(The University Of Choice)

MASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY (MMUST)

UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR

MAIN EXAMINATIONS FOR

FIRST YEAR SECOND SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF SCIENCE (COM, SIK)

COURSE CODE: BCS 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: 17/04/2023 **TIME:** 08:00-10:00AM

Instructions to candidates:
Answer Question one and any other two questions.

Time: 2 hours

This paper consists of 4 printed pages. Please turn

QUESTION ONE (30 MARKS)

a) A pair of fair dice is thrown. Find the probability that the sum is 10 or greater if:

   i) 5 appears on the first die

   ii) 5 appears on at least one die. (6 Marks)

b) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ be Boolean matrices. Find AB and BA.

(4 Marks)

c) Let $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of each of the following statements:

   i) $(\exists x \in A)(x + 3 = 10)$

   ii) $(\exists x \in A)(x + 3 < 5)$

   iii) $(\forall x \in A)(x + 3 < 10)$ (6 Marks)

d) Discuss the following terms:

   i) Stack

   ii) Queues (4 Marks)

e) What is a homogeneous recurrence relation? (2 Marks)

f) Consider the second-order homogeneous recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 2,\ a_1 = 7,$

   i) Find the next three terms of the sequence. (3 Marks)

   ii) Find the general solution. (2 Marks)

   iii) Find the unique solution with the given initial conditions. (3 Marks)

QUESTION TWO (20 MARKS)

a) Consider the third-order homogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$

   i) Find the general solution. (4 Marks)

   ii) Find the solution with initial conditions $a_0 = 3,\ a_1 = 4,\ a_2 = 12.$ (5 Marks)

b) Draw a binary search tree for the set $S = \{1,2,3,4,5,6,7,8,9,10\}$. (5 Marks)

c) Define the following terms as used in Graph theory.

   i) Rooted tree

   ii) Binary tree

   iii) Decision tree

(6 Marks)

QUESTION THREE (20 MARKS)

a) Assume that in a country with currently 100million people has a population growth rate of 1% per year and it also receives a hundred thousand immigrants per year. Find its population in 10 years. (6 Marks)

b) Convert these adjacency matrices into incidence matrices. (6 Marks)

i) $\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$

ii) $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$

c) A fair coin is tossed three times yielding the equiprobable space

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Consider the three events

$$A = \{First\ toss\ is\ heads\} = \{HHH, HHT, HTH, HTT\}$$
$$B = \{Second\ tos\ is\ heads\} = \{HHH, HHT, THH, THT\}$$
$$C = \{Exactly\ two\ heads\ in\ a\ row\} = \{HHT, THH\}$$

Show that A and B and A and C are independent, but B and C are dependent. (8 Marks)

QUESTION FOUR (20 MARKS)

a) Draw the graph with vertices A, B, C, D, E and edges BD, BC, CE, DE. (5 Marks)

b) Find the degree of each vertex in part a) (5 Marks)

c) Find the incidence matrix of the graph in part a). (5 Marks)

d) Negate each of the following statements:

i) $\exists x\ \forall y.\ p(x, y)$;

ii) $\exists x\ \forall y.\ p(x, y)$;

iii) $\exists y\ \exists x\ \forall z.\ p(x, y, z)$.

Use $\neg\forall x\ p(x) \equiv \exists x \neg p(x)$ and $\neg\exists x\ p(x) \equiv \forall x \neg p(x)$. (5 Marks)

QUESTION FIVE (20 MARKS)

a) Prove that the complete graph K_n with $n \geq 3$ vertices have $H = (n-1)!/2$ Hamiltonian circuits. (5 Marks)

b) (i) Write in math notation the following English sentence: "Every number is divisible by 2 or by 3" (use d|n for "n is divisible by d"). (4 Marks)

(ii) For which universe of discourse is it true? (1 Mark)

(iii) For which universe of discourse is it false? (2 Marks)

(iv) State it is true or false if the universe of discourse complex numbers (2 Marks)

c) Form a binary search tree for the data 16, 24, 7, 5, 8, 20, 40 and 3 in the given order. (6 Marks)