Let $A$ be an adjacency matrix of a graph which represents all possible transitions between songs in some music library. $A$ is a weighted and directed graph. Let $A$ be the weighted transition from song $i \to j$ and where $A_{ij} \in [0,1]$. This weight represents something like a personal preference or emotional response to the transition. Its convenient to refer to it as a continuity score as in if we like the transition $i \to j$ then $A_{ij}$ should be weighted higher.

Continuity scores are updated periodically. Let $\frac{dC(i,j)}{dt}$ be a function which evaluates how some connection will be updated over a single time-step.

A continuous transition is defined as listening to all of song $i$ and then all of song $j$, where in between there may be either **none** or **some arbitrary number** of songs skipped. The event $i \to j$ **preserves** continuity while $i \to k \to \ldots \to j$ **restores** continuity.

Let the **continuous session** be defined as the sequence of songs played which **preserve** or **restore** continuity over the course of a listening session such as $S = (s_0, s_1, s_2, \ldots, s_n)$. We want to encode this into the graph weights.

Let $i, j$ be any 2 songs in a session and lets say we listen to $i$ before $j$. Now suppose $i, j \in S$ and let $\Delta_{ij}$ be the number of steps through $S$ from $i \to j$. If there are $n$ songs in a session then $\Delta_{ij} \in [1, n-1]$. Let the change in continuity score of the transition $i \to j$ per time-step be defined as $C_s(i,j) = e^{-\left(\frac{\Delta_{ij}-1}{\sqrt{n}}\right)}$. This ensures that $C_s(i,j) \in (0,1]$. Influence decays exponentially as distance along the continuous session increases at a speed which depends on the length of the session. Then in general for any $i, j$ in a session

$$C_s(i,j) = \begin{cases} e^{-\left(\frac{\Delta_{ij}-1}{\sqrt{n}}\right)} & \text{if } i, j \in S \\ 0 & \text{else} \end{cases}$$

Consider an arbitrary transition from $i, j \in S$ where there exist an arbitrary number of skipped songs in between $i, j$ such as the event $i \to k \to \ldots \to j$ . Let $K$ be the set of intermediate songs between $i \to j$ defined as $K = \{k_0, k_1, k_2, \ldots\}$. It's necessary that we weaken the intermediate connections within this set which is $O(|K|^2)$ . As $K$ grows larger we should weaken the connections more. For any $k_i, k_j \in \text{some } K$ we can penalize $k_i \to k_j$ with $e^{-|K|^2/|S|} - 1$.

We also want to penalize any connections that breaks continuity that is of the form $i \to K \to j$ where $K \neq \emptyset$. This penalization can be weighted by instantaneous engagement as in the percentage of the song $k \in K$ that was listened to before skipping. Let $E(k)$ be the percentage of the song listened to and $\bar{E}_K = \sum_{k \in K} E(k)$ is the average engagement in $K$. Now we can define a continuity penalty function $\Gamma(K) = (e^{-|K|^2/|S|} - 1)(1 - \bar{E}_K)$. In general for any $i, j$ in a session

$$C_w(i,j) = \begin{cases} e^{-|K|^2/|S|} - 1 & \text{if } i,j \in K \\ \Gamma(K) & \text{if } (i \notin K \wedge j \in K) \vee (j \notin K \wedge i \in K) \\ 0 & \text{else} \end{cases}$$

Finally we need to consider individual song level engagement. For any song $i \in S$ we evaluate its **historical engagement** . Let $f(i)$ be the total historical plays of song $i$ and let $g(i)$ be the historical number of times the song was finished. Then we can update the bidirectional relationship between songs based on their historical engagement alone defined as the function

$$C_e(i,j) = \frac{g(i)}{f(i)} \frac{g(j)}{f(j)}$$

The entire formula for the update equation edge in the session $S$ is defined as

$$\frac{dC(i,j)}{dt} = C_s(i,j) + C_w(i,j) + C_e(i,j)$$

Now at each time step we do $A'_{ij} = A_{ij} + \eta \frac{dC(i,j)}{dt}$ where $\eta \leq 1$ is a learning rate parameter. Finally the session graph is projected into the rest of the library and for each mutated row of the graph library $A$ we apply an activation function on the updated weights and turn them into logits with

$$\bar{A} = \mathrm{softmax}(\max(0, \bar{\mathrm{A}}))$$