From point mass to floating base dynamics

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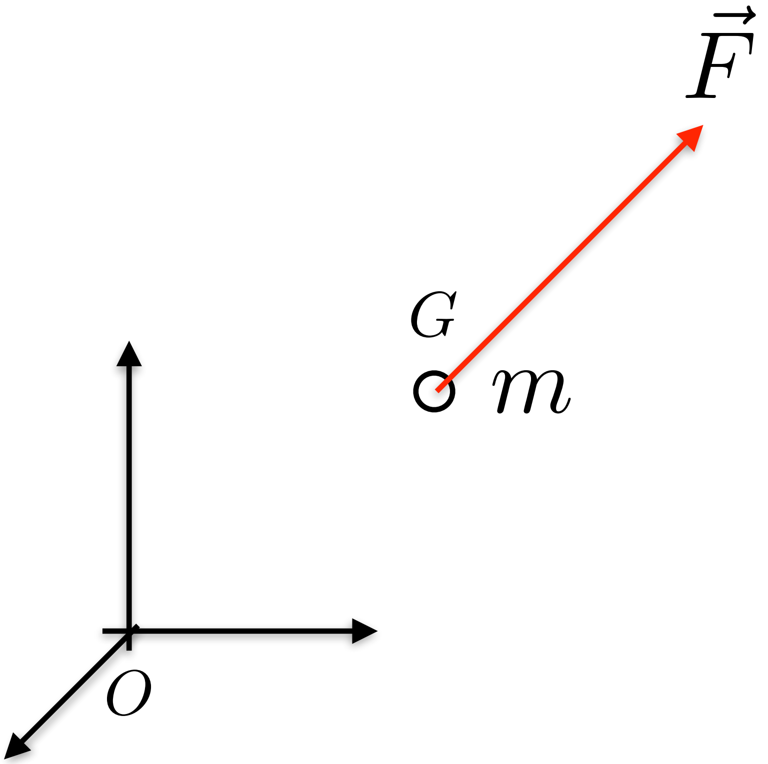
The point mass equations of motion

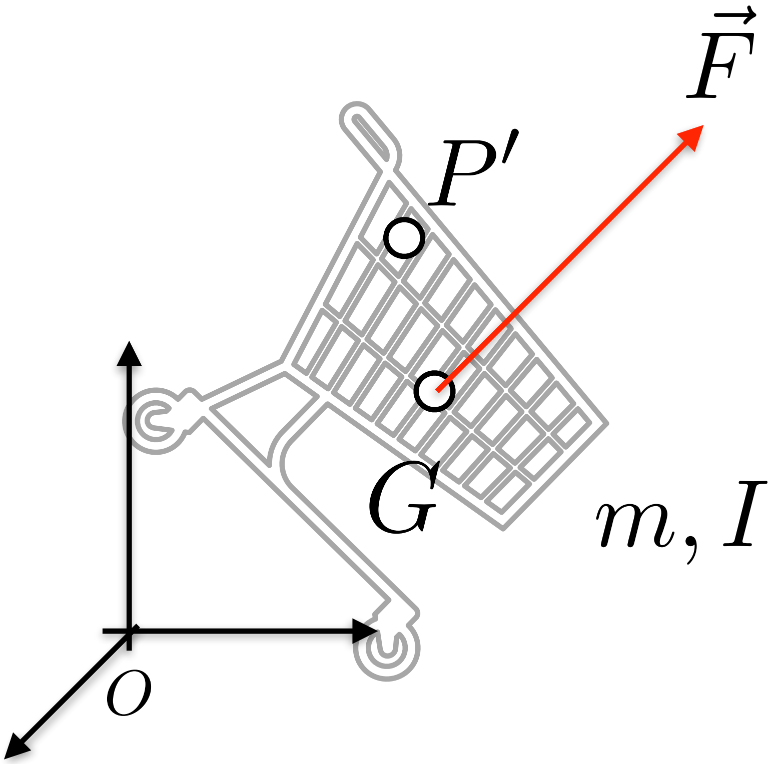


$$m\vec{a} = \vec{F}$$

$$\vec{v} := \frac{d}{dt}\vec{OG}$$

$$\boxed{\frac{d}{dt}(m\vec{v}) = \vec{F}}$$

The rigid body equations of motion 1/2



$$I = -\int_V \rho S(r)^2 dV$$

$$r = P' - G$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} m\vec{v} \\ I\vec{\omega} \end{pmatrix} = \begin{pmatrix} \vec{F} \\ \vec{M} \end{pmatrix}}$$

The rigid body equations of motion 2/2

$$\frac{d}{dt}\begin{pmatrix} m\vec{v} \\ I\vec{\omega} \end{pmatrix} = \begin{pmatrix} \vec{F} \\ \vec{M} \end{pmatrix} \Rightarrow \frac{d}{dt}\begin{pmatrix} m & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \vec{v} \\ \vec{\omega} \end{pmatrix} = \begin{pmatrix} \vec{F} \\ \vec{M} \end{pmatrix}$$

$$\frac{d}{dt}\mathbb{M}\nu = \tau \Rightarrow \mathbb{M}\dot{\nu} + C\nu = \tau$$

$$\boxed{\mathbb{M}\dot{\nu} + C\nu + g = \tau}$$

Forces and torques (tau) do not contain gravity

Robot Dynamics

$$\boxed{f = ma \quad \text{for the robot?}}$$

Robot Dynamics

$$\boxed{\frac{d}{dt}\frac{\partial}{\partial \dot{q}}\mathcal{L} - \frac{\partial}{\partial q}\mathcal{L} = \tau}$$

- $\mathcal{L} = T - U$: Lagrangian
- $q, \dot{q}, \ddot{q}$: joints' positions, velocities, and accelerations
- τ joint torques

$$\Downarrow$$

$$\boxed{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - J^{\top}F_{ext} = \tau}$$

- M, C, g: mass and Coriolis matrices, gravity torques
- F_{ext}, J: **vectorized** external forces and its Jacobian

Robot Dynamics Terminology

$$\boxed{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - J^\top F_{ext} = \tau}$$

Important Facts

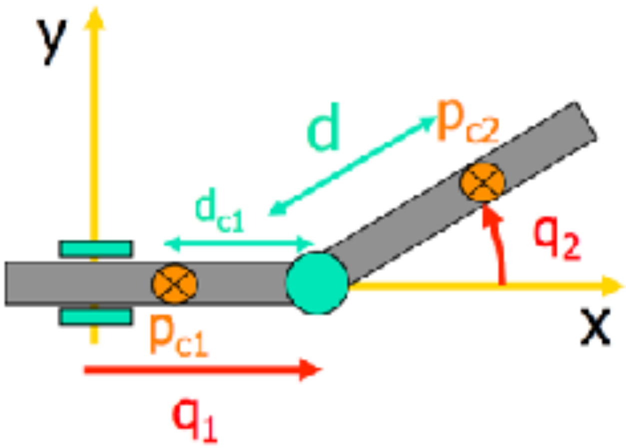
$$\tau = \mathrm{invDyn}(q, \dot{q}, \ddot{q}, F_{ext})$$

Inverse dynamics

$$\ddot{q} = \mathrm{fwDyn}(q, \dot{q}, \tau, F_{ext})$$

Forward dynamics

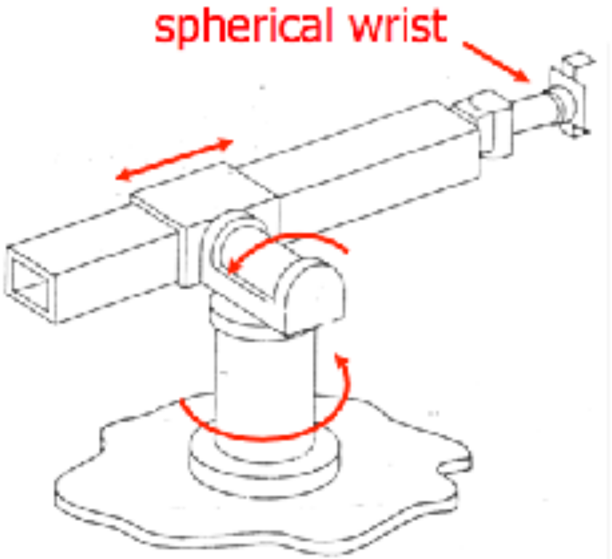
Dynamic Model of PR robot



From http://www.diag.uniroma1.it/~deluca/rob2_en/03_LagrangianDynamics_1.pdf, p23

$$\begin{pmatrix} m_1 + m_2 & -m_2 d \sin(q_2) \\ -m_2 d \sin(q_2) & I_{c_2,zz} + m_2 d^2 \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} -m_2 d \cos(q_2) \dot{q}_2^2 \\ 0 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}$$

Dynamic Model Complexity 1/2



From http://www.diag.uniroma1.it/~deluca/rob2_en/04_LagrangianDynamics_2.pdf\ p2

$B_{11} = m_1 k_{122}^2$

Dynamic Model Complexity 2/2

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

Algorithms to compute dynamic quantities in real time

Passivity of Robot Dynamics

Important Fact

The property

$$\boxed{\begin{aligned} &M = M^\top > 0 \\ &\dot{M} - 2C = -(\dot{M} - 2C)^\top \end{aligned}}$$

implies that without external wrenches, joint torques, and gravity, the kinetic energy conserves

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = 0$$

$$E = \frac{1}{2}\dot{q}^\top M(q)\dot{q} \quad \Rightarrow \quad \dot{E} = 0$$

What is the difference?

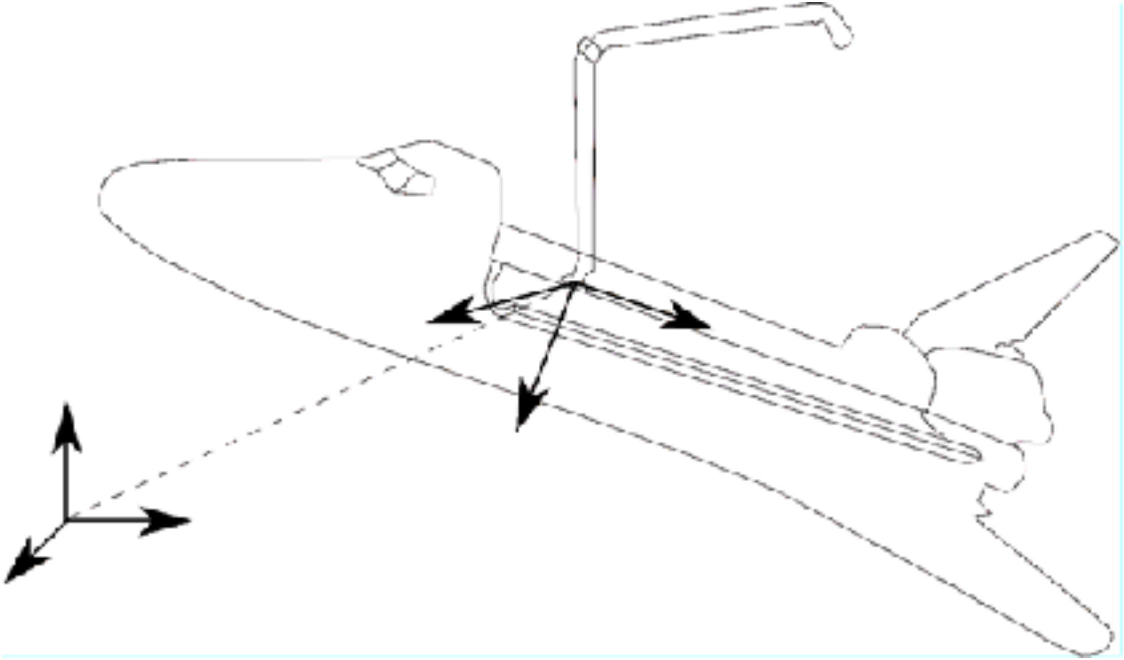
What is the difference?

The system is called **fixed-base** if one of the bodies composing it has a constant pose with respect to the inertial frame

The system is called **floating-base** if none of the bodies composing it has a constant pose with respect to the inertial frame

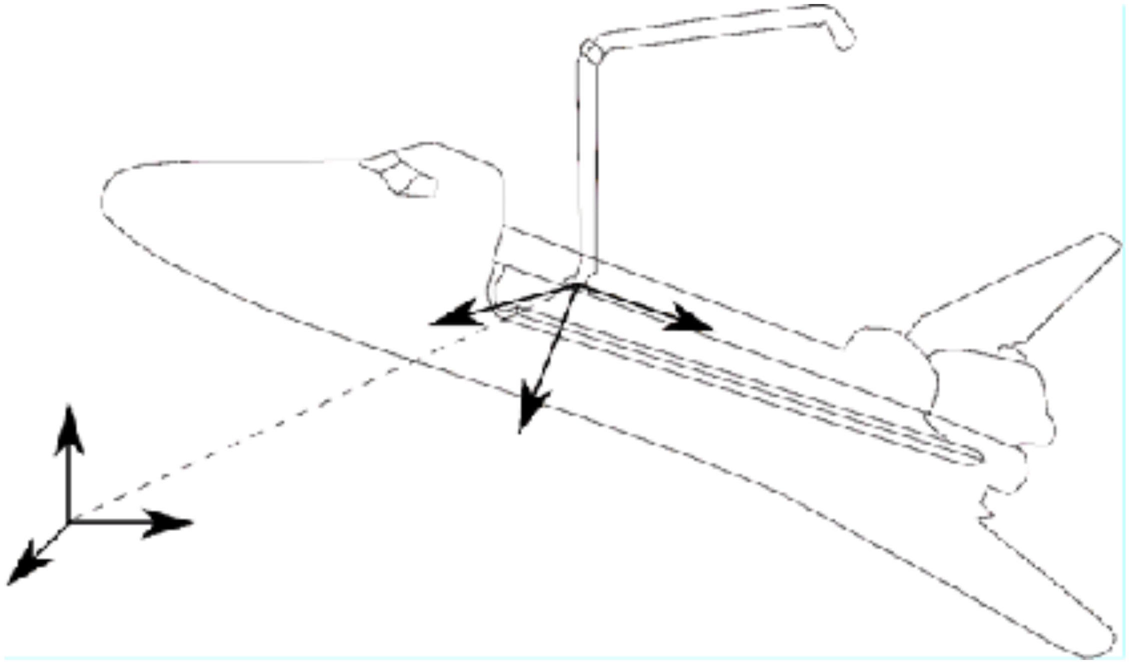
Extension to floating base systems

Pose of the base frame with respect to the inertial frame must be characterised



Extension to floating base systems

Additional six degrees of freedom modelled as additional six fictitious joints



Extension to floating base systems

$$\boxed{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - J^T F_{ext} = \begin{pmatrix} 0_6 \\ \tau \end{pmatrix}}$$

- $q = \begin{pmatrix} q_b \\ q_j \end{pmatrix} \qquad q_b \in \mathbb{R}^6,\ q_j \in \mathbb{R}^{DOF}$
- q_b: base's position and orientation
- q_j: joint positions
- $\tau \in \mathbb{R}^{DOF}$
- J: Jacobian, F_{ext}: vectorized external forces

Extension to floating base systems

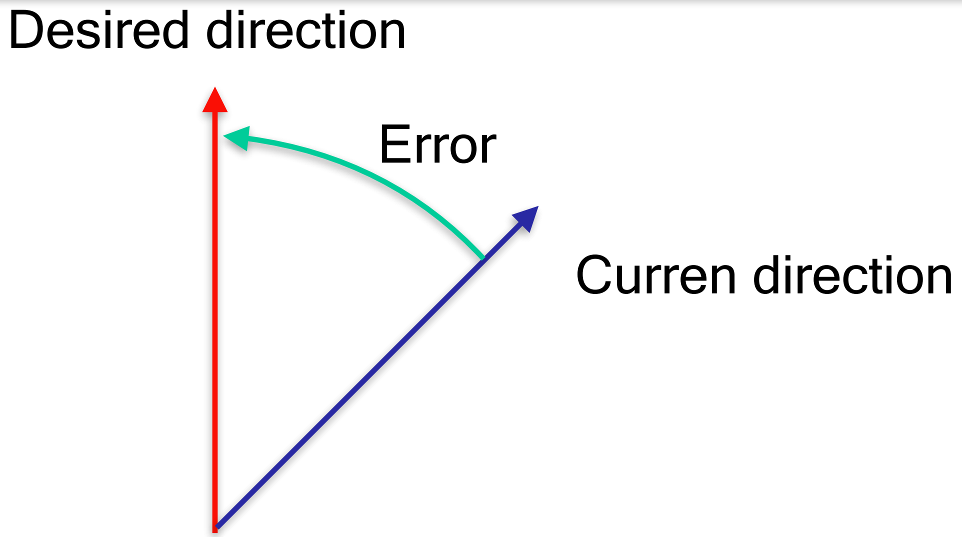
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - J^T F_{ext} = \begin{pmatrix} 0_6 \\ \tau \end{pmatrix}$$

Problem: $q_b \in \mathbb{R}^6$ is the orientation and position of the base frame

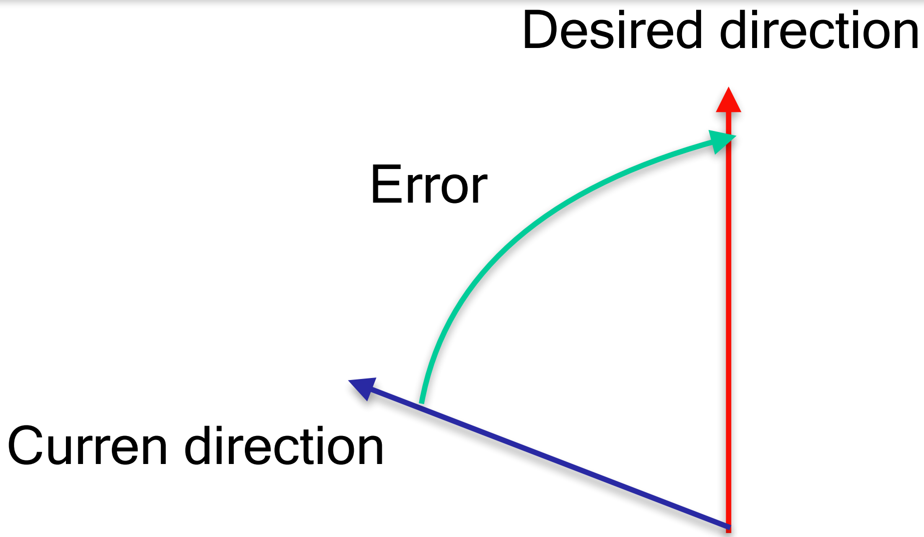
$q_b \in \mathbb{R}^6$ is a local representation of $SE(3) \Rightarrow$

(Forwards) dynamics is not globally defined!

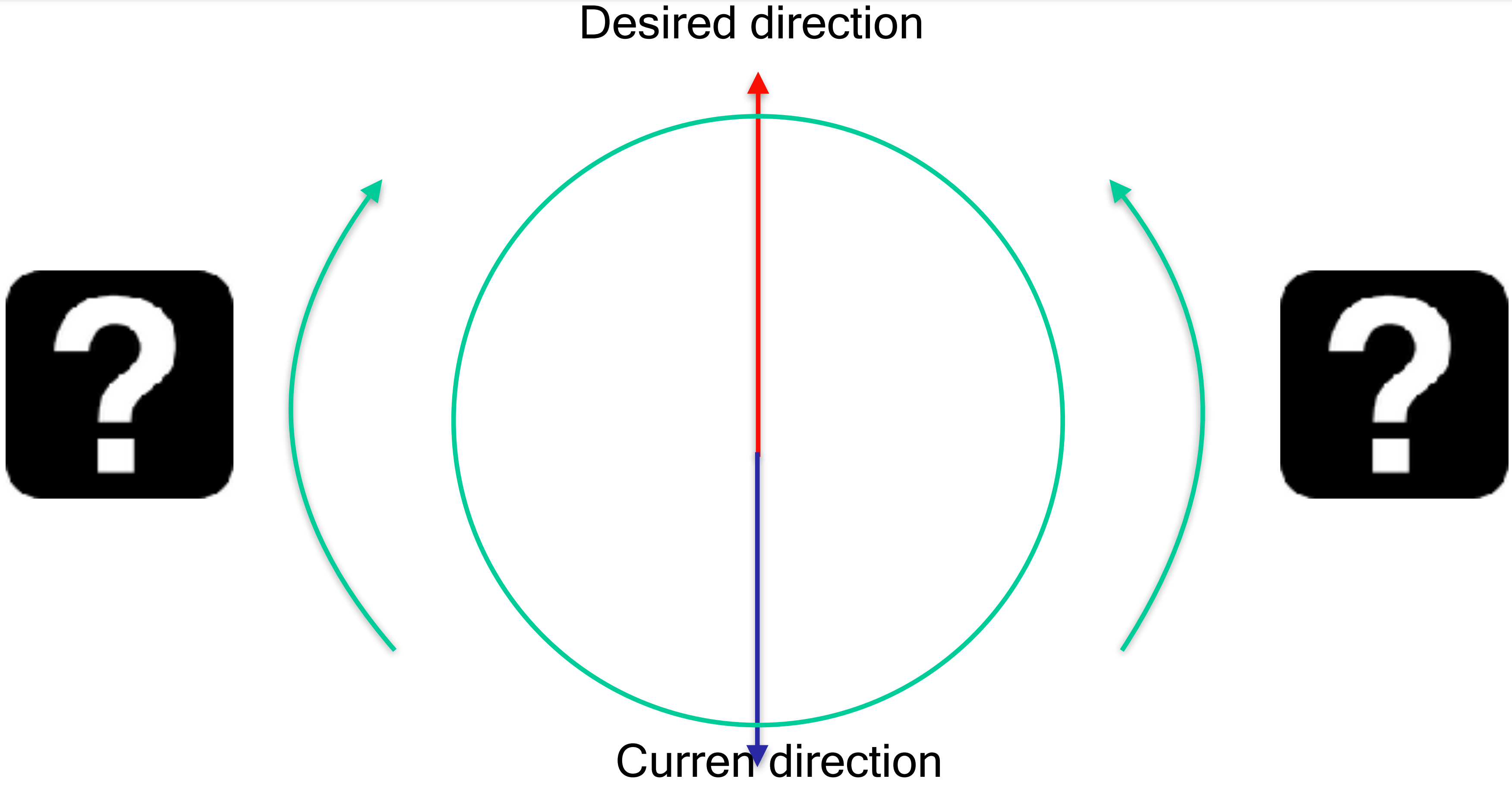
An hint into the topology problem 1/3



An hint into the topology problem 2/3



An hint into the topology problem 3/3



Extension to floating base systems

Configuration space: $$\mathbb{Q} = \mathbb{R}^n$$

Euler-Lagrange for equations of motion

Configuration space: $$\mathbb{Q} = SE(3) \times \mathbb{R}^n$$

Euler-Poincaré for equations of motion

Extension to floating base systems

$$\boxed{M(q)\dot{\nu} + C(q,\nu)\nu + g(q) - J^T F_{ext} = \begin{pmatrix} 0_6 \\ \tau \end{pmatrix}}$$

- $q \in SE(3) \times \mathbb{R}^n$, e.g. $q = ({}^wT_b, q_j)$
- $v \in se(3) \times \mathbb{R}^n$, e.g. $v = (v_b, \dot{q}_j)$
- ${}^wT_b = (p_b, {}^wR_b)$: position and rotation matrix of the base
- q_j: joint positions
- $v_b = (\dot{x}_b, \omega_b)$: linear and angular velocity of base frame