<u>END-SEMESTER ASSIGNMENT</u>

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Abstract

The goal of this assignment are:-

- To break the loop into parts and compute and plot the current elements in x-y plane points of the elements.
- To do vector operations and finding norm.
- To compute and plot the magnetic field B along the z-axis.
- Fit the field to a fit of the type

$$|B| = cz^b$$

Data Given To Solve

Radiation from a loop antenna of length λ.A long wire carries a current through a loop of wire.

$$I = \frac{4\pi}{\mu}\cos(\phi)\exp(j\omega t) \tag{1}$$

The wire is on the xy plane and centered at the origin. The radius of the loop(a) is 10cm. The calculation of the vector potential

$$\vec{A}(r,\phi,z) = \mu/4\pi\int\frac{I(\phi)\vec{\phi}e^{-jkR}ad\phi}{R} \tag{2}$$

$\vec{R} = \vec{r} - \vec{r'}$ and k=0.1.$\vec{r}$ is the point where we want the field, is at r_i,ϕ_j,z_k and $\vec{r'}$ is the point on the loop, $\vec{r'} = a\cos(\phi_l')\vec{x} + a\cos(\phi_l')\vec{y}$ The equation is now reduced to :

$$\vec{A_{ijk}} = \sum_{l=0}^{N-1}\frac{\cos(\phi_l')\exp(-jkR_{ijkl})dl'}{R_{ijkl}} \tag{3}$$

this is valid for any (x_i, y_j, z_k) and every current element is summed. Now finally finding the magnetic field i.e

$$\vec{B} = \nabla \times \vec{A} \tag{4}$$

Then plotting the magnetic field along the Z-axis, plot it and then fit the data to$|\vec{B}| = cz^b$.

Question-1

Pseudocode:-
1.Create a 3 by 3 by 1000 meshgrid of co-ordinates of points.
2.Obtain r_l and and I arrays and plot the current elements.
3.Obtain dl array.
4.Define a function-calc(l)
5.Compute R_{ijkl} i.e. the distance of each and every space point from lth current element. After this operation using vectorization ,we will find an array which contains the distances of each and every space point from lth current element
6.Perform vectorized operation and find $\vec{A_{ijklx}}$ and $\vec{A_{ijkly}}$ using equation.A_{ijklx} and $A_{ijkly}$are the arrays containing the x and y components of vector potential at each and every space point due to lth current element respectively.
7.calc(l) function has values of Rijkl,Ax,Ay due to lth current element.
8.Return Ax,Ay
9.Run for loop for 'N' iterations-in each of the loop calcuate x,y components of vector potential due to each current element using calc function and add to Ax,Ay.
10.B_ZMagnetic Field is found by vector operations.
11.plot the required data.
12.Use least square approach for best fit values of required variables.

Question-2

Breaking the volume into a 3 by 3 by 1000 mesh ,with mesh points seperated by 1cm. This is done by numpy function meshgrid and the python code snippet is shown below :-

```
# x is assigned 3 points from -1 to 1 seperated by 1 cm.
x=np.linspace(-1,1,3)
# y is assigned 3 points from -1 to 1 seperated by 1 cm.
y=np.linspace(-1,1,3)
# z is assigned 1000 points from 1 to 1000 seperated by 1cm.
z=np.arange(1,1001,1)
# Breaking the volume into a 3 by 3 by 1000 mesh.
```

```
X,Y,Z=np.meshgrid(x,y,z)
# rijk is numpy array that have 9000 set of points.
rijk =np.zeros((3,3,1000,3))
# x coordinate of rijk is stored in X.
rijk[:,:,:,0]=X
# y coordinate of rijk is stored in Y.
rijk[:,:,:,1]=Y
# z coordinate of rijk is stored in Z.
rijk[:,:,:,2]=Z
```

The obtained grid by the above code is used in the calculation of curl to find magnetic field.

Question-3

Breaking the loop into 100 sections and then the current vectors are plotted in the x-y plane at the points of the elements. The python code snippet is shown below:-

```
# phil is the angle made by lth part of loop with origin.
phil=np.linspace(0,2*np.pi,101)
phil=phil[:-1]
# initialising an array with 2 rows and 100 columns for current values.
I=np.zeros((2,N))
# initialising an array with 2 rows and 100 columns for position values.
r=np.zeros((2,N))
# x component of current for each element.
I[0]=-1e7*((np.cos(phil)))*(np.sin(phil))
# y component of current for each element.
I[1]=1e7*((np.cos(phil)))*(np.cos(phil))
# x component of position for each current element.
r[0]=a*(np.cos(phil))
# y component of position for each current element.
r[1]=a*(np.sin(phil))
# creating and naming the figure window
py.figure(0)
# plotting the current vectors using quiver function
py.quiver(r[0],r[1],I[0],I[1],scale=1e8,label='current vectors')
# naming x-label to the plot
py.xlabel('X $\longrightarrow$ ')
# naming y-label to the plot
py.ylabel('Y $\longrightarrow$ ')
# giving title to the plot
py.title('Current Flow Through The Loop')
```

```
# displaying square axes
py.axis('square')
# enabling grid in the axes
py.grid('True')
# describing the elements of the plot
py.legend ()
```

The plot for the current vectors at described points is shown below:

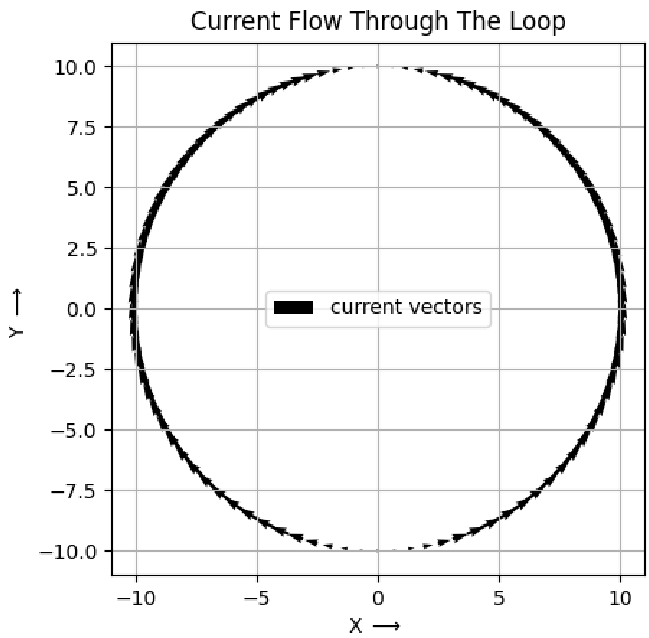

Figure 1: Current flow in the loop

The python code snippet for current elements at decribed points is shown below:-

```
# creating and naming the figure window
py.figure(1)
#plotting the current elements
py.plot(r[0],r[1],'bo',label='current elements')
# naming x-label to the plot
py.xlabel('X $\longrightarrow$ ')
# naming y-label to the plot
py.ylabel('Y $\longrightarrow$ ')
# giving title to the plot
py.title('Current Elements In The Loop')
# displaying square axes
py.axis('square')
# enabling grid in the axes
py.grid('True')
# describing the elements of the plot
py.legend()
# displaying the output plot
py.show()
```

The plot for current elements of the loop is shown below:-

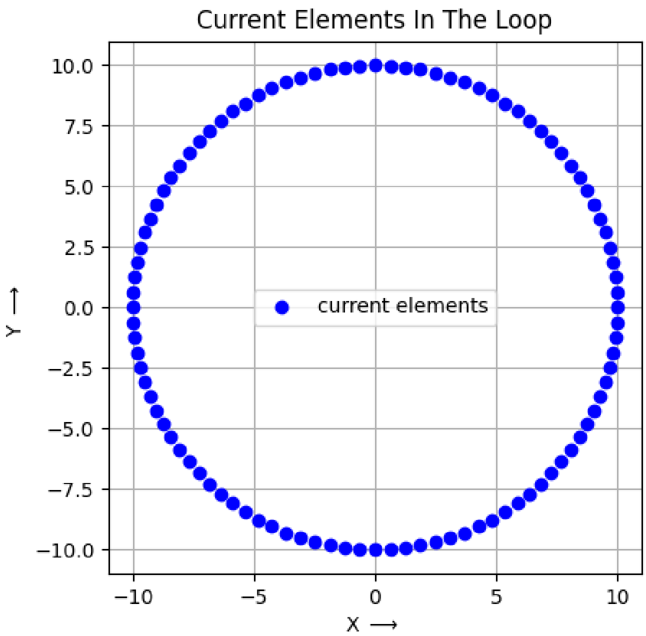

Figure 2: Current elements of the loop

Question-4

The vectors rl,dl are found corresponding to the divided sections.The mathematical equation used to determine $\vec{r_l}$ and $\vec{dl}$ is

$$\vec{r_l} = a * cos(\phi_l)\vec{x} + a * sin(\phi_l)\vec{y}$$

$$\vec{dl} = \frac{2a\pi}{N}(-\sin(\phi_l)\vec{x} + \cos(\phi_l)\vec{y})$$

The python snippet code for obtaining the above is shown below:-

```
# Initialising the dl vector with all place zeros
dl = np.zeros((2,N))
# defining the rl vector
rl=np.c_[a*(np.cos(phil)),a*(np.sin(phil)),np.zeros(N)]
# x coordinate of dl vector
dl[0] = -(2*np.pi*a/N)*np.sin(phil)
# y coordinate of dl vector
dl[1] = (2*np.pi*a/N)*np.cos(phil)
# assigning k equal to 0.1 given in question.
k=0.1
```

Question-5 and 6

We define a function that calculates

$$R_{ijkl} = |r_{ijk} - r'_l|$$

for all r_{ijk}. Then we compute the vector potential for the current and absolute value of current.

$$\vec{A_{ijk}} = \sum_{l=0}^{N-1} \frac{\cos(\phi'_l)\exp(-jkR_{ijkl})dl'}{R_{ijkl}} \tag{5}$$

$$\vec{A_{ijk}} = \sum_{l=0}^{N-1} \frac{|\cos(\phi'_l)|\exp(-jkR_{ijkl})dl'}{R_{ijkl}} \tag{6}$$

The defined function will be in vectorized form since the variables used are vectors. And then we extend the function to return the variables that are required to add to vector potential. The mathematical equations for the variables are:

$$A_{xl} = \frac{((cos(\phi_l))exp((-jk)(R_{ijkl}))(dl[0][l]))}{R_{ijkl}}$$

$$A_{yl} = \frac{((cos(\phi_l))exp((-jk)(Rijkl))(dl[1][l]))}{Rijkl}$$

The computation of Magnetic Field for absolute values of current.

$$A_{xl} = \frac{(|(cos(\phi_l)|)exp((-jk)(R_{ijkl}))(dl[0][l]))}{R_{ijkl}}$$

$$A_{yl} = \frac{(|(cos(\phi_l)|)exp((-jk)(Rijkl))(dl[1][l]))}{Rijkl}$$

So the function returns R$_{ijkl}$, A_{xl}, Ayl

The following code describe it:

```
# defining the function calc
def calc(l):
    # finding Rijkl i.e the norm rijkrl
    Rijkl=np.linalg.norm(rijk-rl[l],axis=-1)
    # Defining the x component of A where the current values are taken.
    Axl=((np.cos(phil[l]))*np.exp((-1j)*k*(Rijkl))*(dl[0][l]))/Rijkl
    # Defining the y component of A where the current values are taken.
    Ayl=((np.cos(phil[l]))*np.exp((-1j)*k*(Rijkl))*(dl[1][l]))/Rijkl
    # Defining the x component of A where absolute values of current.
    Axl1=(np.abs(np.cos(phil[l]))*np.exp((-1j)*k*(Rijkl))*(dl[0][l]))/Rijkl
    # Defining the y component of A where absolute values of current.
    Ayl1=(np.abs(np.cos(phil[l]))*np.exp((-1j)*k*(Rijkl))*(dl[1][l]))/Rijkl
    # when the function is called, then these values return.
    return Axl,Ayl,Axl1,Ayl1
```

Question-7

From the above function, Vector potential A_{ijkl} is computed. The required potential is a summation over the range,we use for the loop and add the function outputs to the potential computed in the previous loop.

The python code snippet is shown below:-

```
# assigning both the initial values of x,y components of magnetic potential to 0.
Axl=Ayl=0
# for i value ranging from 1 to N.
for i in range(N):
    # calling the calc function for every value from 1 to N.
    dxl,dyl=calc(i)
    # incrementing Axl
    Axl+=dxl
    # incrementing Ayl
    Ayl+=dyl
    # incrementing Axl1
    Axl1+=dxl1
    # incrementing Ayl1
    Ayl1+=dyl1
```

Question-8

The mathematical equation for finding the Magnetic Field(B_z) along the z-axis for both cases. The equation is:

$$B_z(z) = (Ay(\delta x, 0, z) + Ax(0, \delta y, z) - Ay(-\delta x, 0, z) + Ax(0, \delta y, z))/4\delta x\delta y$$

The python code snippet is shown below:-

```
# computing the Bz values by vectorized operation given in question
Bz=(Ayl[1,2,:]-Ayl[1,0,:]-(Axl[2,1,:]-Axl[0,1,:]))/4
# computing the Bz1 values by vectorized operation given in question.
Bz1=(Ayl1[1,2,:]-Ayl1[1,0,:]-(Axl1[2,1,:]-Axl1[0,1,:]))/4
```

Question-9

The Loglog plot for Magnetic Field(B_z) vs z is plotted for both the cases. The python code snippet for plot of Bz:

```
# creating and naming the figure window
py.figure(2)
# plotting Bz vs z in loglog plot
py.loglog(z,np.abs(Bz),label= 'Magnetic Field Bz')
```

```
# naming x-label to the plot
py.xlabel('z $\longrightarrow$ ')
# naming y-label to the plot
py.ylabel('Bz $\longrightarrow$ ')
# giving title to the plot
py.title('Magnetic Field Variation w.r.to z')
# enabling grid in the axes
py.grid('True')
# describing the elements of the plot
py.legend()
# displaying the output plot
py.show()
# creating and naming the figure window
py.figure(3)
# plotting Bz1 vs z in loglog plot
py.loglog(z,np.abs(Bz),label= 'Magnetic Field Bz')
# naming x-label to the plot
py.xlabel('z $\longrightarrow$ ')
# naming y-label to the plot
py.ylabel('Bz $\longrightarrow$ ')
# giving title to the plot
py.title('Magnetic Field Variation w.r.to z')
# enabling grid in the axes
py.grid('True')
# describing the elements of the plot
py.legend()
# displaying the output plot
py.show()
```

The plot of the Bz due to a circular loop in x-y plane is as shown:

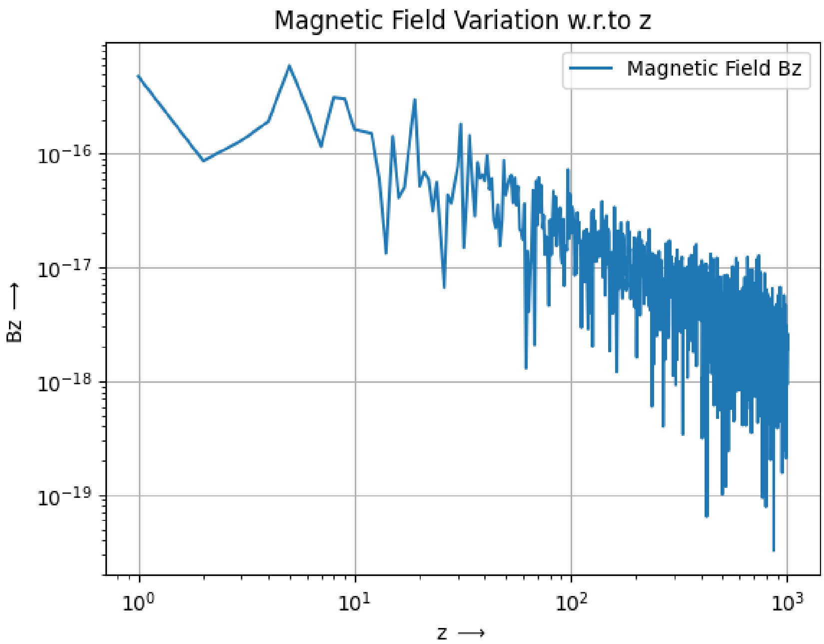

Figure 3: Magnetic Field variation with z

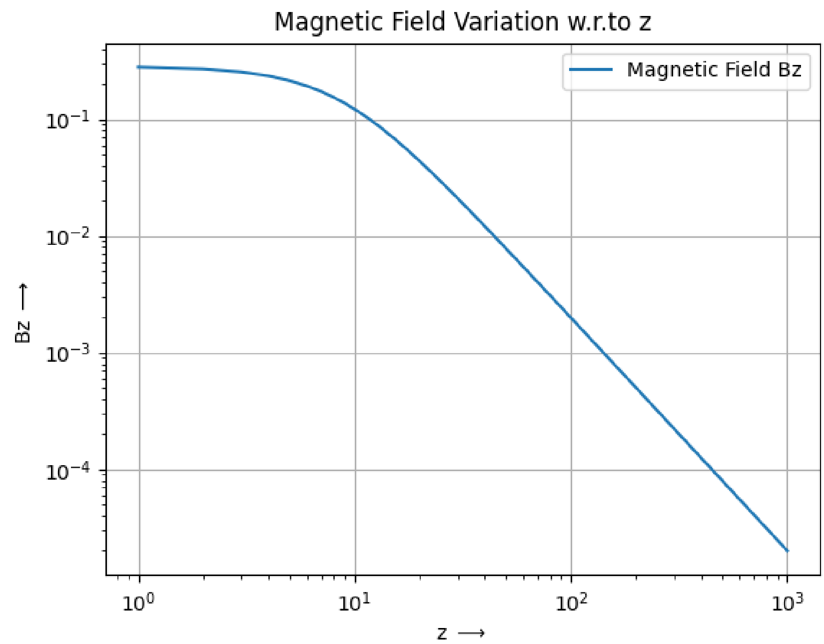

Figure 4: Magnetic Field variation with z

Question-10

Fit the obtained values of magnetic field values to

$$|B| = cz^b$$

by using least square method and print the values of c and b for both the cases. The python code snippet is shown below:-

```
# defining the estimate function to find the best fit values of c,b
def estimate(data):
    # assigning the values to the matrix.
    M=np.c_[np.ones(len(data)),np.log(np.arange(1001-len(data),1001,1))]
    #using np.linal.lstsq for obtaining best fit values
    est=np.linalg.lstsq(M,np.log(data),rcond=None)[0]
    # returning the required quantities c,b
    return np.exp(est[0]),est[1]
# calling the estimate function.
c,b = estimate(np.abs(Bz))
# printing the best fit value of c when current values are taken to compute Bz.
print('best fit value of c is :',c)
# printing the best fit value of b when current values are taken to compute Bz.
print('best fit value of b is :',b)
c,b = estimate(np.abs(Bz1))
# printing the best fit value of c abs values of current are taken to compute Bz.
print('best fit value of c is :',c)
# printing the best fit value of b abs values of current are taken to compute Bz.
print('best fit value of b is :',b)
```

The obtained values of c and b when absolute values of current are taken to compute Bz are: c=11.213894465872784 and b=-1.9057405100404816/newline
The obtained values of c and b when current values are taken to compute Bz are: c=1.3026862936464004e-15 and b=-0.9616597391492582

Question-11

- In the expression given in question, the loop and magnetic field due to current elements are symmetric about z-axis. As a result, the magnetic field on z-axis becomes zero. But we got very small values of field because we took magnetic vector potential as summation instead of integration.

- The magnetic field due to symmetrical elements adds up instead of cancelling out, by taking absolute value of current into account. As a result, there will be a non zero magnetic field along the z-axis. The plots for Bz in both cases are shown above.The decay constant obtained here is -1.9(approximately -2)

- The magnetic field varies with a decay constant of -3, as the current is constant in static case. The variation in decay constant is due to not considering exponential term in the current in magneto-statics. By fitting the data same as above, a decay constant of -2.8(approximately equal to -3) is obtained.