Date 7/9/21
Page
L&W

Unit 1. Set Theory

* Set of Natural Number

$N = \{1, 2, 3, 4 \ldots\}$

* Set of integers

$Z = \{\ldots -2, -1, 0, 1, 2, 3 \ldots\}$

$Z^+ = \{1, 2, 3, 4 \ldots\}$

$Z^- = \{\ldots -2, -1\}$.

* Non Negative ~~natu~~ integer $= \{0, 1, 2, 3 \ldots\}$

* Set of rational number.

$Q = \{ p/q \mid p \in z, \text{ and } q = 0\}$

eg :- $p = 2$ $q = 3$ $= 2/3$

* Set of real number

$R = \cancel{0}$

* Set of complex number

$C = x + iy, a + ib \ldots$

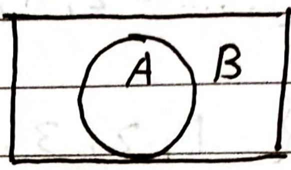
* Types of Sets.

* Empty set = ϕ = null set

* Sub set:

The set A is said to be subset of B if and only if every element of A is present in B.

$A \subseteq B$



$A \subseteq B$

B = super set of A

i) Every set is subset of itself

$A \subseteq A$

ii) Empty set is subset of any set.

* Size of set

Number~~ing~~ of distinct elements present in the set.

suppose S is the set

$|S| = 2$

eg: S denotes the +ive int less than or equal to 10

$|S| = 10$

for empty set $|\phi| = 1$

Date / /
Page

* Power Set

Given set s, then power set of s is the set of all subsets of s.

Ex: $S = \{0, 1, 2\}$

subset of s = ϕ, S, $\{0\}$, $\{0,1\}$, $\{0,2\}$
$\{1,2\}$, $\{1\}$, $\{2\}$.

$$P(S) = \{\phi, \{1\}, \{2\}, \{0\}, \{0,1\}, \{1,2\}, \{0,2\}, \{0,1,2\}.\}$$

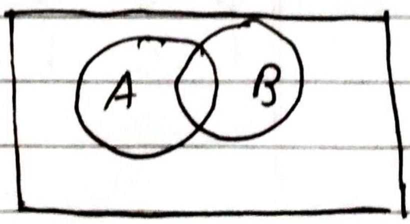
$$|P(S)| = 8 = 2^3$$

General form $\Rightarrow 2^n$

* Union

Let A & B be two sets then A union B is set of those elements that are either in A or in B or in Both.

$\therefore \Rightarrow A \cup B$



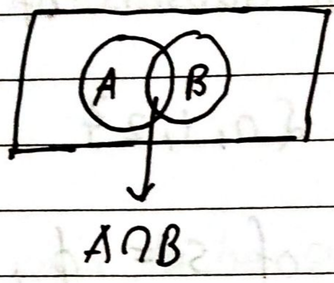
ex:- $A = \{1,3,4,5\}$
$B = \{2,4,6,7\}$

$$A \cup B = \{1,2,3,4,5,6,7\}$$

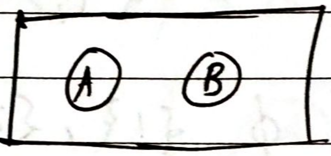
* Intersection

Let A, B be sets

$A \cap B$ =



$A \cap B$ =



$= \phi$

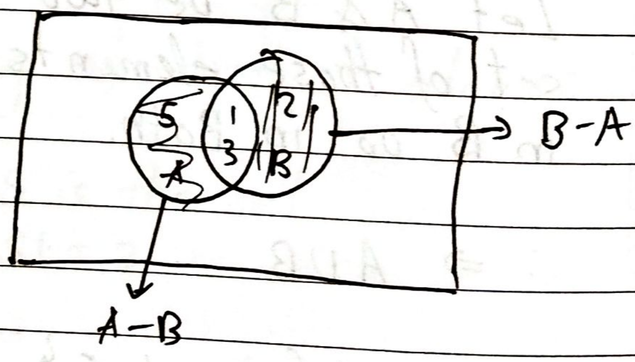
* Difference of sets

let $A, \& B$ be sets

$A - B$ = elements that are in A but not in B

$A = \{1, 3, 5\}$ $\Rightarrow$ $A - B = \{5\}$
$B = \{1, 2, 3\}$

$A - B$



Computer representation of sets.

Ex:- Let $U = \{1,2,3,4,5,6,7,8,9,10\}$

i) What bit string represent the subset of all odd integer in V?

$A = \{1,3,5,7,9\}$

1	2	3	4	5	6	7	8	9	10
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
1	0	1	0	1	0	1	0	1	0

Group of 4 from right

10 1010 1010

ii) What bit string represent the subset of all even integer in V?

$B = \{2,4,6,8,10\}$

1	2	3	4	5	6	7	8	9	10
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
0	1	0	1	0	1	0	1	0	1

01 0101 0101

iii) What bit string represent the subset of all even integers in ~~$\forall$?~~ not exceedding 5 in V?

$C = \{1, 2, 3, 4, 5\}$

$$\begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

11 1110 0000

$A \cup B = (10\ 1010\ 1010) \cup (01\ 0101\ 0101)$

$= (11\ 1111\ 1111)$

$A \cap B = (10\ 1010\ 1010) \cap (01\ 0101\ 0101)$

$= (00\ 0000\ 0000) = \phi$

Product of set / cartesian Product

let A & B be the sets

$A = \{a_1, a_2 \dots a_n\}$ $|A| = n$

$B = \{b_1, b_2 \dots b_m\}$ $|B| = m$

Then cartesian product of two sets A and B is denoted by $A \times B$ and is defined as

$$A \times B = \{ (a_i, b_j) / a_i \in A, b_j \in B \}$$
$$i = 1,2,3 \ldots n \quad j = 1,2,3 \ldots m$$

Ex: $A = \{1,2,3\}$, $B = \{4,5,6\}$

$$A \times B = \{ (1,4)\ (1,5)\ (1,6)\ (2,4)\ (2,5)\ (2,6)\ (3,4)\ (3,5)\ (3,6) \}.$$

$$|A \times B| = 9$$

$$B \times A = \{ (4,1)\ (5,1)\ (6,1)\ (4,2)\ (5,2)\ (6,2)\ (4,3)\ (5,3)\ (6,3) \}$$

$$|B \times A| = 9$$

$A \times B \neq B \times A$ (elements not equal

* If $|A| = n$ & $|B| = m$ then $|A \times B| = |B \times A| = m \times n$

Covering of sets.

Ex:- 1)

$A = \{1,2,3,4\}$

$A_1 = \{1,2\}$ $A_2 = \{2,3\}$ $A_3 = \{3,4\}$

$A_4 = \{2,3,4\}$

$A_1 \cup A_2 \cup A_3 \cup A_4 = A$

$\therefore$ The collection $\{A_1, A_2, A_3, A_4\}$ is called cover of A.

2)

$A = \{1,2,3,4\}$

$A_1 = \{1,2\}$ $A_2 = \{3,4\}$

$\Rightarrow$ $A_1 \cap A_2 = \phi$

$A_1 \cup A_2 = A$

Every partition is covering but not every covering is partition.

Relation

let $A = \{1,2\}$ $B = \{3,4,5\}$

$A \times B = \{(1,3)\ (1,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5)\}$

$P(A \times B) = 2^6$

let A & B be sets A binary relation from A to B is a subset of $A \times B$.

Ex:- 1

$A = \{1\}$ $B = \{2,3\}$

$A \times B = \{(1,2)\ (1,3)\}$

$|A| = 1$ $|B| = 2$ $|A \times B| = 2$

i) $R_1 = \phi$

ii) $R_2 = A \times B = \{(1,2)\ (1,3)\}$

iii) $R_3 = \{(1,2)\}$

iv) $R_4 = \{(1,3)\}$

$\therefore$ $P(A \times B) = 2^2 = 4.$

Number of elements in the power set $= |P(A \times B)|$
$= 2^{nm}$

$\therefore$ Number of relation from A to B $= 2^{nm}$

let A & B non empty sets

Relation is subset of its cartesian product

Page

$R \subseteq A \times B$

$|A| = m \qquad |B| = n$

if $A = B$ i.e $|A| = n$

$A = \{1,2,3\} \qquad B = \{1,2,3\}$

$R \subseteq A \times A$

No. of relation from A to $A = 2^{n \times n} = 2^{n^2}$

Type of Relation

i) Reflexive Relation

A Relation R on set A is called reflexive if $(a,a) \in R$ for every element $a \in A$.

Ex :- $A = \{1,2,3,4\}$
$A = \{1,2,3,4\}$

$$A \times A = \left\{ \begin{matrix} (1,1) & \ldots\ldots & (1,4) \\ (2,1) & \ldots\ldots & (2,4) \\ (3,1) & & (3,4) \\ (4,1) & & (4,4) \end{matrix} \right\}$$

* Same ordered pair should include them only reflexive.

Date / /
Page

1) $R_1 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

$(1,1)\ (2,2)\ (3,3)\ (4,4) \in R_1$

i.e $(a,a) \in R_1$ for every element $a \in A$

$\therefore R_1$ is Reflexive relation.

2) $R_2 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (3,1)\ (4,1)\}$

R_2 is reflexive.

3) $R_3 = \{(1,1)\ (2,2)\ (3,3)\}$

$(4,4) \notin R_3$

R_3 is not reflexive relation

4) $R_4 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\}$

$(5,5) \notin A \times A$

It is not in cartesion product

$\therefore R_4$ is not a relation.

5) $R_5 = \{A \times A\}$

$\therefore R_5$ is a reflexive relation.

2) Symmetric relation
A relation R on set A is called symmetric If $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$

Ex: $A = \{1,2,3,4\}$

$R_1 = \{(1,2)\ (2,1)\ (3,1)\ (1,3)\}$

R_1 is symmetric relation

Ex: $R_2 = \{(1,1)\ (2,1)\ (1,2)\ (3,3)\ (1,3)\}$

$(1,3) \in R_2$ But $(3,1) \notin R_2$

$\therefore$ R_2 is not symmetric relation

3) Anti symmetric relation
A Relation R on set A is called antisymmetric Rel^n If $(a,b) \in R$ then $(b,a) \notin R$ for $b \neq a$.

Ex: $A = \{1,2,3,4\}$

$R_1 = \{(1,2)\ (1,3)\ (1,4)\}$ But $(2,1) \notin R_1$
$(3,1) \notin R_1$
$(4,1) \notin R_1$

R_1 is antisymmetric relation

4) Transitive relation

A relation R on set A is called transitive If whenever is $(a,b) \in R$ and $(b,c) \in R$ The $(a,c) \in R$ for all $a,b,c \in A$.

Ex: i) $A = \{1,2,3,4\}$

$R_1 = \{(1,2)\ (2,3)\ (1,3)\}$

→

If $(1,2)$ and $(2,3) \in R_1$ Then $(1,3) \in R_1$

$\therefore$ R_1 is transitive relation

Ex: ii) $R_2 = \{(1,3)\ (3,1)\ (1,1)\ (3,2)\}$

→

If $(1,3)$ & $(3,2) \in R_2$ Then $(1,2) \notin R_2$

$\therefore$ R_2 is not transitive Relation.

Ex: iii) $R_3 = \{(1,4)(4,1)\ (1,1)\ (4,3)\ (1,3)\}$

→

1) $(1,4)\ (4,1) \in R_3 \Rightarrow (1,1) \in R_3$
2) $(1,4)\ (4,3) \in R_3 \Rightarrow (1,3) \in R_3$
3) $(1,1)\ (1,3) \in R_3 \Rightarrow (1,3) \in R_3$
4) $(4,1)\ (1,3) \in R_3 \Rightarrow (4,3) \in R_3$
5) $(4,1)\ (1,1) \in R_3 \Rightarrow (4,1) \in R_3$

$\therefore$ R_3 is transitive relation.

Page

Ex : iv) $R_4 = A \times A$

$\rightarrow$ R_4 is transitive

Ex : v) $R_5 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

$\rightarrow$ yes it is anti sym.

✱ ✱ $A = \{1,2,3,4\}$

i) $R_1 = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

Ref $\rightarrow$ Yes it is reflexive
Sym $\rightarrow$ Yes it is symmetric
Anti Sym $\rightarrow$ Yes it is antisymmetric
Transitive $\rightarrow$ Yes it is transitive.

ii) $R_2 = A \times A$

Ref $\rightarrow$ ✓
Sym $\rightarrow$ ✓
Anti $\rightarrow$ X
Tran $\rightarrow$ ✓

iii) $R_3 = \{(1,2)\ (1,3)\ (1,1)\ (3,3)\ (4,1)\}$

Ref $\rightarrow$ X
Sym $\rightarrow$ X
Anti $\rightarrow$ ✓
Tran $\rightarrow$ ✓

iv) $R_4 = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,4)\}$

Ref $\rightarrow$ X
Sym $\rightarrow$ X
Anti $\rightarrow$ X
Trans $\rightarrow$ X

Date / /

Page

5) Equivalence Relation

A relation R on set A is called an equivalent relation If it is reflexive symmetrical and transitive.

Ex: $A = \{1,2,3,4\}$

$R_1 = \{(1,1)(2,2)(3,3)(4,4)\}$

$R_2 = A \times A$

6) Partial order Relation

A relation R on set A is called partial order If it is reflexive antisymmetric, transitive.

Ex: $A = \{1,2,3,4\}$

$R_1 = \{(1,1)(2,2)(3,3)(4,4)\}$

$R_2 = A \times A$

It is not partial order reln.

7) Representing Relation:

i) Representing relation using matrices:

Suppose that R is relation from $A = \{a_1, a_2 \ldots a_n\}$ to $B = \{b_1, b_2 \ldots b_n\}$ Then Relation R can be represented by the matrix $M_R = [M_{ij}]_{m \times n}$

where M_{ij} $\begin{bmatrix} 1 \text{ if } (1,1) \in R \\ 0 \text{ if } (1,1) \notin R \end{bmatrix}$

$$M_R = \begin{bmatrix} m_{11} & m_{12} \ldots & m_{1n} \\ m_{21} & m_{22} \ldots & m_{2n} \\ \vdots & & \\ m_{m1} & m_{m2} \ldots & m_{mn} \end{bmatrix}_{m \times n}$$

Ex: i)

$A = \{1, 2, 3\}$ $B = \{4, 5\}$

$A \times B = \{(1,4)\ (1,5)\ (2,4)\ (2,5)\ (3,4)\ (3,5)\}$

$R = \{(1,5)\ (2,5)\ (3,5)\}$

$$M_R = \begin{bmatrix} M_{14} & M_{15} \\ M_{24} & M_{25} \\ M_{34} & M_{35} \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

2)

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

Date / /
Page L&W

$$\to \quad M_R = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}_{4\times 4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Representing Relation Using Diagraphs

A directed graph or diagraph, consist of set V of vertices together with set E of ordered pairs of elements of V called edges.

The vertex a is called intial vertex of the edge (a,b) and the vertex b is called the terminal vertex of this edge an edge of the form (a,a) .

a

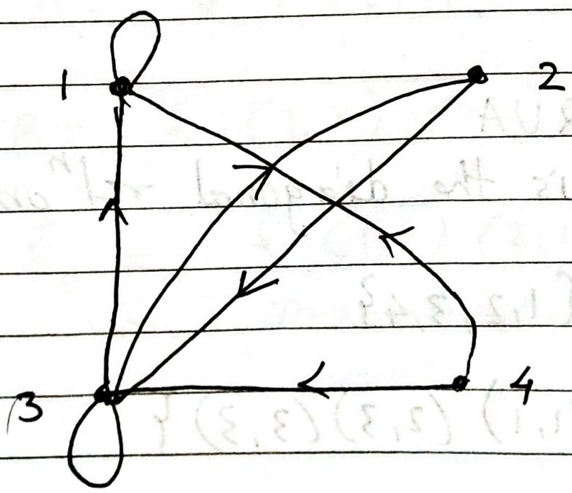
Ex: $A = \{1,2,3,4\} \Rightarrow$ (V) Vertices set

loop vertex

$R = \{(1,1)\ (2,3)\ (3,1)\ (3,2)\ (4,1)\ (4,3)\ (3,3)\}$

(E) ⇒ set of edges.

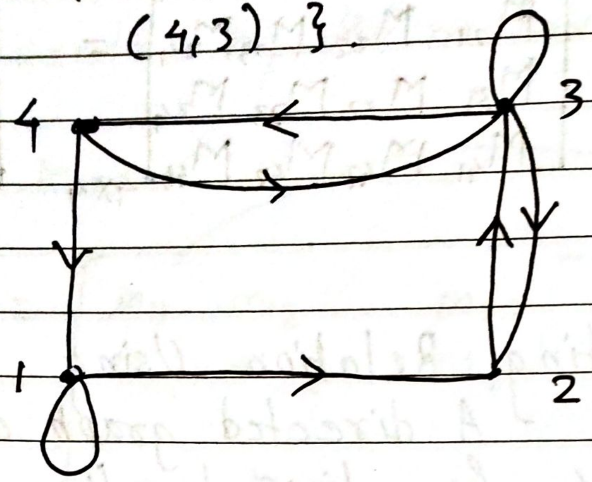
intial vertex / terminal vertex



2) $A = \{1,2,3,4\}$

$R = \{(1,1)\ (2,3)\ (3,3)\ (4,1)\ (3,2)\ (3,4)\ (1,2)$
$(4,3)\}$.

$\rightarrow$



Closure of relation.

$A = \{1,2,3,4\}$

$R = \{(1,1)\ (2,2)\ (1,2)\ (3,2)\ (4,4)\}$

$(3,3) \notin R$

$\bar{R}_r = \{(1,1)\ (2,2)\ (1,2)\ (3,2)\ (4,4)\ (3,3)\}$
↳ reflexive closure of relation R

$\bar{R}_r = R \cup \Delta$
where Δ is the diagonal relⁿ on set A

Ex:- $A = \{1,2,3,4\}$

$A = \{(1,1)\ (2,3)\ (3,3)\}$

$\overline{R_r} = R \cup \Delta$

where $\Delta = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

2) $A = \{1,2,3,4\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (3,2)\}$

→

$(2,3) \notin R$

$\overline{R_s} = \{(1,1)\ (1,2)\ (2,1)\ (3,2)\ (2,3)\}$

↳ symmetric closure of R.

$\overline{R_s} = R \cup R^{-1}$

↳ Inverse of relⁿ R.

Ex: $A = \{1,2,3,4\}$

$R = \{(1,1)\ (2,3)\ (2,1)\ (2,4)\ (3,3)\ (3,2)\ (4,4)\}$

→ $R^{-1} = \{(1,1)\ (3,2)\ (1,2)\ (4,2)\ (3,3)\ (2,3)\ (4,4)\}$

$R_s = R \cup R^{-1}$

$= \{(1,1)\ (2,3)\ (3,2)\ (2,1)\ (1,2)\ (4,2)\ (2,4)\ (3,3)\ (4,4)\}$.

3) $A = \{1,2,3,4\}$.

$R = \{(1,2)\ (2,1)\}$

$\overline{R_t} = \{(1,2)\ (2,1)\ (1,1)$

↳ Transitive clousure of R.

Date 18/09/21
Page

Warshal Algorithm.

Find the transitive clousre of relation R when
A = {1,2,3,4}
R = {(1,2) (2,1) (2,3) (3,4) (4,1)}

→

$$M_R = \begin{bmatrix} 0 & 1_{(1,2)} & 0 & 0 \\ 1_{(2,1)} & 0 & 1_{(2,3)} & 0 \\ 0 & 0 & 0 & 1_{(3,4)} \\ 1_{(4,1)} & 0 & 0 & 0 \end{bmatrix}_{4\times 4}$$

∵ Express given in matrix form.

$$W_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

∵ Write M_R as it is.

C_1 = column(1) = {2,4} R_1 = Row(C_1) = {2}

$C_1 \times R_1$ = {(2,2)(4,2)}
↳ Replace 1 on this place in W_0 for W_1, If already 1 then keep it.

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1_{(2,2)} & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1_{(4,2)} & 0 & 0 \end{bmatrix}$$

C_2 = {1,2,4} R_2 = {1,2,3}

Date / /
Page

$C_2 \times R_2 = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)$
$(4,1)\ (4,2)\ (4,3)\}.$

$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$C_3 = \{1,2,4\}$ $R_3 = \{4\}$

$C_3 \times R_3 = \{(1,4)\ (2,4)\ (4,4)\}$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$C_4 = \{1,2,3,4\}$ $R_4 = \{1,2,3,4\}$

$$C_4 \times R_4 = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4) \end{matrix} \right\}$$

$= A \times A$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\bar{R}_\ell = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4) \end{Bmatrix} = A \times A$$

2) $A = \{1,2,3,4\}$
$R = \{(2,1)\ (2,3)\ (3,1)\ (3,4)\ (4,1)\ (4,3)\}$

$\Rightarrow$

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}_{4\times 4}$$ $\rightarrow$ Depend on A

$$\omega_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$C_1 = \{2,3,4\}$ $R_1 = \{\ \} = \phi$
$C_1 \times R_1 = \phi$

$$\omega_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$C_2 = \{\ \} = \phi$ $R_2 = \{1,3\}$
$C_2 \times R_2 = \phi$

Date / /

Page

$$\omega_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$C_3 = \{2,4\}$ $R_3 = \{1,4\}$

$C_3 \times R_3 = \{(2,1)\ (2,4)\ (4,1)\ (4,4)\}$.

$$\omega_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & \underline{1} \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & \underline{1} \end{bmatrix}$$

$C_4 = \{2,3,4\}$ $R_4 = \{1,3,4\}$

$C_4 \times R_4 = \{(2,1)\ (2,3)\ (2,4)\ (3,1)\ (3,3)\ (3,4)\ (4,1)\ (4,3)\ (4,4).\}$

$$\omega_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & \underline{1} & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\bar{R}_t = \left\{ \begin{matrix} (2,1) & (2,3) & (2,4) \\ (3,1) & (3,3) & (3,4) \\ (4,1) & (4,3) & (4,4) \end{matrix} \right\}$$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_1 = \{1, 3\}$ $\qquad$ $R_1 = \{1, 3\}$

$C_1 \times R_1 = \{(1,1)\ (1,3)\ (3,1)\ (3,3)\}$

$$W_1 = \begin{bmatrix} 1_{(1,1)} & 0 & 1_{(1,3)} & 0 \\ 0 & 1 & 1 & 0 \\ 1_{(3,1)} & 0 & 1_{(3,3)} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

★ Partial Ordering or Partial order real^n.

A Relation R on set A is said to be spart. order relation if R is reflexive, antisymmetric and transitive.

Ex: $A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (1,3)\ (4,2)\}$

finite or infinte (set)

(A, R) Patial order relation on set A

$\left(A = \{1,2,3,4\} \quad R = \left\{ \begin{matrix} (1,1)\ (1,2)\ (1,3)\ (4,2) \\ (2,2)\ (3,3)\ (4,4) \end{matrix} \right\} \right)$

Ex: 2

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$R = \{(a,b) \mid a \leq b\}$

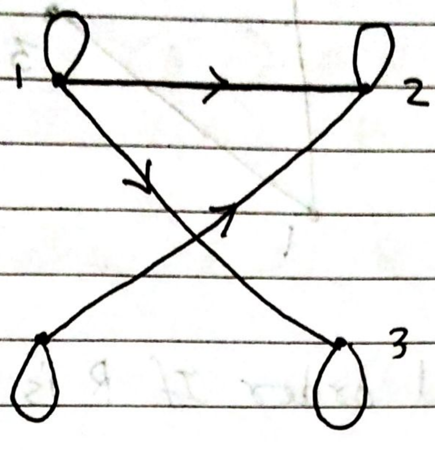
Ex:

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (1,3)$
$(4,2)\}$

Date / /
Page

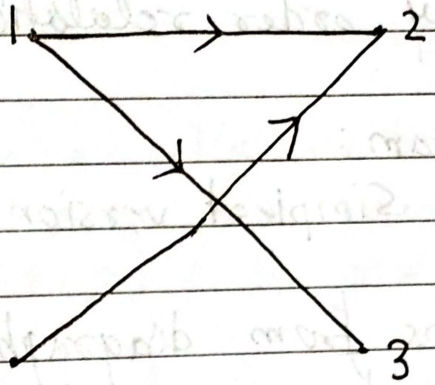
Diagraph



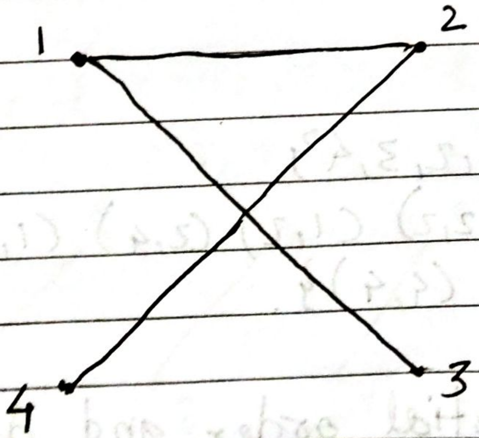
Remove loops

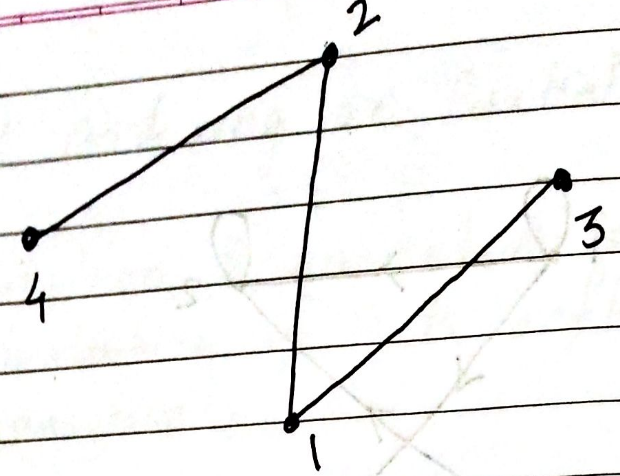
Remove transitive pair

Remove arrow 4



If here (1,2)(2,3) pair present you have to remove transitive pair (1,3)





Here diagram hosediagram

* If R is partial order If R is reflexive, Antisymmetric transitive.
  (A, R) - Poset or Partially ordered set where R is partially order relation.

* Hasse Diagram:
  Simplest version of diagraph

i) Remove loops from diagraph
ii) Delete all edges implied by transitivity property
iii) Omit the arrows
iv) Re-arrange.

Ex.: -1

Let $A = \{1, 2, 3, 4\}$
$R = \{(1,1) (2,2) (1,2) (2,4) (1,3) (3,3) (3,4) (1,4) (4,4)\}$.

S.T R is partial order and draw Hasses diagram.

⇒ Let's check it's transistive.

(a,b) & (b,c) ∈ R then (a,c) ∈ R.
(1,2) & (2,4) ∈ R then (1,4) ∈ R
⋮
All satisfied so R is transistive.

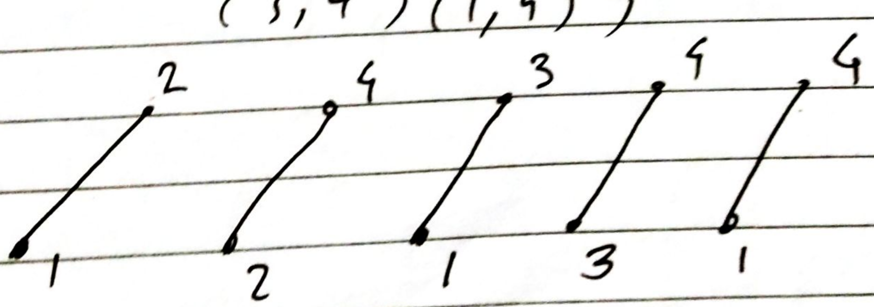
Let's check it's reflexive

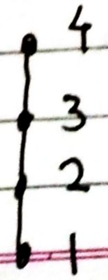
R (1,1) (2,2) (3,3) (4,4) ∈ R so it's reflexive.

Let's check it's antisymmetric.
Let's check for inverse
if (a,b) ∈ R then (b,a) ∉ R.

(2,4) ∈ R then (4,2) ∉ R
⋮
All satisfied so R is antisymmetric

$R = \{ \cancel{(1,1)}\ \cancel{(2,2)}\ \cancel{(3,3)}\ \cancel{(4,4)}\ (1,2)\ (2,4)\ (1,3)$
$(3,4)\ (1,4) \}$



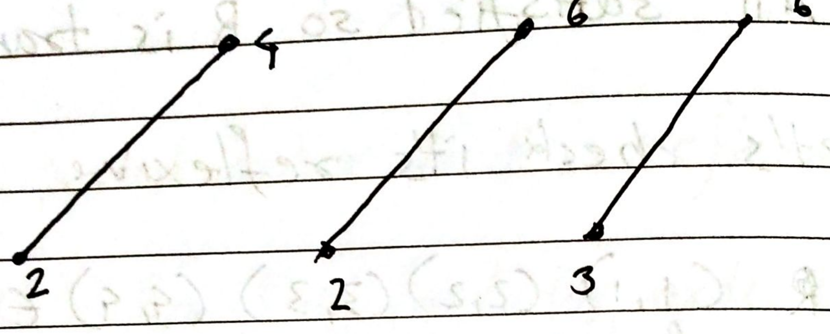


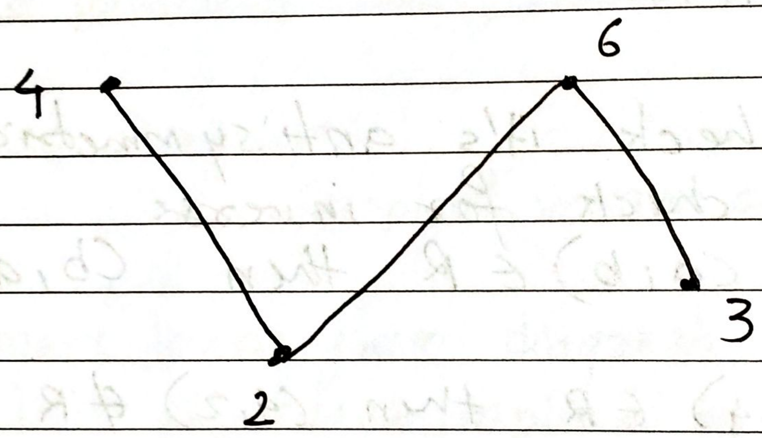
Ex: 2

$A = \{2, 3, 4, 6\}$

$R = \{(2,2)\ (2,4)\ (2,6)\ (3,3)\ (3,6)\ (4,4)\ (6,6)\}$

$\rightarrow \quad R = \{\cancel{(2,2)}\ (2,4)\ (2,6)\ \cancel{(3,3)}\ (3,6)\ \cancel{(4,4)}\ \cancel{(6,6)}\}$





Date 27/09/21

Page

Ex: $A = \{1,2,3,4\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (3,1)\ (4,1)$
$(4,3)\ (4,2)\ (3,2)\}$

→ First remove reflexive pair.

$R = \{\cancel{(1,1)}\ \cancel{(2,2)}\ \cancel{(3,3)}\ \cancel{(4,4)}\ (1,2)\ (3,1)\ \cancel{(4,1)}\ (4,3)\}$
$\cancel{(4,2)}\ \cancel{(3,2)}$

$(3,1)\ (1,2) \rightarrow (3,2)$ remove

$(4,1)\ (1,2) \rightarrow (4,2)$ transitive

$(4,3)\ (3,1) \rightarrow (4,1)$ pair.

$R = \{(1,2)\ (3,1)\ (4,3)\ \cancel{(3,2)}\}$

$4 < 3 < 1 < 2$