Вариант 29

① $2x\,dx - y\,dy = x^2y\,dy - xy^2dx$

$$y^2 = C(x^2+1)-2$$

$2x\,dx - y\,dy = x^2y\,dy - xy^2dx$

$(-x^2y-y)\,dy = (-xy^2-2x)\,dx$

$(-x^2-1)\,y\,dy = x(-y^2-2)\,dx$

$-\dfrac{y\,dy}{y^2+2} = -\dfrac{x\,dx}{x^2+1}$

$\int -\dfrac{y}{y^2+2}\,dy = \int -\dfrac{x}{x^2+1}\,dx$

$\ln(y^2+2) = \ln(x^2+1)+C$

$y^2+2 = e^C(x^2+1)$

$y^2 = C(x^2+1)-2$

$$\begin{cases} u+2 \to u+2=0 \to \frac{y}{x}+2=0 \to x=-\frac{y}{2} \\ u+5 \to u+5=0 \to \frac{y}{x}+5=0 \to x=-\frac{y}{5} \end{cases}$$

$$\int \frac{1}{u^2+7u+10}\,du = \int \frac{1}{3x}\,dx$$

$$\ln(u+2) - \ln(u+5) = \ln(x) + C$$

$$\frac{u+2}{u+5} = e^{C} x$$ Обратная замена

$$\frac{\frac{y}{x}+2}{\frac{y}{x}+5} = Cx$$

$$y = \frac{2x - 5Cx^2}{Cx-1}$$

$x = -\frac{y}{2}$ при $C=0$

$x = -\frac{y}{5}$ при $C=\infty$

③ $y' = \dfrac{6y-6}{4y+5x-9}$

$(y-1)^5 = C(x-1)^6\left(\dfrac{4(y-1)}{x-1} - 1\right)^6, \; x = 4y-3$

$y' = \dfrac{6y-6}{4y+5x-9}$

$\begin{cases} 4y+5x-9=0 \\ 6y-6=0 \end{cases} = \begin{cases} x=1 \\ y=1 \end{cases}$

$y = y_1 + 1 \quad | \quad x = x_1 + 1$

$y' = y_1'$

$y_1' = \dfrac{6y_1}{4y_1+5x_1}$

$\dfrac{dy_1}{dx_1} = \dfrac{6y_1}{4y_1+5x_1}$

$dy_1 = \dfrac{6y_1 dx_1}{4y_1+5x_1}$

$1 = \dfrac{6ky_1}{4ky_1+5kx_1}$

$$u = \frac{y_1}{x_1} \quad \begin{array}{l} y_1 = u x_1 \\ dy_1 = u\,dx_1 + x_1\,du \end{array}$$

$$u\,dx_1 + x_1\,du = \frac{6u\,dx_1}{4u+5}$$

$$x_1\,du = \left(\frac{6u}{4u+5} - u\right) dx_1$$

$$-\frac{(4u+5)\,du}{4u^2-u} = \frac{dx_1}{x_1}$$

$$\begin{cases} u - \frac{1}{4} \to u - \frac{1}{4} = 0 \to \frac{y-1}{x-1} - \frac{1}{4} = 0 \to x = 4y-3 \\ u \to u = 0 \to \frac{y-1}{x-1} = 0 \to y = 1 \end{cases}$$

$$\int -\frac{4u+5}{4u^2-u}\,du = \int \frac{1}{x_1}\,dx_1$$

$$5\ln\left(\frac{u}{4u-1}\right) - \ln(4u-1) = \ln(x_1) + C$$

$$\frac{u^5}{(4u-1)^6} = e^C x_1$$

Обратная замена

$$u = \frac{y_1}{x_1}$$

$$\frac{y_1^5}{x_1^5\left(\frac{4y_1}{x_1}-1\right)^6}=Cx_1$$

Обратная замена

$y_1 = y - 1 \mid x_1 = x - 1$

$$\frac{(y-1)^5}{(x-1)^5\left(\frac{4(y-1)}{x-1}-1\right)^6}=C(x-1)$$

$$(y-1)^5 = C(x-1)^6\left(\frac{4(y-1)}{x-1}-1\right)^6 \quad x=4y-3$$

(4)

$$y' - 3x^2 y = \frac{x^2(x^3+1)}{3}$$

$$y = Ce^{x^3} - \frac{x^3}{9} - \frac{2}{9}$$

$$y' - 3x^2 y = \frac{x^2(x^3+1)}{3}$$

$$3y' - 9x^2 y = x^2(x^3+1)$$

$$y' - 3x^2 y = \frac{x^5}{3} + \frac{x^2}{3}$$

$$y' + a(x) y = b(x)$$

где $a = -3x^2$ ; $b = \frac{x^5}{3} + \frac{x^2}{3}$

$$y = uv$$

$$y' = uv' + u'v - 3uvx^2 = \frac{x^5}{3} + \frac{x^2}{3}$$

$$u'v + u(v' - 3vx^2) = \frac{x^5}{3} + \frac{x^3}{3}$$

$$v' - 3vx^2 = 0$$

$$v' = 3vx^2$$

$$\frac{dv}{dx} = 3vx^2$$

$$dv = 3vx^2 dx$$

$$\frac{dv}{v} = 3x^2 dx$$

$$\int \frac{1}{v}\,dv = \int 3x^2\,dx$$

$$\ln(v) = x^3$$

$$v = e^{x^3}$$

$$u'v + u(v' - 3vx^2) = \frac{x^5}{3} + \frac{x^2}{3}$$

$$v = e^{x^3} \quad u\,(v' - 3vx^2) = 0$$

$$u'e^{x^3} = \frac{x^5}{3} + \frac{x^2}{3}$$

$$3u'e^{x^3} = x^5 + x^2$$

$$u' = \frac{x^5 + x^2}{3e^{x^3}}$$

$$\frac{du}{dx} = \frac{x^5 + x^2}{3e^{x^3}}$$

$$du = \frac{(x^5 + x^2)\,dx}{3e^{x^3}}$$

$$\int 1\,du = \int \frac{x^5 + x^2}{3e^{x^3}}\,dx$$

$$u = C - \frac{x^3 + 2}{9e^{x^3}} \qquad y = \frac{y}{v} \,\Big|\, v = e^{x^3}$$

$$\cancel{u = C - \frac{x^3+3}{}} \qquad y = \frac{9Ce^{x^3} - x^3 - 2}{9}$$

$$y = Ce^{x^3} - \frac{x^3}{9} - \frac{2}{9} \quad \text{при} \quad \begin{matrix} x = 0 \\ y = 0 \end{matrix}$$

$$0 = C - \frac{2}{9} \rightarrow C = \frac{2}{9}$$

$$y = \frac{1}{\frac{1}{2} + C} \to 1 + 2C = 1 \to C = 0$$

Ответ $y = \frac{1}{\frac{1}{2 \cdot e^{-x}}} = 2 \cdot e^{x}$

⑥ $(3x^2y + 2x^3)dy + (3xy^2 + 6x^2y + 3x^3)dx = 0$

$\cancel{x^2 = C(6x^4y^2 + 8x^5y + 3x^6)}$

$(3x^2y + 2x^3)dy + (3xy^2 + 6x^2y + 3x^3)dx = 0$

$3y^2dx + x(3ydy + 6ydx) + x^2(2dy + 3dx) = 0$

$(3xy + 2x^2)dy = (-3y^2 - 6xy - 3x^2)dx$

$3k^2xy + 2k^2x^2 = -3k^2y^2 - 6k^2xy - 3k^2x^2 \to k^2$

$u = \frac{y}{x}$ | $y = ux$

$dy = udx + xdu$

$(3u+2)x^2(udx + xdu) = (-3u^2 - 6u - 3)x^2dx$

$(3u+2)(udx + xdu) = (-3u^2 - 6u - 3)dx$

$(3ux + 2x)du = (-6u^2 - 8u - 3)dx$

$(3u+2)xdu = (-6u^2 - 8u - 3)dx$

$$-\frac{(3u+2)du}{6u^2 + 8u + 3} = \frac{dx}{x}$$

$$\int -\frac{3u+2}{6u^2 + 8u + 3}du = \int \frac{1}{x}dx$$

$$y = \frac{2e^{x^3}}{9} - \frac{x^3}{9} - \frac{2}{9}$$

$$y = \frac{2e^{x^3}}{9} - \frac{x^3}{9} - \frac{2}{9} \quad x_0 = 0;\ y_0(0) = 0$$

5) $2y' + 2xy = (1+x)\cdot e^{-x}\cdot y^2 \quad (:y^2) \to$

$2y^{-2}\cdot y' + 2x\cdot y^{-1} = (x+1)\cdot e^{-x} \quad t = y^{-1} \ t' = -y^{-2}\cdot y' \to$

$y^{-2}\cdot y' = -t' \to$

$-2t' + 2xt = (x+1)e^{-x}$

$t' - xt = -\frac{1}{2}(x+1)\cdot e^{-x} \quad t = u\cdot v \quad t' = u'v + v'u$

$u'v + v'u - uv = -\frac{1}{2}(x+1)\cdot e^{-x}$

$u'v + u(v' - vx) = -\frac{1}{2}(x+1)\, e^{-x} \quad \frac{dv}{dx} = vx$

$u'\cdot e^{\frac{x^2}{2}} = -\frac{1}{2}(x+1)\cdot e^{-x} \quad \frac{dv}{v} = x\,dx$

$u = \int -\frac{1}{2}\cdot e^{-\frac{x^2}{2} - x}(x+1)\,dx = \frac{1}{2}$ $\quad \ln v = \frac{x^2}{2}$

$\int e^{-\frac{x^2}{2} - x}\, d\left(-\frac{x^2}{2} - x\right) = \frac{1}{2} e^{-\frac{x^2}{2} - x} + C \quad v = e^{\frac{x^2}{2}}$

$t = u\cdot v = \left(\frac{1}{2}e^{-\frac{x^2}{2} - x} + C\right) e^{\frac{x^2}{2}} =$

$= \frac{1}{2}\cdot e^{-x} + C\cdot e^{\frac{x^2}{2}} = \frac{1}{y} \to$

$y = \dfrac{1}{\frac{1}{2}e^{-x} + C\cdot e^{\frac{x^2}{2}}}$ при $x = 0 \ y = 2 \to$

$$\frac{x^3\,dx}{z^3} - \frac{3x^4\,dz}{z^4} = 4\,dx$$

$$-\frac{3x^4\,dz}{z^4} = \left(4 - \frac{x^3}{z^3}\right)dx$$

$$v = \frac{z}{x} \quad\Bigg|\quad \begin{aligned} z &= vx \\ dz &= v\,dx + x\,dv \end{aligned}$$

$$-\frac{3(v\,dx + x\,dv)}{v^4} = \left(4 - \frac{1}{v^3}\right)dx$$

$$-\frac{3x\,dv}{v^4} = \left(\frac{2}{v^3} + 4\right)dx$$

$$\frac{dv}{4v^4 + 2v} = -\frac{dx}{3x}$$

$$\begin{cases} v + \dfrac{1}{\sqrt[3]{2}} \to v + \dfrac{1}{\sqrt[3]{2}} = 0 \to \dfrac{1}{\sqrt[3]{u}\,x} + \dfrac{1}{\sqrt[3]{2}} = 0 \to \dfrac{\sqrt[3]{2}}{\sqrt[3]{4}} = x \\[2ex] v + \dfrac{\sqrt{3}i - 1}{2\sqrt[3]{2}} \to v + \dfrac{\sqrt{3}i - 1}{2\sqrt[3]{2}} = 0 \to \dfrac{1}{\sqrt[3]{u}\,x} + \dfrac{\sqrt{3}i - 1}{2\sqrt[3]{2}} = 0 \to u = -\dfrac{2}{x^3} \end{cases}$$

$$-\frac{\ln(6u^2+8u+3)}{4}=\ln(x)+C$$

$$\frac{1}{\sqrt[4]{6u^2+8u+3}}=e^C x$$

$$\frac{1}{\sqrt[4]{\frac{6y^2}{x^2}+\frac{8y}{x}+3}}=Cx$$

$$x^2=C(6x^4y^2+8x^5y+3x^6)$$

⑦ $x^4y''+x^3y'=4$

~~$y=C\ln(x)+\frac{1}{x^2}+C_1$~~

$x^4y''+x^3y'=4$

$y'=u$

$u'x^4+ux^3=4$

$u'x^4+ux^3=4$

$$\frac{x^4du}{dx}+ux^3=4$$

$u=\frac{1}{z^3}$ $\quad du=-\frac{3dz}{z^4}$

$$\frac{x^3}{z^3}-\frac{3x^4dz}{z^4dx}=4$$

$$\int \frac{1}{4v^4+2v}\,dv = \int -\frac{1}{3x}\,dx$$

$$\frac{\ln(v)}{2} - \frac{\ln(2v^3+1)}{6} = c - \frac{\ln(x)}{3}$$

$$\frac{\sqrt{u}}{\sqrt[6]{2u^3+1}} = \frac{e^c}{\sqrt[3]{x}}$$

$$\frac{\sqrt{z}}{\sqrt{x}\,\sqrt[6]{\frac{2\cancel{z}^3}{x^3}+1}} = \frac{C}{\sqrt[3]{x}}$$

$$\frac{1}{\sqrt[6]{u}\,\sqrt[6]{\frac{2}{u x^3}+1}\,\sqrt{x}} = \frac{C}{\sqrt[3]{x}}$$

$$u = \frac{C}{x} - \frac{2}{x^3}, \quad \frac{\sqrt[3]{z}}{\sqrt[3]{u}} = -x$$

$$y' = \frac{C}{x} - \frac{2}{x^3}; \quad \frac{dy}{dx} = \frac{C}{x} - \frac{2}{x^3}$$

$$dy = \left(\frac{C}{x} - \frac{2}{x^3}\right)dx$$

$$\int dy = \int \frac{C}{x} - \frac{2}{x^3}\,dx; \quad y = C\ln(x) + \frac{1}{x^2} + C_1$$

$$y = C\ln(x) + \frac{1}{x^2} + C_1$$

$$\int 1\,dx = \int \frac{1}{2C}\,dp$$

$$x = \frac{p}{2C} + C_1$$

$$x = \frac{p}{2C} + C_1$$

$$y = \frac{C^2x^2 - 2C^2C_1x + C^2C_1^2 - 1}{C}$$

$$y^2 = Cx^2 - 2CC_1x + CC_1^2 - \frac{1}{C}$$

$$y^2 = C(x^2 - 2C_1x + C_1^2) - \frac{1}{C}$$

(8) $y^3 y'' + 1 = 0$

$\cancel{y^2 = C(x^2 - 2C_1 x + C_1^2) - \frac{1}{C}}$

$y^3 y'' + 1 = 0$

$u u' y^3 + 1 = 0$

$u u' y^3 = -1$

$u u' = -\frac{1}{y^3}$

$\frac{u\,du}{dy} = -\frac{1}{y^3}$

$u\,du = -\frac{dy}{y^3}$

$\int u\,du = \int -\frac{1}{y^3}\,dy$

$u^2 = \frac{1}{y^2} + C$

$y'^2 = \frac{1}{y^2} + C$ $\qquad v = y^2$

$y^2 y'^2 = Cy^2 + 1$ $\qquad v' = 2yy' \quad \Big| \quad y' = \frac{v'}{2y}$

$\frac{v'^2}{4} = Cv + 1$ $\qquad v'^2 = 4Cv + 4$

$v = \frac{p^2}{4C} - \frac{1}{C}$ $\qquad dv = \frac{p\,dp}{2C}$

$p\,dx = \frac{p\,dp}{2C}$ $\qquad dx = \frac{dp}{2C}$

(9) $y^{IV} - 6y''' + 9y'' = 3x-1$

$\cancel{y = C_1 x e^{3x} + C_2 e^{3x} + \frac{x^3}{18} + \frac{x^2}{18} + C_3 x + C_4}$

$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = f(x)$

$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = 0$

$a_0 \lambda^n + a_1 \lambda^{n-1} + \dots + a_{n-1} \lambda + a_n = 0$

$\lambda^4 - 6\lambda^3 + 9\lambda^2 = 0$

$(\lambda - 3)^2 \lambda^2 = 0$

$(\lambda - 3)^2 \to \lambda_{1,2} = 3 \quad k = 2 \quad T: (C_1 x + C_2) e^{3x}$

$\lambda^2 \to \lambda_{3,4} = 0 \quad k = 2 \quad T: C_3 x + C_4$

$\bar{y} = \sum P_{k-1}(x) e^{ax} \sin \beta x + Q_{k-1}(x) e^{ax} \cos \beta x$

$\lambda = a \pm \beta i$

$P_{k-1}(x), Q_{k-1}(x) \to C_1 + \dots + C_k x^{k-1}$

$\bar{y} = (C_1 x + C_2) e^{3x} + C_3 x + C_4$

$e^{ax}(P_m(x) \cos \beta x + Q_m(x) \sin \beta x)$

$y_1 = x^s e^{ax}(R_m(x) \cos \beta x + T_m(x) \sin \beta x)$

Частное решение для $3x-1$

$a + \beta i = 0 \to s = 2$

$y_0 = x^2 (Ax + B)$

$y_0'' = 6Ax + 2B$

$y_0''' = 6A$

$y_0^{IV} = 0$

$54Ax + 18B - 36A = 3x - 1$

$$\begin{cases} 54A = 3 \\ 18B - 36A = -1 \end{cases} = \begin{cases} A = \frac{1}{18} \\ B = \frac{1}{18} \end{cases}$$

$$y_0 = \left(\frac{x}{18} + \frac{1}{18}\right)x^2$$

$$y = (C_1 x + C)e^{3x} + \left(\frac{x}{18} + \frac{1}{18}\right)x^2 + C_3 x + C_2$$

$$y = C_1 x e^{3x} + Ce^{3x} + \frac{x^3}{18} + \frac{x^2}{18} + C_3 x + C_2$$

(10) $y''' - y'' - 9y' + 9y = (12 - 16x)e^{x}$

$\cancel{y = C e^{3x} + x^2 e^{x} - xe^{x} + C_1 e^{x} + \frac{C_2}{e^{3x}}}$

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = f(x)$$

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = 0$$

$$a_0 \lambda^n + a_1 \lambda^{n-1} + \ldots + a_{n-1} \lambda + a_n = 0$$

$$\lambda^3 - \lambda^2 - 9\lambda + 9 = 0$$

$$(\lambda - 3)(\lambda - 1)(\lambda + 3) = 0$$

$\lambda - 3 \to \lambda_1 = 3$ $\quad k = 1 \quad$ $\tau: Ce^{3x}$

$\lambda - 1 \to \lambda_2 = 1$ $\quad k = 1 \quad$ $\tau: C_1 e^{x}$

$\lambda + 3 \to \lambda_3 = -3$ $\quad k = 1 \quad$ $\tau: \frac{C_2}{e^{3x}}$

$$\bar{y} = \sum P_{k-1}(x) e^{ax} \sin\beta x + Q_{k-1}(x) e^{ax} \cos\beta x$$

$$\lambda = a \pm \beta i$$

$$P_{k-1}(x),\ Q_{k-1}(x) \to C_1 + \ldots + C_k x^{k-1}$$

$$\bar{y} = Ce^{3x} + C_1 e^{x} + \frac{C_2}{e^{3x}}$$

$$e^{ax}\left(P_m(x)\cos\beta x + Q_m(x)\sin\beta x\right)$$

$$y_1 = x^s e^{ax}\left(R_m(x)\cos\beta x + T_m(x)\sin\beta x\right)$$

Частное решение для $(12 - 16x)e^{x}$

$a + \beta i = 1 \to s = 1$

$$y_0 = x(Ax + B)e^{x}$$

$$y_0' = (Ax^2 + (B+2A)x + B)e^x$$

$$y_0'' = (Ax^2 + (B+4A)x + 2B + 2A)e^x$$

$$y_0''' = (Ax^2 + (B+6A)x + 3B + 6A)e^x$$

$$(4A - 8B)e^x - 16Axe^x = (12 - 16x)e^x$$

$$\begin{cases} 4A - 8B = 12 \\ -16A = -16 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = -1 \end{cases}$$

$$y_0 = (x-1)xe^x$$

$$y = Ce^{3x} + x^2e^x - xe^x + C_1e^x + \frac{C_2}{e^{3x}}$$

(11) $y'' + y = 3\sin(4x) + 2\cos(4x)$

$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = f(x)$

$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = 0$

$a_0 \lambda^n + a_1 \lambda^{n-1} + \dots + a_{n-1} \lambda + a_n = 0$

$\lambda^2 + 1 = 0$

$\tilde{y} = \sum P_{k-1}(x) e^{ax} \sin\beta x + Q_{k-1}(x) e^{ax} \cos\beta x$

$\lambda = a \pm \beta i$

$\tilde{y} = C_1 \sin(x) + C \cos(x)$

$e^{ax}(P_m(x)\cos\beta x + Q_n(x)\sin\beta x)$

$y_i = x^s e^{ax}(R_m(x)\cos\beta x + T_m(x)\sin\beta x)$

частное решение где

$3\sin(4x) + 2\cos(4x)$

$a + \beta i = 4i \Rightarrow s = 0$

$y_0 = B\sin(4x) + A\cos(4x)$

$y_0'' = -16B\sin(4x) - 16A\cos(4x)$

$-15B\sin(4x) - 15A\cos(4x) = 3\sin(4x) + 2\cos(4x)$

$\begin{cases} -15B = 3 \\ -15A = 2 \end{cases} \Rightarrow \begin{cases} A = -\frac{2}{15} \\ B = -\frac{1}{5} \end{cases}$

$y_0 = -\frac{\sin(4x)}{5} - \frac{2\cos(4x)}{15}$

$$y = -\frac{\sin(4x)}{5} - \frac{2\cos(4x)}{15} + C_1 \sin(x) + C\cos(x)$$

$\cancel{y = \frac{\sin(4x)}{5}}$

(12) $y'' + 100y = 20\sin(10x) - 30\cos(10x) - 200e^{10x}$

$\cancel{y = -\frac{3x\sin(10x)}{2} + C_1 \sin}$

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = 0$$

$$a_0 \lambda^n + a_1 \lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n = 0$$

$$\Downarrow$$

$$\lambda^2 + 100 = 0$$

$\lambda^2 + 100 \to \lambda_{1,2} = \pm 10i \quad k = 1 \quad T: C_1 \sin(10x) + C\cos(10x)$

$$y = \sum P_{k-1}(x) e^{\alpha x} \sin\beta x + Q_{k-1}(x) e^{\alpha x} \cos\beta x$$

$\lambda = \alpha \pm \beta i$

$P_{k-1}(x), Q_{k-1}(x) \to C_1 + \ldots + C_k x^{k-1}$

$y = C_1 \sin(10x) + C\cos(10x)$

$f_1 + \ldots + f_p = 20\sin(10x) - 30\cos(10x) - 200e^{10x}$

$f_1, \ldots, f_p = 20\sin(10x) - 30\cos(10x), -200e^{10x}$

$$e^{\alpha x}(P_m(x)\cos\beta x + Q_m(x)\sin\beta x)$$

$$y_i = x^s e^{\alpha x}(R_m(x)\cos\beta x + T_m(x)\sin\beta x)$$

~~$S$~~ $C$ если $\alpha+\beta i$

$20\sin(10x)-30\cos(10x)$

$\alpha+\beta i=10i \to S=1$

$y_0=x(B\sin(10x)+A\cos(10x))$

$y_0''=(-100Bx-20A)\sin(10x)+(20B-100Ax)\cos(10x)$

$20B\cos(10x)-20A\sin(10x)=20\sin(10x)-30\cos(10x)$

$$\begin{cases}-20A=20\\20B=-30\end{cases} = \begin{cases}A=-1\\B=-\frac{3}{2}\end{cases}$$

$$y_0=x\left(-\frac{3\sin(10x)}{2}-\cos(10x)\right)$$

частное решение для $-200e^{10x}$

$\alpha+\beta i=10 \to S=0$

$y_1=Ae^{10x}$

$y_1''=100Ae^{10x}$

$200Ae^{10x}=-200e^{10x}$

$200A=-200 \to A=-1$

$y_1=-e^{10x}$

$$y=C_1\sin(10x)+x\left(-\frac{3\sin(10x)}{2}-\cos(10x)\right)+C\cos(10x)-e^{10x}$$

$$y=-\frac{3x\sin(10x)}{2}+C_1\sin(10x)-x\cos(10x)+C\cos(10x)-e^{10x}$$