Вариант 29 $\in [0,2]$ $h=0,5$ Метод Эйлера

$y' = 0,2y + 0,4x^2 + 1,2$

$y(0) = 1 \Rightarrow x_0 = 0$

$\underline{x_0 = 0 \quad y_0 = 1}$

$f(x_0, y_0) = f(0,1) = 0,2 + 1,2 = 1,4$

$hf(x_0, y_0) = 0,5 \cdot 1,4 = 0,7$

$\underline{x_1 = 0,5}$ $y_1 = y_0 + hf(x_0, y_0) = 1 + 0,7 = 1,7$

$f(x, y) = f(0,5; 1,7) = 0,2 \cdot 1,7 + 0,4 \cdot (0,5)^2 + 1,2 =$

$= 1,64$; $hf(x_1, y_1) = 0,5 \cdot (1,64) = 0,82$

$\underline{x_2 = 1}$ $y_2 = y_1 + hf(x_1, y_1) = 1,7 + 0,82 = 2,52$

$A(x_2, y_2) = f(1; 2,52) = 0,2 \cdot 2,52 + 0,4 \cdot (1)^2 + 1,2 =$

$= 2,104$; $hf(x_2, y_2) = 0,5 \cdot (2,104) = 1,052$

$\underline{x_3 = 1,5}$ $y_3 = y_2 + hf(x_2, y_2) = 2,52 + 1,052 =$

$= 3,572$, $f(x_3, y_3) = f(1,5; 3,572) = 0,2 \cdot 3,572 +$

$+ 0,4 \cdot (1,5)^2 + 1,2 = 2,8144$; $hf(x_3, y_3) = 0,5 \cdot 3,572 =$

$= 1,786$

$x_4 = 2$ $y_4 = y_3 + hf(x_3, y_3) = 3,572 + 1,786 = 5,358$

$f(x_4, y_4) = 0,2 \cdot 5,358 + 0,4 \cdot (2)^2 + 1,2 = 3,8716$

$hf(x_4, y_4) = 0,5 \cdot 3,8716 = 1,9358$

Усовершенствованный метод Эйлера

$y_{i+1} = y_i + \Delta y_i$ где $\Delta y_i = hf\left(x_i + \frac{h}{2}, y_i + \frac{h}{2} f(x_i, y_i)\right)$

$y' = 0{,}2y + 0{,}4x^2 + 1{,}2 \qquad h = 0{,}5$

$x_0 = 0 \quad y_0 = 1$

$f(x_0, y_0) = 0{,}2 + 1{,}2 = 1{,}4, \quad hf(x_0, y_0) = 0{,}5 \cdot 1{,}4 = 0{,}7$

$x_1 = 0{,}5$

$y_1 = y_0 + hf(0{,}25; 1{,}35) = 1 + 0{,}5(0{,}2 \cdot 1{,}35 + 0{,}4 \cdot (0{,}25)^2 + 1{,}2 = 1{,}185; \quad f(0{,}5; 1{,}185) = 0{,}2 \cdot 1{,}185 + 0{,}4 \cdot (0{,}5)^2 + 1{,}2 = 1{,}527$

~~$hf(x_1, y_1) = 0{,}5 \cdot 1{,}527 = 0{,}7635$;~~

~~$x_2 = 1$~~

~~$y_2 = y_1 + hf(x_1, y_1) = 1{,}185 + 0{,}7635 = 1{,}8985$~~

~~$f(x_2; y_2) = 0{,}2 \cdot 1{,}8985 + 0{,}4 \cdot 1^2 + 1{,}2 = 1{,}9797$~~

~~$hf(x_2, y_2) = 0{,}5 \cdot 1{,}9797 = 0{,}98985$~~

~~$x_3 = 1{,}5$~~

~~$y_3 = y_2 + hf(x_2, y_2) = 1{,}8985 + 0{,}98985 = 2{,}88835$~~

~~$f(x_3; y_3) = 0{,}2 \cdot 2{,}88835 + 0{,}4 \cdot (1{,}5)^2 + 1{,}2 = 2{,}67767$~~

~~$hf(x_3, y_3) = 0{,}5 \cdot 2{,}67767 = 1{,}338835$~~

~~$x_4 = 2; \quad y_4 = y_3 + hf(x_3, y_3) = 2{,}88835 + 1{,}338835$~~

$x_2 = 1$

$y_2 = y_1 + hf(0{,}75;\ 1{,}51675) = 1{,}135 + 0{,}5(0{,}2 \cdot 1{,}51676 + 0{,}4 \cdot (0{,}75)^2 + 1{,}2 = 1{,}999175$

$f(x_2, y_2) = f(1;\ 1{,}999175) = 0{,}2 \cdot 1{,}999175 + 0{,}4 \cdot 1^2 + 1{,}2 = 1{,}999835$

$x_3 = 1{,}5$

$y_3 = y_2 + hf(1{,}25;\ 2{,}9990925) = 1{,}999175 + 0{,}5(0{,}2 \cdot 2{,}9990925 + 0{,}4 \cdot (1{,}25)^2 + 1{,}2 = 3{,}211\,58425$

$f(1{,}5;\ 3{,}211\,58425) = 0{,}2 \cdot 3{,}211\,58425 + 0{,}4 \cdot 1{,}5^2 + 1{,}2 = 2{,}742\,31685$

$x_4 = 2$

$y_4 = y_3 + hf(1{,}75;\ 4{,}582\,742\,675) = 3{,}211\,58425 + 0{,}5(0{,}2 \cdot 4{,}582\,742\,675 + 0{,}4 \cdot (1{,}75)^2 + 1{,}2) = 4{,}88936$

$f(x_4, y_4) = f(2;\ 4{,}40786) = 0{,}2 \cdot 4{,}88936 + 0{,}4 \cdot 2^2 + 1{,}2 = 3{,}776472$

Метод Рунге-Кутта III порядка

$y' = 0{,}2y + 0{,}4x^2 + 1{,}2 \quad x \in [0;2] \quad h = 0{,}5$

$y_0 = 1 \quad x_0 = 0$

$x_1 = 0{,}5$; $k_1 = f(x_0; y_0) = 0{,}2 \cdot 1 + 0{,}4 \cdot 0^2 + 1{,}2 = 1{,}4$

$k_2 = f\left(x_0 + \frac{h}{2}; y_0 + \frac{hk_1}{2}\right) = f(0{,}25; 1{,}35) = 0{,}2 \cdot 1{,}35 + 0{,}4 \cdot 0{,}25^2 + 1{,}2 = 1{,}495$

$k_3 = f(x_0 + 0{,}5; y_0 - h \cdot k_1 + 2 \cdot h \cdot k_2) = f(0{,}5; 1{,}795) =$

$= 0{,}2 \cdot 1{,}795 + 0{,}4 \cdot 0{,}5^2 + 1{,}2 = 1{,}659$

$y_1 = y_0 + \frac{h}{2} \cdot (k_1 + 4k_2 + k_3) = 3{,}25975$

$x_2 = 1 \quad k_1 = f(x_1; y_2) = 0{,}2 \cdot 3{,}25975 + 0{,}4 \cdot 0{,}5^2 + 1{,}2 =$

$= 2{,}0195$; $k_2 = f(0{,}75; 3{,}764625) = 0{,}2 \cdot 3{,}764625 +$

$+ 0{,}4 \cdot (0{,}75)^2 + 1{,}2 = 2{,}177925$; $k_3 = f(1; 4{,}397925) =$

$= 0{,}2 \cdot 4{,}397925 + 0{,}4 \cdot 1^2 + 1{,}2 = 2{,}479585$

$y_2 = 3{,}25975 + \frac{0{,}5}{6} \cdot (2{,}0195 + 4 \cdot 2{,}177925 + 2{,}479585) =$

$= 6{,}56244625$

$x_3 = 1{,}5 \quad k_1 = f(x_2; y_2) = f(1; 6{,}56244625) = 0{,}2 \cdot 6{,}56244625 +$

$+ 0{,}4 \cdot 1^2 + 1{,}2 = 2{,}912489225$; $k_2 = f(1{,}25; 7{,}290568625) =$

$= 0{,}2 \cdot 7{,}290568625 + 0{,}4 \cdot 1{,}25^2 + 1{,}2 = 3{,}283113771125$

$k_3 = f(1{,}75; 8{,}389315353375) = 0{,}2 \cdot 8{,}389315353375 + 0{,}4 \cdot 1{,}75^2 + 1{,}2 =$

$= 4{,}102863067675$

$y_3 = 6{,}562446 + \frac{0{,}5}{6} \cdot (k_1 + 4k_2 + k_3) = 8{,}20345$

$x_4 = 2$

$k_1 = f(x_3, y_3) = f(1{,}5;\ 8{,}20345) = 0{,}2 \cdot 8{,}20345 + 0{,}4(1{,}5)^2 + 1{,}2 =$

$= 3{,}74069$; $k_2 = f(1{,}75;\ 9{,}1386225) = 0{,}2 \cdot 9{,}1386225 +$

$+ 0{,}4 \cdot (1{,}75)^2 + 1{,}2 = 4{,}2527245$

$k_3 = f(2;\ 10{,}5858295) = 0{,}2 \cdot 10{,}5858295 + 0{,}4 \cdot 4 + 1{,}2 =$

$= 4{,}9051179$

$y_4 = 8{,}20345 + \frac{0{,}5}{6} \cdot (3{,}74069 + 4 \cdot 4{,}2527245 + 4{,}9051179) =$

$= 10{,}941151$

Метод Рунге-Кутта IV порядка

$y' = 0{,}2y + 0{,}4x^2 + 1{,}2$ $y(0) = 1$ $y_0 = 1$ $x_0 = 0$

$y_{m+1} = y_m + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = f(x_m, y_m)$

$k_2 = f(x_m + \frac{h}{2}, y_m + \frac{hk_1}{2})$

$k_3 = f(x_m + \frac{h}{2}, y_m + \frac{hk_2}{2})$

$k_4 = f(x_m + h, y_m + hk_3)$

$x_1 = 0{,}5$

$y_1 = 1{,}37664$

$k1 = 0{,}7$

$k2 = 0{,}7775$

$k3 = 0{,}749875$

$k4 = 0.829988$

$x_2 = 1$

$k1 = 0.787664$

$k2 = 0.889548$

$k3 = 0.894692$

$k4 = 1.02713$

$y_2 = 1.82524$

$x_3 = 1{,}5$

$k_1 = 0{,}982524$

$k_2 = 1.14415$

$k_3 = 1.15223$

$k_4 = 1.34775$

$y_3 = 2.40216$

$x_3 = 2$

$k_1 = 1.29022$

$k_2 = 1.51723$

$k_3 = 1.52858$

$k_4 = 1.79307$

$y_4 = 3.16674$