



(University of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

(MMUST)

UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

(MAIN EXAMINATION)

FIRST YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE AND EDUCATION

(SST, SME, SMT, BES, ETS, ETC, ETM, ETE, COM, SIT,
SIC, SPA, EDA and EDS)

COURSE CODE: MAT 121

COURSE TITLE: DIFFERENTIAL CALCULUS

DATE: 20/12/2020

TIME: 3:00 – 5: 00 PM

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (COMPULSORY) and any other TWO questions.
- Do not write on the question paper.

Time 2 Hours

MMUST observes ZERO tolerance to examination cheating

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE**[30 MARKS]**

- a) State the Mean Value Theorem [2 marks]
- b) Determine the derivative of $f(x) = x^2 + 4x$ from first principles [3 marks]
- c) Find $\frac{dy}{dx}$ if $\sin(x + y) = x^2 \cos xy$ [4 marks]
- d) Compute
- i) $\lim_{x \rightarrow \infty} \frac{3x^2 + 7}{5x^2 + 7x - 4}$ [3 Marks]
- ii) $\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x^2 - 9}$ [3 marks]
- e) The parametric equation for a hyperbola are $x = 2 \sec \theta$, $y = 4 \tan \theta$. Find $\frac{dy}{dx}$ [3 marks]
- f) Determine and distinguish all the turning points of the curve $y = x^3 - 3x^2 - 1$ [4 marks]
- g) Find the derivative of $y = x^{\sqrt{x}}$ [3 marks]
- h) Determine the equation of the normal line to the graph of $x^2 - 3xy + 2y^2 - 2x - 4 = 0$ at point $(1, -1)$ [5 marks]

QUESTION TWO**[20 MARKS]**

- a) Evaluate the derivative of $y = \frac{x^2 + x - 2}{(x^2 + 4)^3}$ [3 marks]
- b) Find the inverse of the function $f(x) = \frac{x+1}{x+2}$ [2 marks]
- c) Given the function $f(x) = \sqrt{4 - x^2}$ and $g(x) = x^2 + 4$. Find
- i. The domain and range of $f(x)$ [3 marks]
- ii. $g(f(x))$. [2 marks]
- d) If $x^4 + y^4 = 16$, find y'' . [5 marks]
- e) A man walks along a straight path at a speed of 4 m/s. A searchlight is located on the ground 20 m from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 m from the point on the path closest to the searchlight? [5 marks]

QUESTION THREE**[20 MARKS]**

- a) Using $\varepsilon - \delta$ definition of limits prove that $\lim_{x \rightarrow 3} (x^2 + x - 4) = 8$ [4 marks]
- b) If $y = x^{-1}$, show that $x^3 y'' + x^2 y' - xy = 0$ [4 marks]
- c) Find the derivative of the function $f(x) = e^{\sin 2x^3}$ [3 marks]
- d) Find y' of the following functions
- i. $x^3 + y^3 = 4xy$ [3 marks]
- ii. $x \sin y + \cos 3y = \sin 2y$ [3 marks]
- e) Find $\frac{d}{dx} \ln \left[\frac{x+1}{\sqrt{x-2}} \right]$ [3 marks]

QUESTION FOUR**[20 MARKS]**

- a) We would like to create a rectangular fenced off area for our dog to play in. We will use 600 feet of fencing material. A building will be on one side of the playpen (so won't need any fencing). Determine the dimensions of the playpen that will enclose the largest area. [5 marks]
- b) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$ [2 marks]
- c) Let f and g be functions defined by $f(x) = 3x^2 - x + 10$ and $g(x) = 1 + x^2$. Determine
- i. $f \circ g(2)$ [2 marks]
 - ii. $g \circ g^{-1}(x)$ at $x = 5$ [3 marks]
- d) Differentiate $f(x) = (2x^3 + 1)^5$ [3 marks]
- e) Ladder 10 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 m/min, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 m from the wall? [5 marks]

QUESTION FIVE**[20 MARKS]**

- a) The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s in meters.
- i. Find the acceleration at time t . [1 mark]
 - ii. When is the time t when the velocity is zero [2 marks]
- b) Prove that $\lim_{x \rightarrow 0} \frac{|x|}{x}$ does not exist [4 marks]
- c) Find the derivative of $y = (x^2 + 1)^3(x^3 + 3)$ [4 marks]
- d) Suppose that it costs $c(x) = x^3 - 6x^2 + 15x$ dollars to produce x radiators when 8 to 10 radiators are produced, and that $r(x) = x^3 - 3x^2 + 12x$ gives a revenue for selling x radiators a day. A shop currently produces 10 radiators a day.
- i. How much extra will it cost to produce one more radiators a day? [3 marks]
 - ii. What is the estimated increase in revenue for selling 11 radiators a day? [3 marks]
- e) Sketch the graph of $y = (x - 2)^2(x + 1)^3(x - 1)$ by finding the intercepts and its limits as $x \rightarrow \infty$ and as $x \rightarrow -\infty$ [3 marks]