

| АФ                | supp<br>нормировка  | Уравнение<br>преобразование Фурье<br>ряд Фурье  | Связь с другими АФ   | Ссылки |
|-------------------|---|---|--|--------|
| $\text{up}(x)$    | $[-1, 1]$<br>$\int_{-1}^1 \text{up}(x) = 1$<br>$\text{up}(0) = 1$   | $y'(x) = 2y(2x + 1) - 2y(2x - 1)$<br>$\widehat{\text{up}}(t) = \prod_{k=1}^{\infty} \text{sinc}\left(\frac{t}{2^k}\right) = \prod_{m=1}^{\infty} \cos^m(t 2^{-m-1})$<br>$\text{up}(x) = \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{\text{up}}(\pi k) \cos(\pi k x) =$<br>$= \frac{1}{2} + \sum_{k=1}^{\infty} \left( \left( \prod_{m=1}^{\infty} \cos^m\left(\frac{\pi k}{2^{m+1}}\right) \right) \cos(\pi k x) \right)$  | q  | w e    |
| $\text{fup}_n(x)$ | $\left[-\frac{n+2}{2}; \frac{n+2}{2}\right]$<br>$\int_{\text{supp}} \text{fup}_n(x) = 1$<br>$\text{fup}_n(0) = ?$ | $y'(x) = \frac{1}{2^{n-1}} \sum_{k=0}^{n+2} (C_{n+1}^k - C_{n+1}^{k-1}) y(2x + \frac{n+2}{2} - k)$<br>$\widehat{\text{fup}}_n(t) = \left(\text{sinc}\left(\frac{t}{2}\right)\right)^n \cdot \prod_{k=1}^{\infty} \text{sinc}\left(\frac{t}{2^k}\right) =$<br>$= \left(\text{sinc}\left(\frac{t}{2}\right)\right)^{n+1} \cdot \prod_{k=2}^{\infty} \text{sinc}\left(\frac{t}{2^k}\right)$<br>$\text{fup}_n = \frac{2}{n+2} \left( \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{\text{fup}}_n\left(\frac{2\pi k}{n+2}\right) \cos\left(\frac{2\pi k x}{n+2}\right) \right)$ | $\text{fup}_0(x) \equiv \text{up}(x)$<br>$\text{fup}_n(x) = 2 \text{up}(2x) * \Theta_n(x)$<br>$\text{fup}_n(x) = \text{up}(x) * \Theta_{n-1}(x)$ | w e    |
| $\text{up}_m(x)$  | $[-1, 1]$<br>$\int_{-1}^1 \text{up}_m(x) = 1$<br>$\text{up}_m(0) = 1$   | $y'(x) = 2 \sum_{k=1}^m (y(2mx + 2m - 2k + 1) - y(2mx - 2k + 1))$<br>$\widehat{\text{up}}_m(t) = \prod_{k=1}^{\infty} \frac{\sin^2\left(\frac{mt}{(2m)^k}\right)}{\frac{mt}{(2m)^k} m \sin\left(\frac{t}{(2m)^k}\right)} = \prod_{k=1}^{\infty} \frac{\text{sinc}^2\left(\frac{mt}{(2m)^k}\right)}{\text{sinc}\left(\frac{t}{(2m)^k}\right)}$<br>$\text{up}_m(x) = \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{\text{up}}_m(\pi k) \cos(\pi k x)$  | $\text{up}_1(x) \equiv \text{up}(x)$   | Старец |
| $\text{cup}(x)$   | $[-2, 2]$<br>$\int_{-2}^2 \text{cup}(x) = 1$<br>$\text{cup}(0) = \int_{-1}^1 \text{up}^2(x) dx$                   | $y''(x) = 2y(2x + 2) - 4y(2x) + 2y(2x - 2)$<br>$\widehat{\text{cup}}(t) = \prod_{k=1}^{\infty} \text{sinc}^2\left(\frac{t}{2^k}\right) = \prod_{m=1}^{\infty} \cos^{2m}(t 2^{-m-1})$<br>$\text{cup}(x) = \frac{1}{2} \left( \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{\text{cup}}\left(\frac{\pi k}{2}\right) \cos\left(\frac{\pi k x}{2}\right) \right)$  | $\text{cup}(x) = \text{up}(x) * \text{up}(x)$  | w e    |

| АФ  | Нормировка  | ФДУ ряд Фурье   | Связь с другими АФ   | Ссылки |
|---|---|---|--|--------|
| $h_a(x)$  | $\left[-\frac{1}{a-1}, \frac{1}{a-1}\right]$<br>$\int_{\text{supp}} h_a(x) = 1$<br>$h_a(0) = \frac{a}{2}, \quad a \geq 2$ | $y'(x) = \frac{a^2}{2} (y(ax+1) - y(ax-1))$<br>$\widehat{h}_a(t) = \prod_{k=1}^{\infty} \text{sinc}\left(\frac{t}{a^k}\right)$<br>$h_a(x) = (a-1) \left( \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{h}_a((a-1)\pi k) \cos((a-1)\pi kx) \right)$   | $h_2(x) \equiv \text{up}(x)$   | [1]    |
| $\Xi_n(x)$  | $[-1, 1]$<br>$\int_{-1}^1 \Xi_n(x) = 1$   | $y^{(n)}(x) = (n+1)^{n+1} 2^{-n} \sum_{k=0}^n C_n^k (-1)^k y((n+1)x + n - 2k)$<br>$\widehat{\Xi}_n(t) = \prod_{k=1}^{\infty} \text{sinc}^n\left(\frac{t}{(n+1)^k}\right)$<br>$\Xi_n(x) = \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{\Xi}_n(\pi k) \cos(\pi kx)$   | $\Xi_1 \equiv \text{up}(x)$<br>$\Xi_n = \underbrace{h_{n+1} * \dots * h_{n+1}}_n$  | [1]    |
| $ch_{a,n}$  | $\left[-\frac{n}{a-1}, \frac{n}{a-1}\right]$<br>$\int_{-n/(a-1)}^{n/(a-1)} ch_{a,n}(x) = 1$                               | $y^{(n)}(x) = a^{n+1} 2^{-n} \sum_{k=0}^n C_n^k (-1)^k y(ax + n - 2k)$<br>$\widehat{ch_{a,n}}(t) = \prod_{k=1}^{\infty} \text{sinc}^n\left(\frac{t}{a^k}\right)$<br>$ch_{a,n}(x) = \frac{a-1}{n} \left( \frac{1}{2} + \sum_{k=1}^{\infty} \widehat{ch_{a,n}}\left(\frac{a-1}{n}\pi k\right) \cos\left(\frac{a-1}{n}\pi kx\right) \right)$                           | $ch_{a,n} = \underbrace{h_a * \dots * h_a}_n$<br>$ch_{2,1}(x) = \text{up}(x)$<br>$ch_{2,2}(x) = \text{cup}(x)$<br>$ch_{a,1}(x) = h_a$<br>$ch_{n+1,n}(x) = \Xi_n$ | w e    |
| Функции, используемые при построении атомарных  |   |   |  |        |
| Прямоугольный импульс<br>$\varphi(x) = \begin{cases} 1 &  x  \leq \frac{1}{2} \\ 0 &  x  > \frac{1}{2} \end{cases}$ |   | $\hat{\varphi}(t) = \text{sinc}\left(\frac{t}{2}\right)$<br>$\text{supp}(\varphi) = \left[-\frac{1}{2}; \frac{1}{2}\right] \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} \varphi(x) dx = 1$<br>$\varphi(x) = 2 \left( \frac{1}{2} + \sum_{k=1}^{\infty} \text{sinc}(2\pi k) \cos(2\pi kx) \right) \equiv 1$   | q  | w e    |
| B-сплайн<br>$\Theta_n = \underbrace{\varphi * \varphi * \dots * \varphi}_{n+1}$                                     |   | $\widehat{\Theta}_n = \text{sinc}^{n+1}\left(\frac{t}{2}\right)$<br>$\text{supp}(\Theta_n) = \left[-\frac{n+1}{2}; \frac{n+1}{2}\right] \quad \int_{\text{supp } \Theta_n} \Theta_n(x) dx = 1$<br>$\Theta_n = \frac{2}{n+1} \left( \frac{1}{2} + \sum_{k=1}^{\infty} \text{sinc}^{n+1}\left(\frac{2\pi k}{n+1}\right) \cos\left(\frac{2\pi k}{n+1}x\right) \right)$ | q  | w e    |

## Список литературы

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