Algorithmic updates for song transition weights

Let A be an adjacency matrix of a graph which represents all possible transitions between songs in some music library. A is a weighted and directed graph. Let A be the weighted transition from song $i \to j$ and where $A_{ij} \in [0, 1]$. This weight represents something like a personal preference or emotional response to the transition. Its convenient to refer to it as a continuity score as in if we like the transition $i \to j$ then A_{ij} should be weighted higher.

Continuity scores are updated periodically. Let $\dfrac{dC(i,j)}{dt}$ be a function which evaluates how some connection will be updated over a single time-step.

A continuous transition is defined as listening to all of song i and then all of song j, where in between there may be either **none** or **some arbitrary number** of songs skipped. The event $i \to j$ **preserves** continuity while $i \to k \to \ldots \to j$ **restores** continuity.

Let the **continuous session** be defined as the sequence of songs played which **preserve** or **restore** continuity over the course of a listening session such as $S = (s_0, s_1, s_2, \ldots, s_n)$. We want to encode this into the graph weights.

Let i, j be any 2 songs in a session and lets say we listen to i before j. Now suppose $i, j \in S$ and let Δ_{ij} be the number of steps through S from $i \to j$. If there are n songs in a session then $\Delta_{ij} \in [1, n-1]$. Let the change in continuity score of the transition $i \to j$ per time-step be defined as $C_s(i,j) = e^{-\left(\frac{\Delta_{ij}-1}{\sqrt{n}}\right)}$. This ensures that $C_s(i,j) \in (0, 1]$. Influence decays exponentially as distance along the continuous session increases at a speed which depends on the length of the session. Then in general for any i, j in a session

$$C_s(i,j) = \begin{cases} e^{-\left(\frac{\Delta_{ij}-1}{\sqrt{n}}\right)} & \text{if } i,j \in S \cap (i \to j) \\ 0 & \text{else} \end{cases}$$

Consider an arbitrary transition from $i, j \in S$ where there exist an arbitrary number of skipped songs in between i, j such as the event $i \to k \to \ldots \to j$. Let K be the sequence of intermediate transitions between $i, j \in S$ defined as $K = (k_0, k_1, k_2, \ldots)$. It's necessary that we weaken the intermediate connections along this path and as K grows larger we should weaken the connections more. In general for any i, j in a session

$$C_w(i,j) = \begin{cases} e^{-|K|^2/|S|} - 1 & \text{if } i,j \in \text{some } K \\ 0 & \text{else} \end{cases}$$

Finally we need to consider individual song level engagement. For any song $i \in S$ we evaluate its **historical engagement** . Let $f(i)$ be the total historical plays of song i and let $g(i)$ be the historical number of times the song was finished. Then we can update the bidirectional relationship between songs based on their historical engagement alone defined as the function

$$C_e(i,j) = \frac{g(i)g(j)}{f(i)f(j)}$$

Now the entire formula for the update equation edge in the session S is defined as

$$\frac{dC(i,j)}{dt} = C_s(i,j) + C_w(i,j) + C_e(i,j)$$

Now at each time step we do $S'_{ij} = S_{ij} + \eta \frac{dC(i,j)}{dt}$ where $\eta \ll 1$ is a learning rate parameter. Finally each row in S is normalized with the $Softmax$ function to produce probabilities.