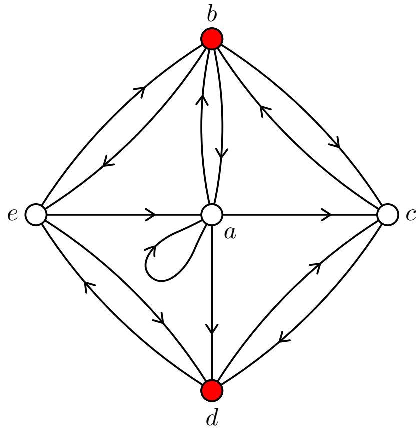
$t = 1$

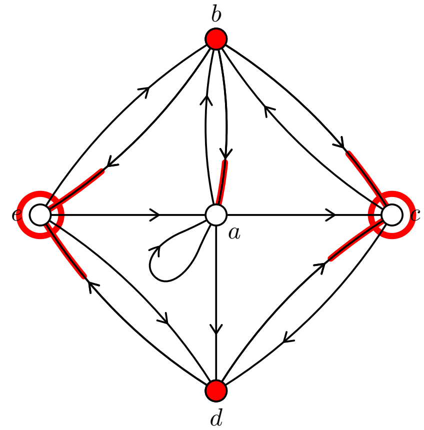


$t = 2$

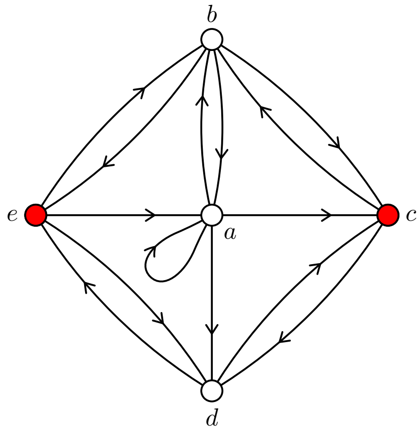


$t = 3$





$t = 4$

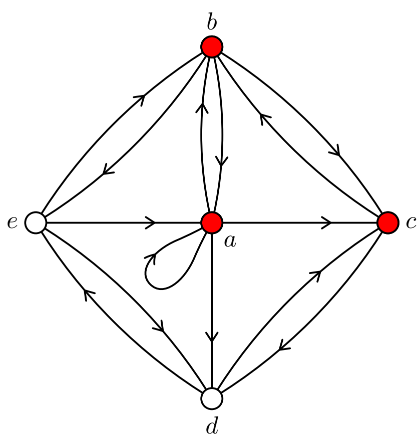


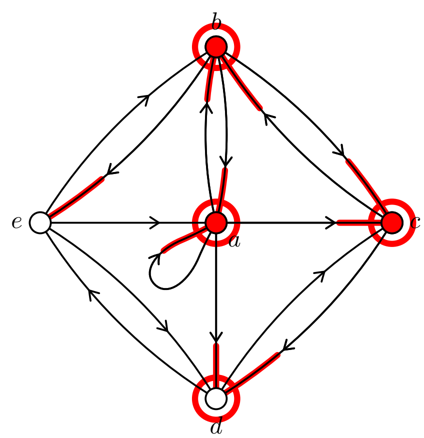
same as $t = 2$ so then we repeat ...

So the attractor is $\quad ce \to bd \to ce \quad$ (where we list the vertices that are 'on' to describe each state): a two-state attractor.

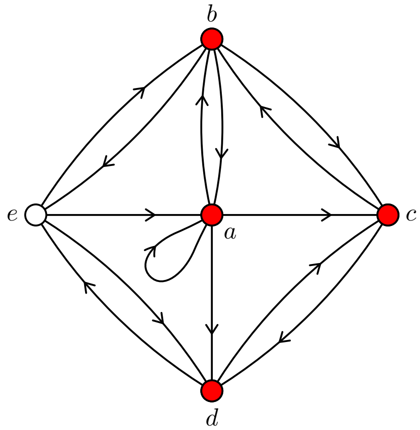
(b) Starting with vertices a, b, c turned on:

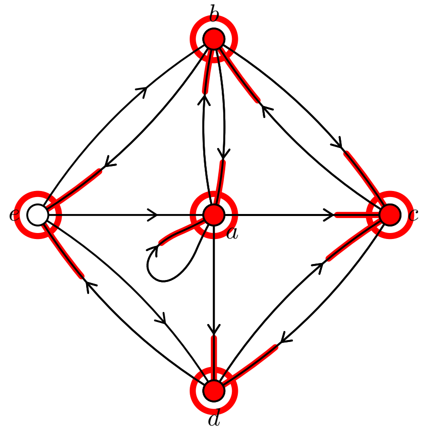
$t = 0$



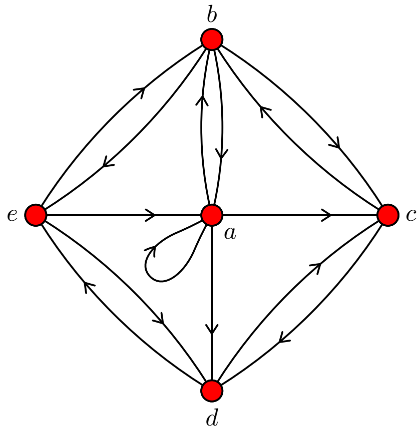


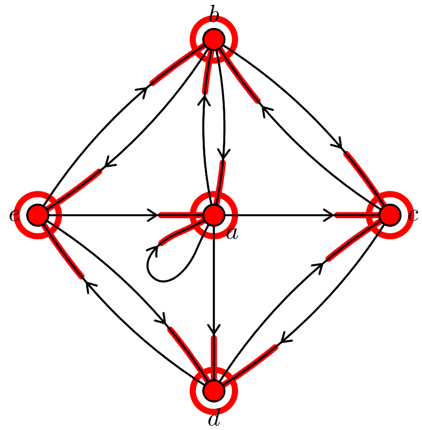
$t = 1$



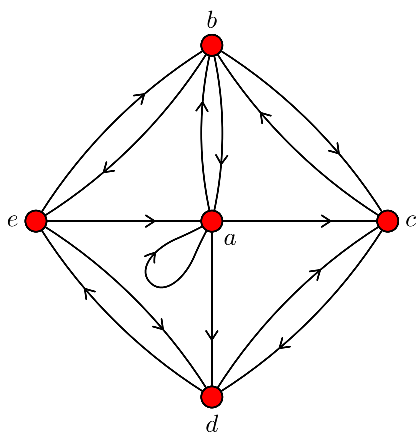


$t = 2$





$t = 3$



same as $t = 2$ so then we repeat ...

So the attractor is $abcde \to abcde$ (where we list the vertices that are 'on' to describe each state): a one-state attractor.