(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

MAIN CAMPUS (REGULAR PROGRAM)

UNIVERSITY EXAMINATIONS FOR
2022/2023 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS
FOR THE DEGREE OF

BACHELOR OF SCIENCE (COM/SIT)

COURSE CODE: MAT 124

COURSE TITLE: INTEGRAL CALCULUS

DATE: FRIDAY 21ˢᵀ APRIL, 2023   TIME: 3.00-5.00 P.M

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO Questions

Time: 2 hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question One (30 Marks)

a. Find the average value of the function $f(x) = x^2 + 3x - 1$ on the interval $[-1, 2]$. (3 Marks)

b. Decompose $\dfrac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x}$ into partial fractions (5 Marks)

c. Evaluate $\int x^2 e^x dx$ (4 Marks)

d. Evaluate $\int x^2 \ln x dx$ (4 Marks)

e. Evaluate the integral $\int e^x \cos x dx$ by parts. (5 Marks)

f. Find $\dfrac{d}{dx} \displaystyle\int_{x^3}^{1} \dfrac{1}{\sqrt{t^4 + 2}} dt$ (4 Marks)

g. Show that the area bounded by the curves $y = 4 - x^2$ and $y = x^2 - 2x$ is 9 sq. units

(5 Marks)

Question Two (20 Marks)

a. Define $I_n = \int \sin^n x dx$. Show that $I_n = \dfrac{-\sin^{n-1} x \cos x}{n} + \dfrac{n-1}{n} \int \sin^{n-2} x dx$. Hence or

otherwise evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin^4 x dx$ (10 Marks)

b. Evaluate $\int \sec(3x + \dfrac{1}{4}) dx$ (5 Marks)

c. Find the area bounded by $y = 4x - x^2$, $x = 1$, $x = 2$ and the x-axis. (5 Marks)

Question Three (20 Marks)

a). Evaluate $\int \dfrac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx$ (7 mks)

b). Find the area bounded by the curves $y = 4 - x^2$ and $y = x^2 - 2x$ (4 mks)

c). Find the $\int \dfrac{x^3}{\sqrt{9 - x^2}} dx$ (4 mks)

d). The curve $y = 3 + 2\sin x$ is rotated about the x-axis from the lines $x = 0$ and $x = \pi$. Find the volume generated by the resulting surface.

(5 mks)

Integral Calculus

2

Question Four (20 Marks)

Evaluate the following integrals

a. $3\int \frac{dx}{x^2+9}$ (3 Marks)

b. Evaluate

$$\int_{-\infty}^{\infty} xe^{-x^2} dx$$ (5 Marks)

c. The curve $y = 3 + 2\sin x$ is rotated about the X-axis from the lines $x = 0$ and $x = \pi$. Find the volume generated by the resulting surface (5 Marks)

d. The arc of the parabola $y^2 = 8x$ from $(0,0)$ to $(2,4)$ is rotated about the x-axis. Find the area of the resulting surface (7 Marks)

Question five (20 Marks)

a. Evaluate the following integrals

i. $\int \cot^5 x \cos ec^2 x dx$ (4 Marks)

ii. $\int \tan^3 x \sec^2 x dx$ (3 Marks)

iii. $\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx$ (5 Marks)

b. Determine the position function $s(t)$ of an object if its velocity is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$ (3 marks)

c. Use partial fractions to evaluate $\int \frac{x^4}{x^2-1} dx$ (5 marks)

END OF EXAMINATION: GOOD LUCK