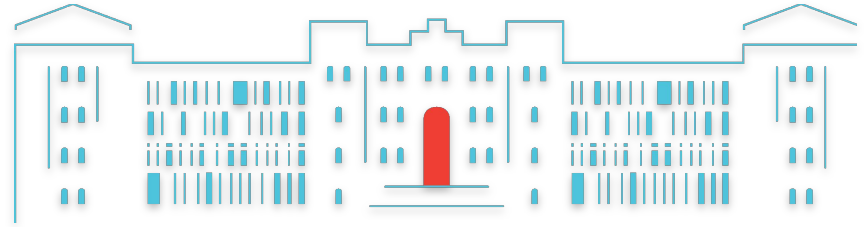
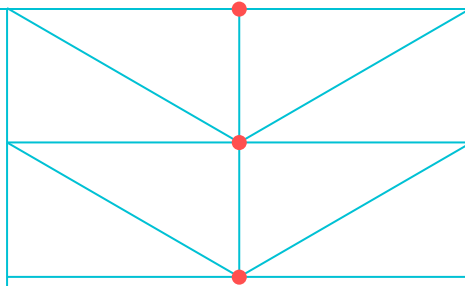


Lego Piece Drawing Experiment



TUHH
Technische
Universität
Hamburg



Group B | 18: Youssef, Valentin, Asad Jamal

Experimental Setup

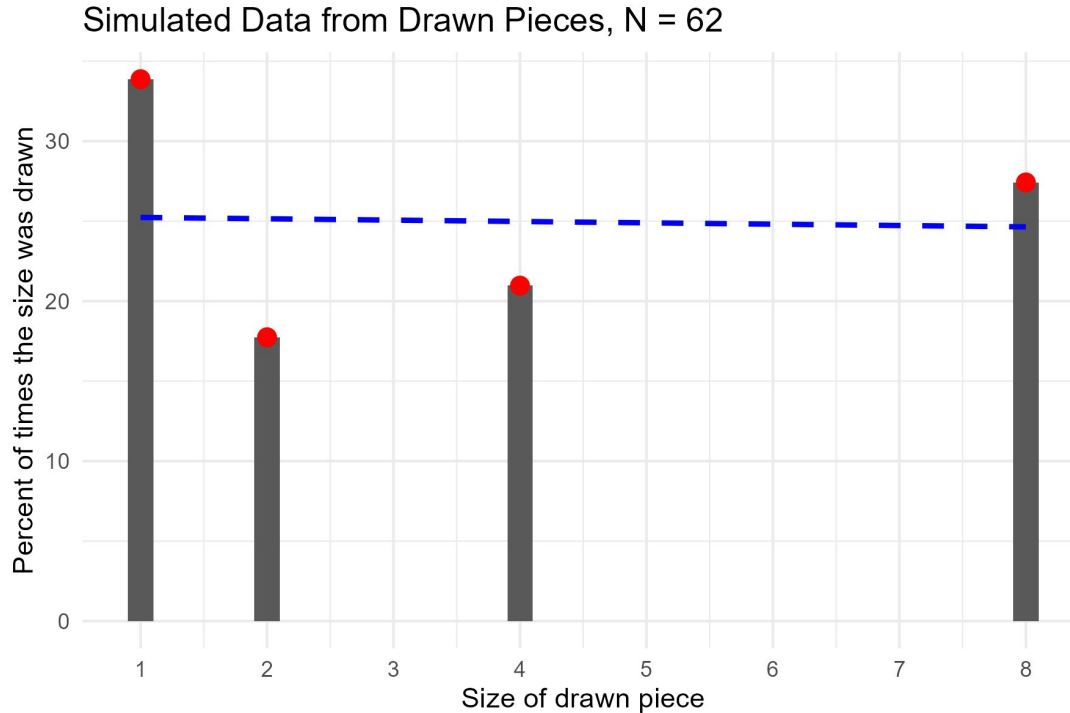


Task: Draw a *random* object from the bag

- **Pieces in bag:**
 $\Omega = \{1, 2, 4, 8\}$
- **Number of draws:**
 $N = 62 \rightarrow$ Unique participants
- **Replacement:**
After drawing, the piece was put back

\rightarrow Assume a linear distribution over Ω

Simulated Uniform Model



We assume a uniform distribution over the solution space Ω

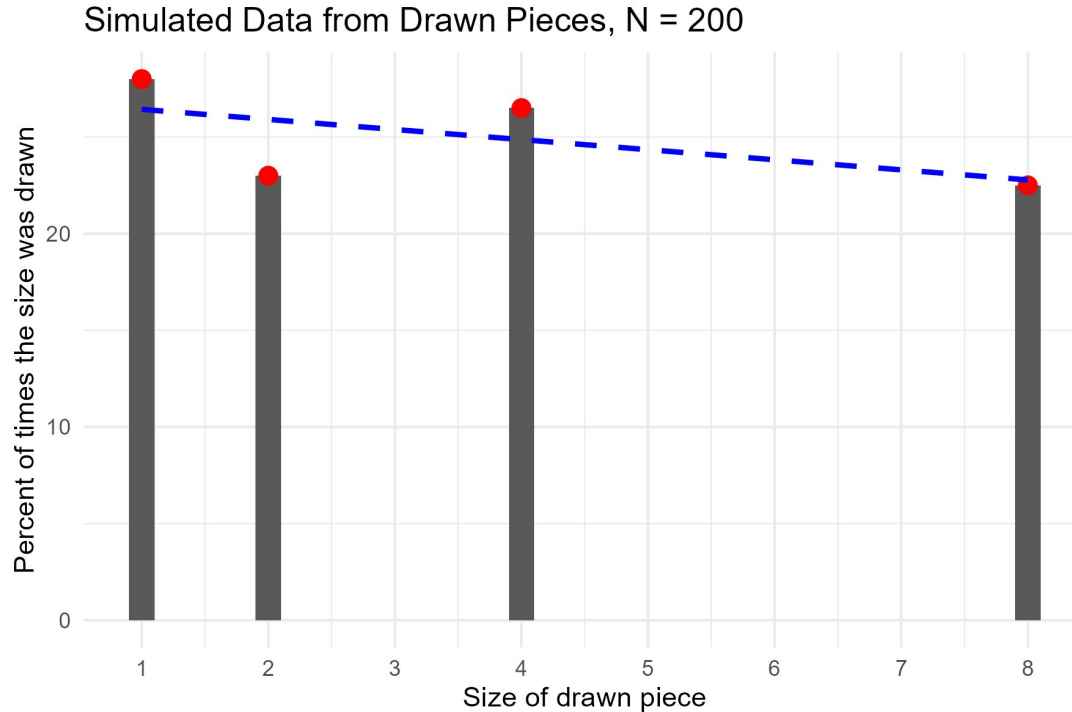
$$\mathbb{P}(X = 1) = 0.25,$$

$$\mathbb{P}(X = 2) = 0.25,$$

$$\mathbb{P}(X = 4) = 0.25,$$

$$\mathbb{P}(X = 8) = 0.25.$$

Simulated Uniform Model



We assume a uniform distribution over the solution space Ω

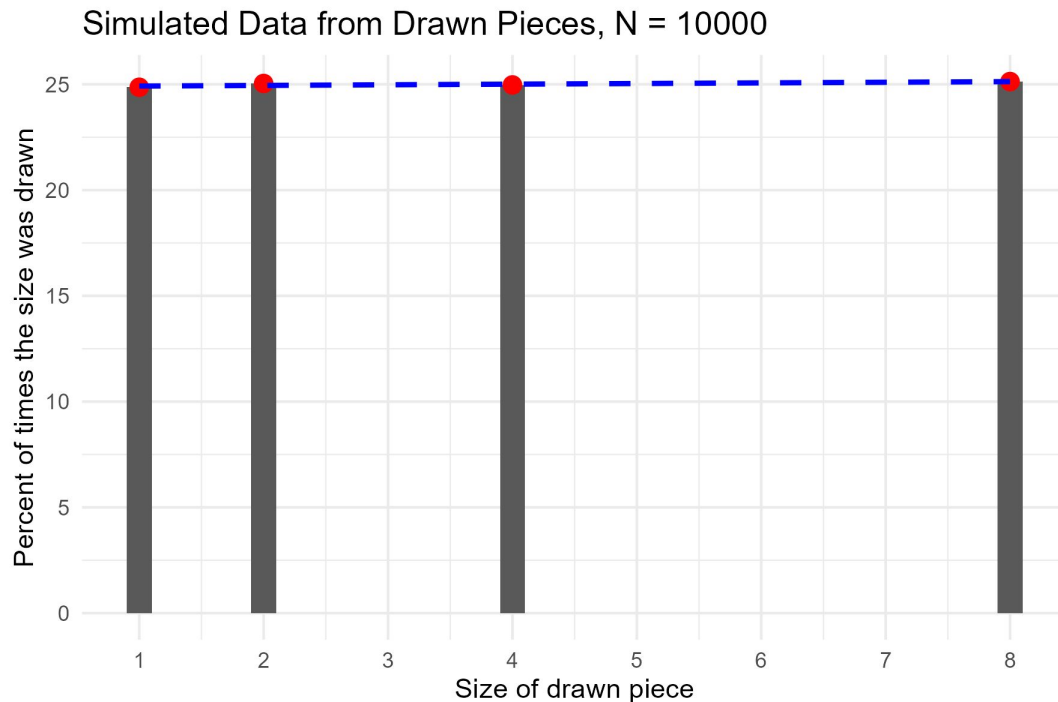
$$\mathbb{P}(X = 1) = 0.25,$$

$$\mathbb{P}(X = 2) = 0.25,$$

$$\mathbb{P}(X = 4) = 0.25,$$

$$\mathbb{P}(X = 8) = 0.25.$$

Simulated Uniform Model



We assume a uniform distribution over the solution space Ω

$$\mathbb{P}(X = 1) = 0.25,$$

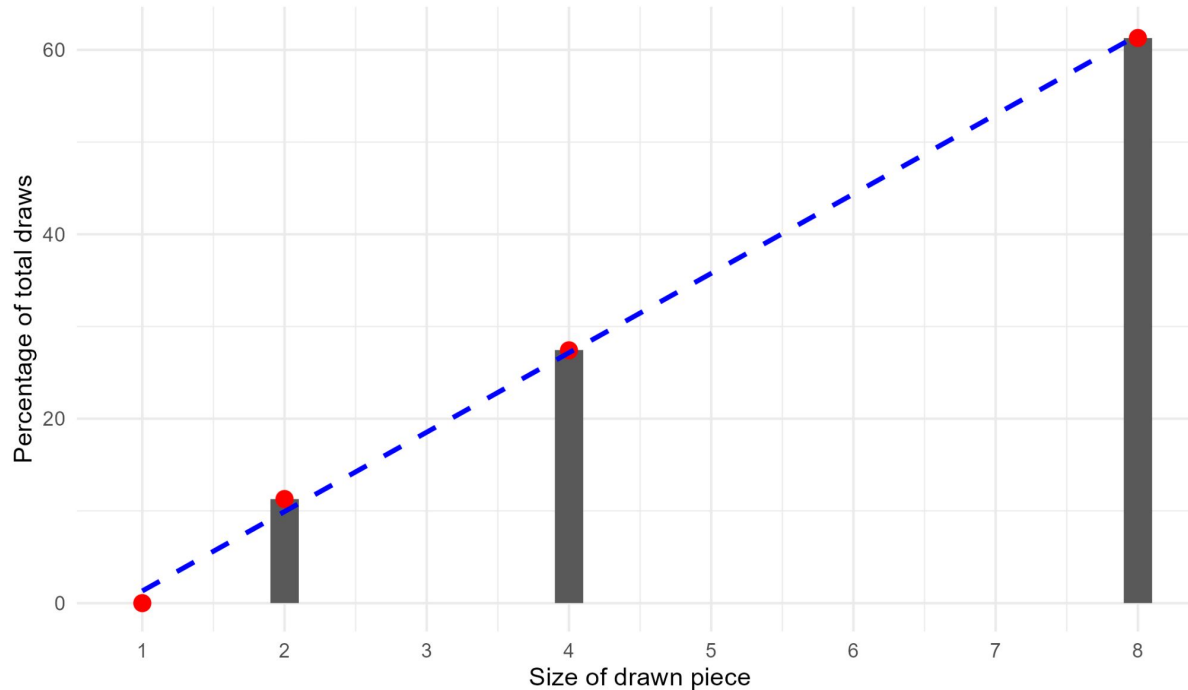
$$\mathbb{P}(X = 2) = 0.25,$$

$$\mathbb{P}(X = 4) = 0.25,$$

$$\mathbb{P}(X = 8) = 0.25.$$

Real Experiment Results

Real Data from Drawn Pieces



- $N_1 = 0$ → 0.00%
- $N_2 = 7$ → 11.3%
- $N_4 = 17$ → 27.4%
- $N_8 = 38$ → 61.3%

Reasons for $N_1 = 0$:

- Difficult to grab it
- Stuck in the corners of the bag?
- People might think it's too small to be considered random

Proportional Model

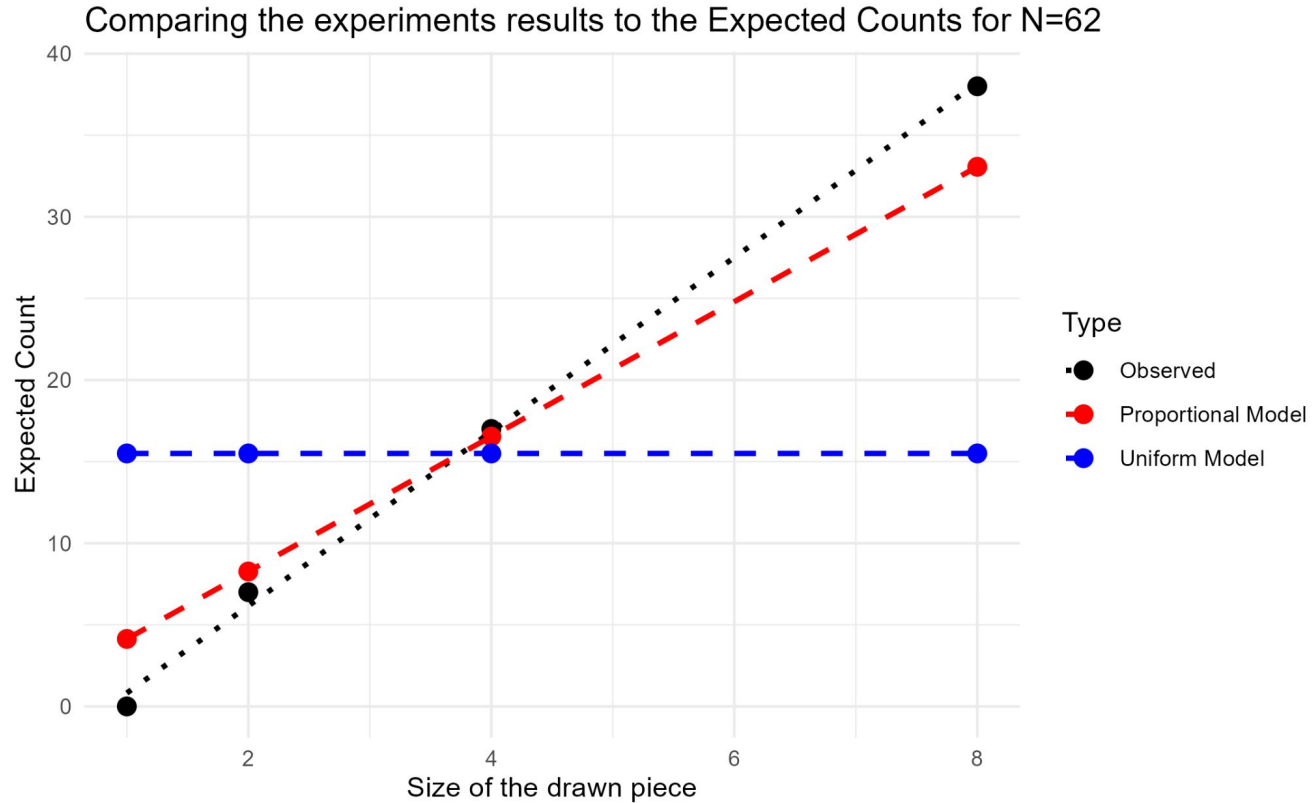
The probability P_i of a piece being drawn is assumed to be proportional to its size s . We have four pieces with sizes $s_i \in \{1, 2, 4, 8\}$

$$s_{\text{total}} = \sum_{i \in \{1, 2, 4, 8\}} s_i = 1 + 2 + 4 + 8 = 15$$

$$P_i = \frac{s_i}{s_{\text{total}}}$$

$$P_1 = \frac{1}{15}, \quad P_2 = \frac{2}{15}, \quad P_4 = \frac{4}{15}, \quad P_8 = \frac{8}{15}$$

Observed vs Models



Chi Squared Test

We compare the observed frequencies with two models:

Uniform model: $P = (0.25, 0.25, 0.25, 0.25)$, Proportional model: $P = (1/15, 2/15, 4/15, 8/15)$.

For the uniform model the chi-squared test yields:

$$\chi^2 = 52.968, \quad \text{df} = 3, \quad p = 1.863 \times 10^{-11}.$$

Since the test statistic is way too large in combination with the p-value being extremely small, we reject the null hypothesis. Thus, the **uniform model does not provide a good fit** to the observed data.

For the proportional model, the test gives:

$$\chi^2 = 5.0766, \quad \text{df} = 3, \quad p = 0.1663.$$

The test statistic is well below the critical value

$$\chi_{0.05, 3}^2 = 7.815,$$

and the p-value is sufficiently large.

Therefore, we fail to reject the proportional model.

The proportional model is a plausible and good fit for the data.