# Lecture 11 - Conditioning in Gaussians

## Goal: Find posterior for voltage problem

1. Isolate the important terms

(shape is only determined by dependence on s, get rid of consts)

$$p(s|v) \propto p(v|s)p(s)$$
$$= \mathcal{N}(v; s, \sigma^2)\,\mathcal{N}(s; 0, 1)$$
$$\propto \exp\left(-\frac{(v-s)^2}{2\sigma^2}\right)\exp\left(-\frac{s^2}{2}\right)$$
$$\propto \exp\left(-\frac{1+\sigma^2}{2\sigma^2}s^2 + \frac{v}{\sigma^2}s\right)$$

$$\mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

↳ All terms that depend on s

2. Notice:

$$\mathcal{N}(s; a, b) = c \cdot \exp\left(-\frac{1}{2b}x^2 + \frac{a}{b}x\right)$$

↳ Any coefficients can be obtained by <u>some</u> choice of a, b.

3. Equate coefficients (skill)

$$-\frac{1}{2b} = -\frac{1+\sigma^2}{2\sigma^2}$$

$$\therefore\ b = \frac{\sigma^2}{1+\sigma^2}$$

$$\frac{v}{\sigma^2} = \frac{a}{b} \implies a = \frac{v}{\sigma^2} b$$

$$\therefore \quad a = \frac{1}{1+\sigma^2} v$$

$$\implies p(s|v) = \mathcal{N}\left(s ; \frac{1}{1+\sigma^2} v , \frac{\sigma^2}{1+\sigma^2}\right)$$