DATE 31 | 12 | 2024

MATRICES

⇒ A matrix is a rectangular array of numbers/symbols/expressions arranged in rows and columns.

Application of matrices:-

(i) Data representation

suppose there is a dataset, with features

	math-score	physics-score	biology-score	
(1)	55	65	75	→ Rows
(2)	65	60	70	→ Rows
(3)	70	60	72	→ Rows
	↓ columns	↓ columns	↓ columns	

This can be represented as

$$\underbrace{\begin{matrix}[55 & 65 & 75] \\ [65 & 60 & 70] \\ [70 & 60 & 72]\end{matrix}}_{\text{A set of vectors}} \xrightarrow{\text{combine}} \underbrace{\begin{bmatrix}55 & 65 & 75 \\ 65 & 60 & 70 \\ 70 & 60 & 72\end{bmatrix}}_{\text{matrix}}$$

matrix is represented as a set of vectors

(ii) Representing images in computer vision

Suppose there is a 3x3 gray scale image with pixel values

Image =

0	128	120
123	122	115
0	118	100

3x3 gray scale image pixels

$\xrightarrow{\text{can be represented as}}$

$$\begin{bmatrix}0 & 128 & 120 \\ 123 & 122 & 115 \\ 0 & 118 & 100\end{bmatrix}$$

3x3 matrix

(iii) Calculating accuracy of a model:-

To calculate accuracy of the model, we use confusion matrix.

feature1 → model → $\hat{y}$ ⇒ predicted output

feature2 → y ⇒ Actual output

we can find difference between predicted and actual outputs and create a confusion matrix to find accuracy of model.

$$\text{confusion matrix} = \begin{bmatrix} 50 & 10 \\ 5 & 35 \end{bmatrix}$$

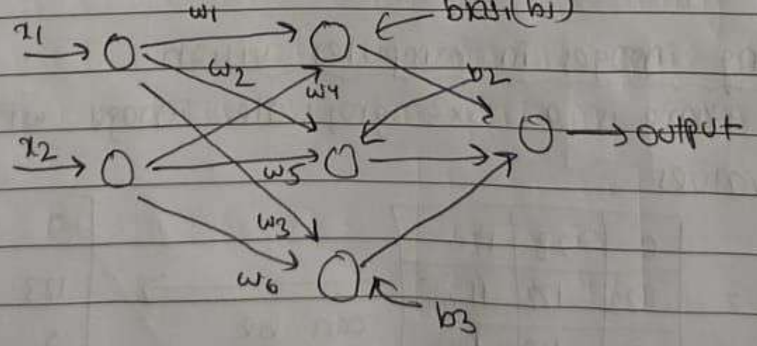
50 ⇒ True positive 5 ⇒ False positive

10 ⇒ False negative 35 ⇒ True negative

$$\boxed{\text{Accuracy} = \frac{\text{True positive} + \text{True negative}}{\text{False positive} + \text{False negative} + \text{True positive} + \text{True negative}}}$$

(iv) Neural networks:-

If forward propogation, we perform matrix operations



$$z = W^T x + b = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$Z = \begin{bmatrix} w_1 & w_4 \\ w_2 & w_5 \\ w_3 & w_6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

So, we use matrix multiplication and matrix addition operations in neural networks

(v) Linear regression:-

suppose, for a dataset,

no. of study hours	IQ	score
4	100	90
5	90	85
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no. of study hours, IQ } Independent feature

score } Dependent / output feature

This is an example of regression

$x_1 \Rightarrow$ no of study hours

$x_2 \Rightarrow$ IQ

The equation will be,

$$y = m_1x_1 + m_2x_2 + c$$

It can be presented as

$\boxed{y = m^T x + c}$, where $m = [m_1 \ m_2]$, $x = [x_1 \ x_2]$

we use matrix multiplication and matrix addition operations in regression.

MATRIX OPERATIONS

⇒ matrix operations help to manipulate and analyze multidimensional data efficiently.
some matrix operations are:-
(i) matrix addition and subtraction
(ii) scalar matrix multiplication
(iii) matrix multiplication.

⇒ Matrix addition and subtraction:-
We add or substract corresponding elements of 2 matrices of same dimensions.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{bmatrix}_{3\times3}$$

$$A+B = \begin{bmatrix} 1+4 & 2+5 & 3+6 \\ 4+7 & 5+8 & 6+9 \\ 7+1 & 8+2 & 9+3 \end{bmatrix} = \begin{bmatrix} 5 & 7 & 9 \\ 11 & 13 & 15 \\ 8 & 10 & 12 \end{bmatrix}$$

⇒ Scalar multiplication:-
It involves multiplying every element of a matrix by scalar.
suppose we have a matrix representing product prices and if we want to increase price of all products by 1.05 times, we can perform scalar multiplication

$$A(\text{original prices}) = \begin{bmatrix} 10 & 20 & 30 \\ 15 & 25 & 35 \\ 20 & 30 & 40 \end{bmatrix}$$

Inflation rate = 1.05

$$B(\text{adjusted}) = A \times \text{inflation rate} = 1.05 \begin{bmatrix} 10 & 20 & 30 \\ 15 & 25 & 35 \\ 20 & 30 & 40 \end{bmatrix} = \begin{bmatrix} 10.5 & 21 & 31.5 \\ 15.75 & \dots & \\ \to & \dots & \end{bmatrix}$$

DATE 31 12 2024

B(adjusted price) = Inflation price · A(original price)

$$= 1.05 \begin{bmatrix} 10 & 20 & 30 \\ 15 & 25 & 35 \\ 20 & 30 & 40 \end{bmatrix} = \begin{bmatrix} 10.5 & 21 & 31.5 \\ 15.75 & 26.25 & 36.75 \\ 21 & 31.5 & 42 \end{bmatrix}$$

If we want to increase/decrease all elements by a factor, then we can achieve it through ~~matrix~~ scalar multiplication.

→ Matrix multiplication

It involves dot product of rows of the first matrix with columns of the second matrix.

$$A_{(m \times n)} \times B_{(n \times p)} = C_{(m \times p)}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$

$$A \times B = A_{(2\times3)} \times B_{(3\times2)} = C_{(2\times2)}$$

$$C = A \times B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 1\cdot7+2\cdot9+3\cdot11 & 1\cdot8+2\cdot10+3\cdot12 \\ 4\cdot7+5\cdot9+6\cdot11 & 4\cdot8+5\cdot10+6\cdot12 \end{bmatrix}$$

$$= \begin{bmatrix} 7+18+33 & 8+20+36 \\ 28+45+66 & 32+50+72 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}_{(2\times2)}$$