Date 23/11/21

4. Graph Theory

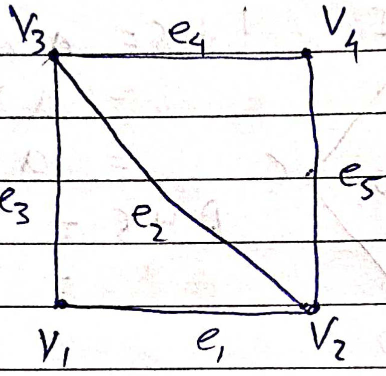
* Graph:

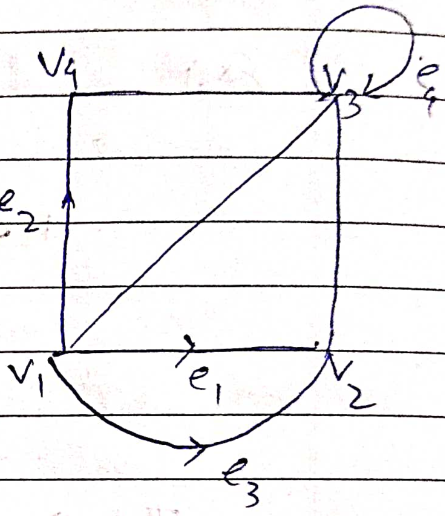
Graph is denoted by $G = (V, E)$

V = Vertices E = edges

consist of V non empty set of vertices & E is the set of edges each edges has either one or two vertices associated with it

$V = \{V_1, V_2, V_3, V_4\}$ $E = \{e_1, e_2, e_3, e_4, e_5\}$



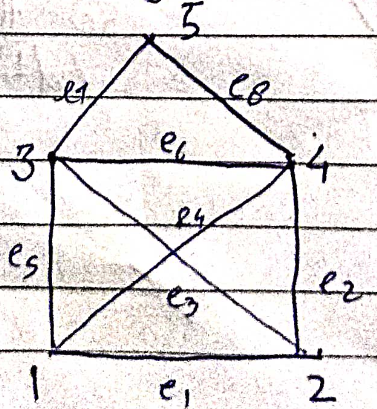


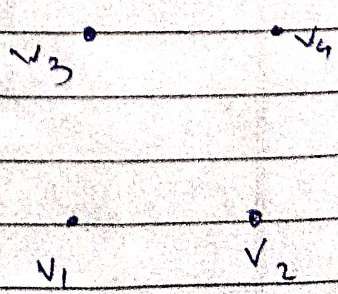
* Types of Graph.

1) Simple graph.

Graph that has neither self loop nor parallel edges

ex:





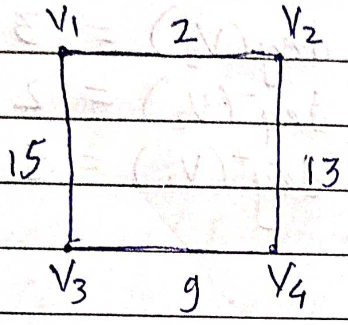
(graph) (Null)

Date / /
Page

2) Weighted Graph.

Let G be the graph will vertex set V and edge set E if each edge or each vertex are associated with positive real number then the graph is called weighted graph.

ex:



3) Degree of the vertex

It is the number of edges incident on the vertex V_i with self loop counted twice is called degree of vertex it is denoted by deg(Vi).

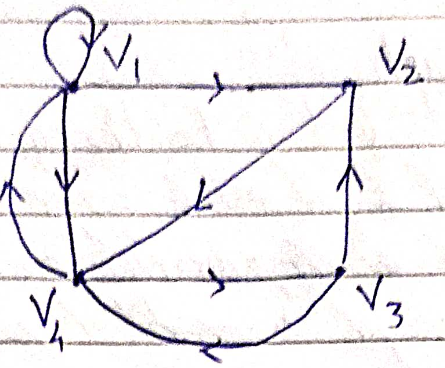
* Indegree of vertex

It is the number of edges as their terminal vertex & denoted by $deg^-(V_i)$

* Outdegree of vertex

It is the number of edges as their intial vertex & denoted by $deg^+(V_i)$.

Ex:



$deg(V_1) = 5 =$
$deg^-(V_1) = 2 +$
$deg^+(V_1) = 3$

$deg(V_4) = 5$	$deg(V_2) = 3$	$deg(V_3) = 3$
$deg^-(V_4) = 3$	$deg^-(V_2) = 2$	$deg^-(V_3) = 1$
$deg^+(V_4) = 2$	$deg^+(V_2) = 1$	$deg^+(V_3) = 2$

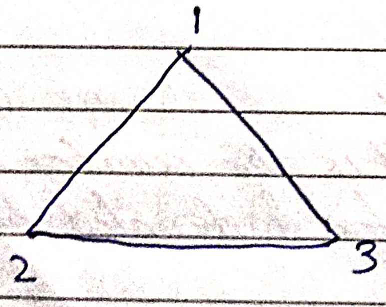
4) Complete Graph:-

It is a simple graph with n vertices if the degree of each vertex is $n-1$ then the graph is called complete graph

Ex.

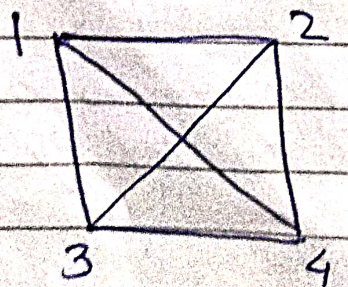
K_2

Ex



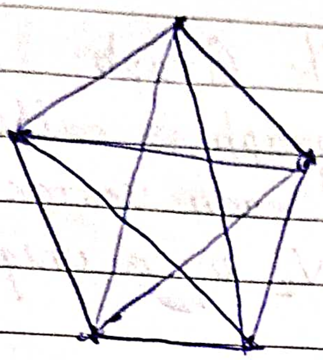
K_3

Ex



K_4

Ex



K_5

5) Bipartite Graph.

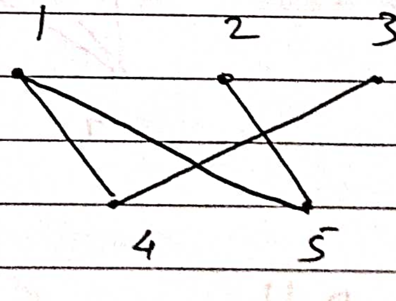
Let G be the graph with vertex set V and edges set E then G is called bipartite Graph. If its vertex set V can be partitioned into two disjoint subsets V_1 and V_2 such that $V_1 \cup V_2 = V$ & $V_1 \cap V_2 = \phi$ and also each edge of G joins vertex of V_1 to a vertex of V_2

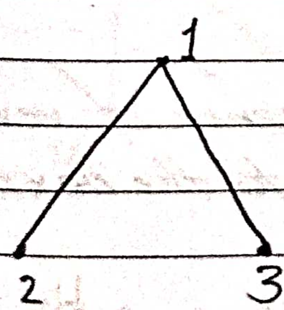
ex: $V = \{1, 2, 3, 4, 5\}$.

$V_1 = \{1, 2, 3\}$

$V_2 = \{4, 5\}$.

$\Rightarrow$



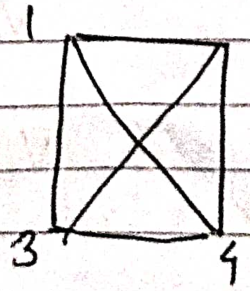


$V_1 = \{1\}$

$V_2 = \{2, 3\}$

$V_1 \cup V_2 = V$

$V_1 \cap V_2 = \phi$.



not bipartite graph.

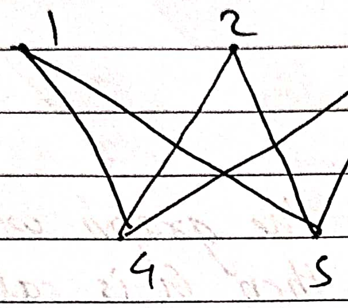
~~complete graphs are not bipartite graph~~

Date / /
Page

Complete Bipartite Graph

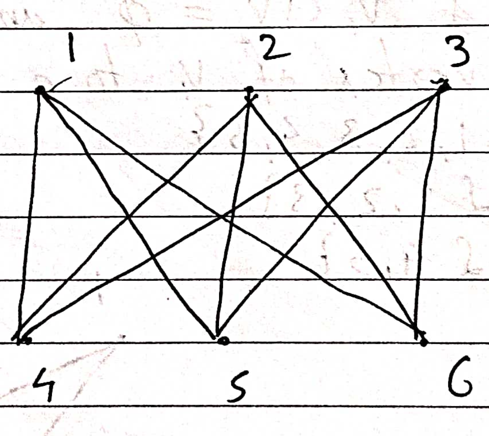
A bipartite graph is called complete bipartite graph if each vertex of V_1 is join to every vertex of V_2 by a unique edge

Ex:



$K_{3,2}$

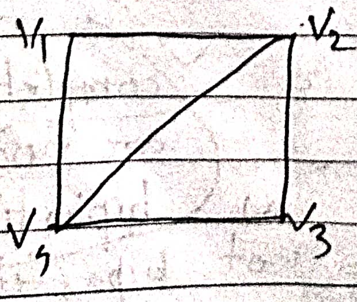
Ex:



$K_{3,3}$.

Path

Path in graph is sequence of vertices $(V_1, V_2 \ldots V_k)$ each adjacent to the next.



V_4, V_2, V_3 path.

V_1, V_3, V_4 not path

★ circuit & cycle

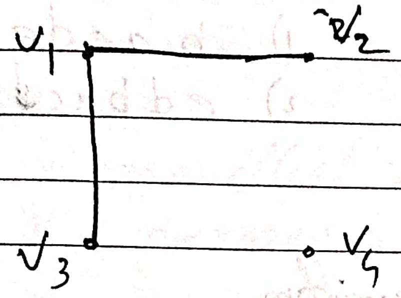
→ $V_1 V_V V_3 V_4 V_1$

→ $V_1 V_2 V_4 V_1$

★ Connected graph:

If there exist path between every pair of vertices.

Ex:

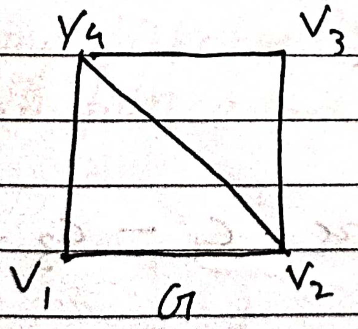


Disconnected graph.

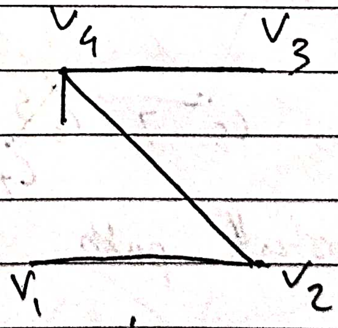
★ Subgraph of Graph

Let $G=(V,E)$ then the graph $G'=(V',E')$ is called subgraph of Graph G.
If $V' \subseteq V$, $E' \subseteq E$.

Ex:



G



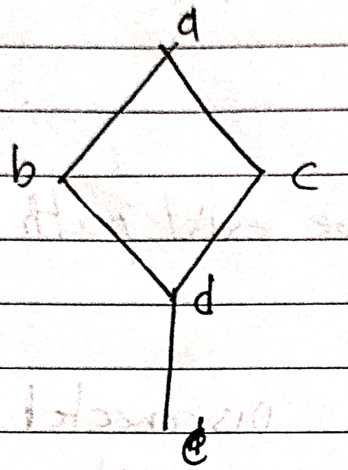
G'

★ Euler Path

Path in graph is euler path if it include every edge exactly once.

Hamiltonian Path:

If it include every vertex exactly once



Euler path

1) e d b a c d

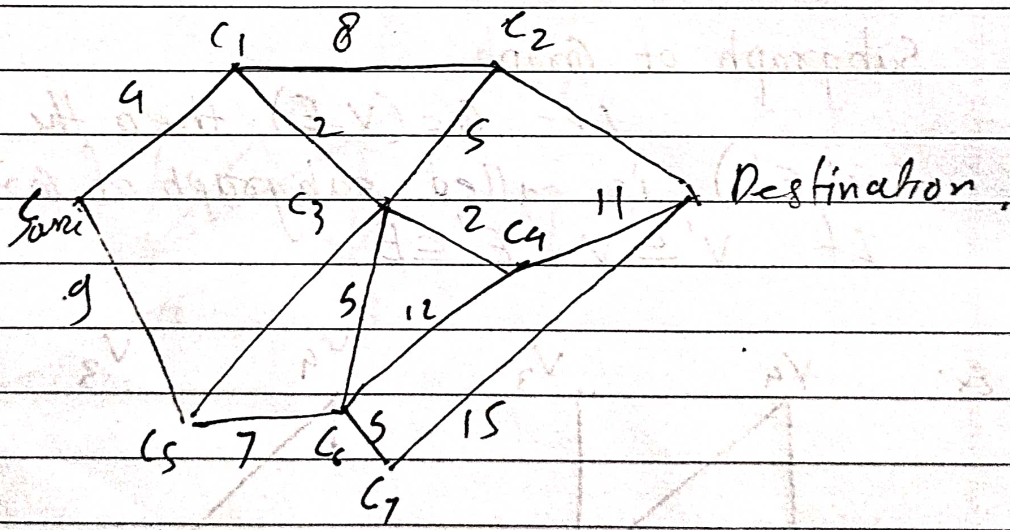
ii) e d c a b d

Hamiltonian path

1) b a c d e

2) e d b a c d

Shortest Path algorithm



shortest path source - C_1 - C_3 - C_4 - D.

Date / /
Page

$G = (V, E)$

Let a and z be any two vertices of graph $L(x)$ - denotes the label of vertex w_{ij} denotes the weight of edge (V_i, V_j)

Step (1)

$p = \phi$ $T = \{$ all vertices of graph $G\}$.

$L(a) = 0$, $L(x) = \infty$, $x \in T$, $x \neq a$.

Step (2)

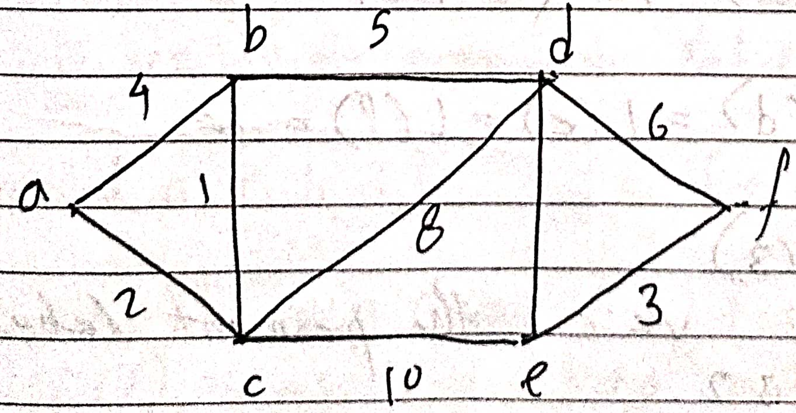
v = smallest label

$v \Rightarrow$ permanent label.

$P \cup v$ $T = \{$All vertices of graph $G - v\}$.

$$L(x) = \min\{ \text{old } L(x), L(v) + w(v, x)\}.$$

Ex. Using Dijkstra's Algorithm to find the shortest path from a to f.



Date / /

Page

step (1)

let $p = \phi$ $\quad T = \{a, b, c, d, e, f\}$

$L(a) = 0 \quad L(b) = L(c) = L(d) = L(e) = L(f) = \infty$

step (2)

let $v = a$ the permant label of a is 0.

$p = \{a\} \quad T = \{b, c, d, e, f\}$

The new label of vertex x in T is given by

$L(x) = \min\{\text{old } L(x),\ L(v) + w(v, x)\}$

$L(b) = \min\{\text{old } L(b),\ L(a) + w(a, b)\}$

$= \min\{\infty,\ 0 + 4\} = 4$

$L(c) = \min\{\infty,\ 0 + 2\} = 2$

~~$L(d) = \min\{\infty, 0+$~~

$L(d) = L(e) = L(f) = \infty$

Step (3)

$v = c$ the promant label of c is 2.

$p = \{a, c\}$ $T = \{b, d, e, f\}$.

$L(d) = \min\{ old(L(d)), L(c) + w(c, d) \}$
$= \min\{ \infty, 2+8 \}$
$= \min\{ \infty, 10 \}$
$= 10$

$L(b) = \min\{ 4, 3 \} = 3.$
$L(e) = 12$
$L(f) = \infty$

step (4)

$v = b$ the peomant label of b is 3

$p = \{a, c, b\}$ $T = \{d, e, f\}$

$L(d) = \min (10, 3+5 \}$
$= 8$
$L(e) = 12$
$L(f) = \infty$

step(5)

$v = d$ the peromaint label of d is 8

$p = \{a, c, b, d\}$ $T = \{e, f\}$

$L(e) = \min (\infty, 8+2 \} = 10$
$L(f) = \min (\infty, 8+6) = 14$

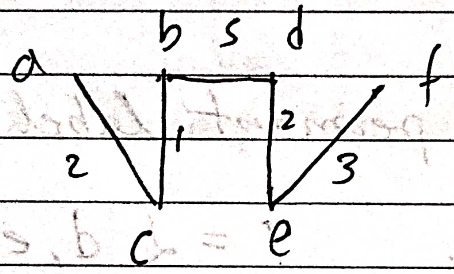
Step 6)

v = e the perment label of e = 10

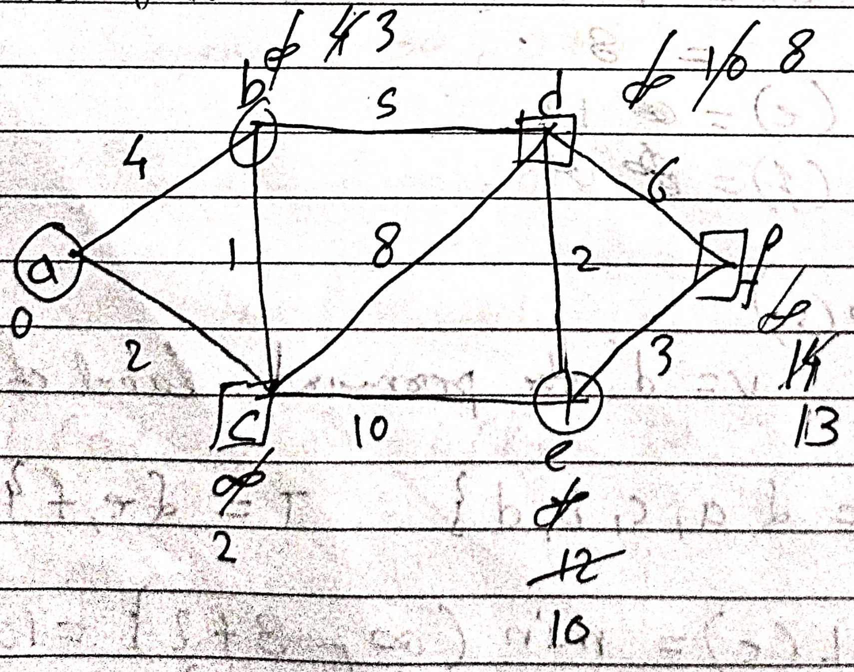
$P = \{a, c, b, d, e\}$ $T = \{f\}$

$L(f) = \{\infty, 10+3\}$
$= 13$

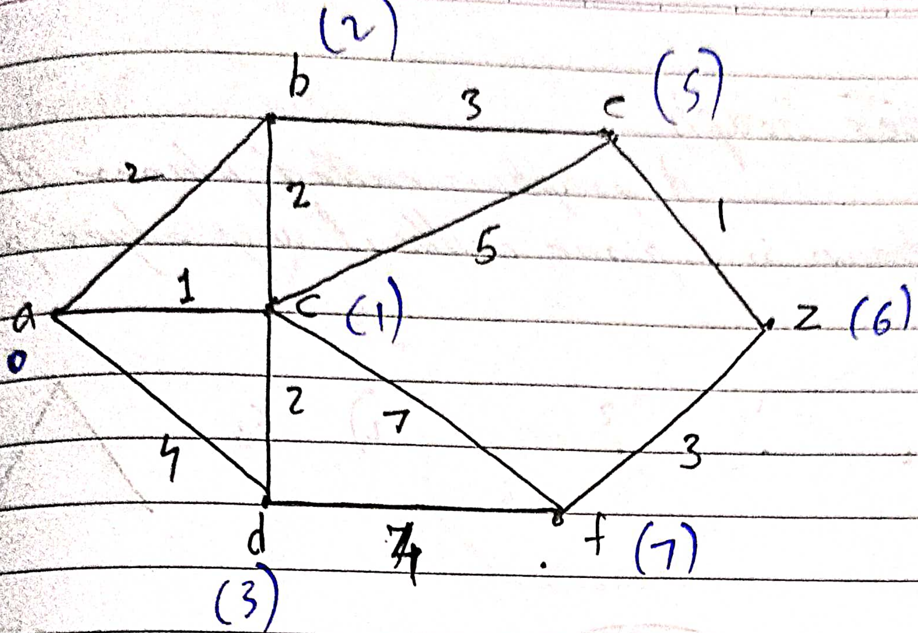
$P = \{a, c, b, d, e, f\}$.



Shotcut trick

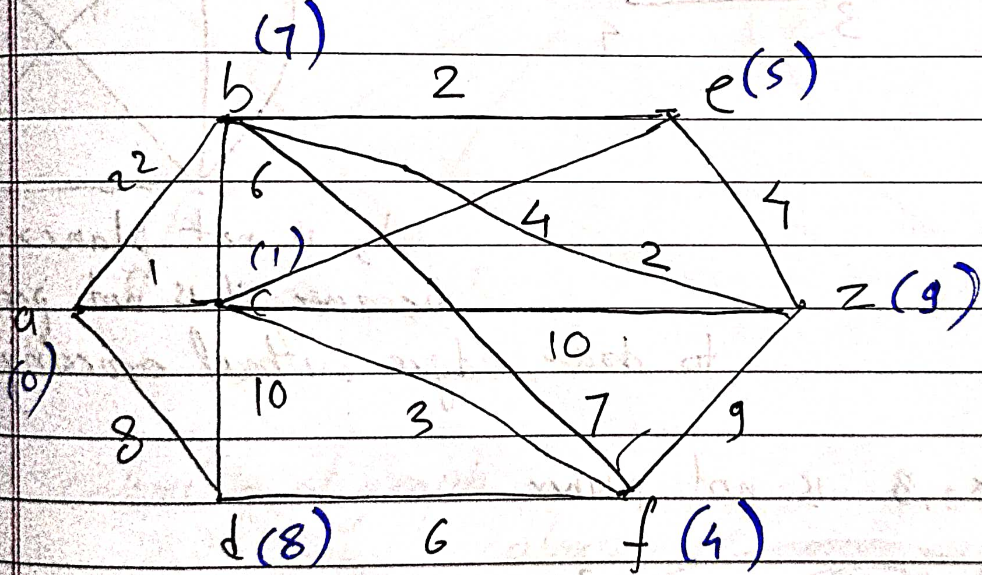


Ex:-



$L(a) = 0 \quad L(b) = 2 \quad L(d) = 3 \quad L(c) = 1$

$L(e) = 5 \quad L(f) = 7 \quad L(z) = 6.$



$L(a) = 0 \quad L(b) = 7 \quad L(e) = 5$

$L(d) = 8 \quad L(c) = 1 \quad L(f) = 4$

$L(z) = 9$

Date 29/11/21

Page

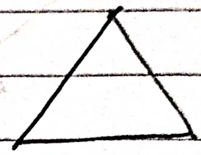
* Planer Graph

A graph is called planer If it can be drawn in a plane without any edges possible

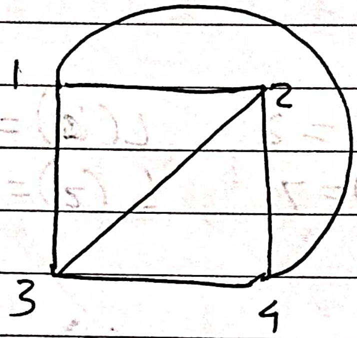
Ex:

1) K_2

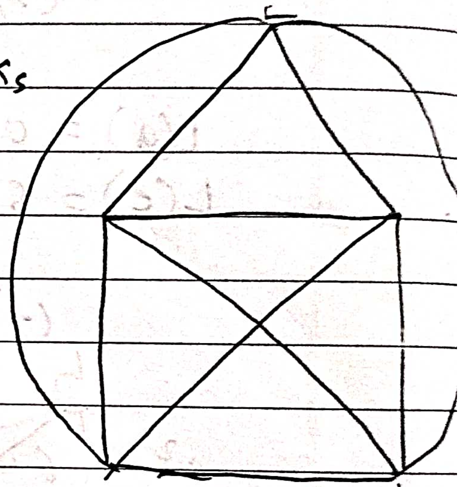
2) K_3



3) K_4

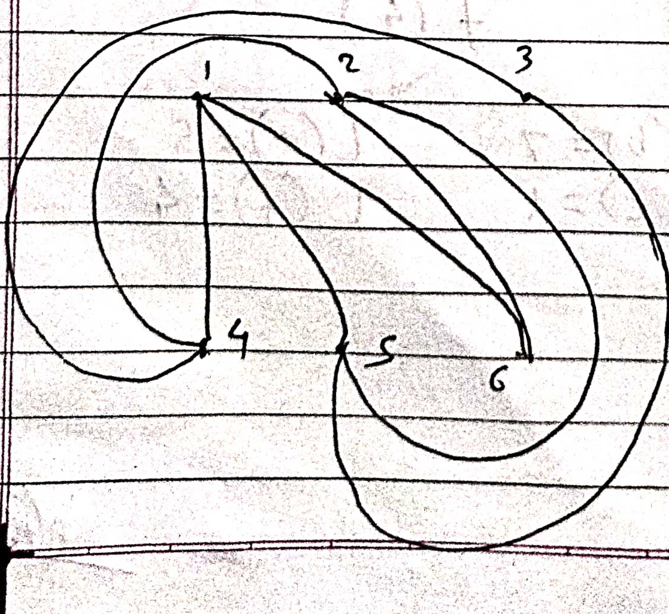


4) K_5



K_5 is not planer because it is not possible to draw edges without crossing.

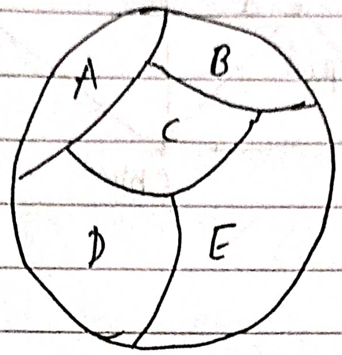
5) $K_{3,3}$ is not planer

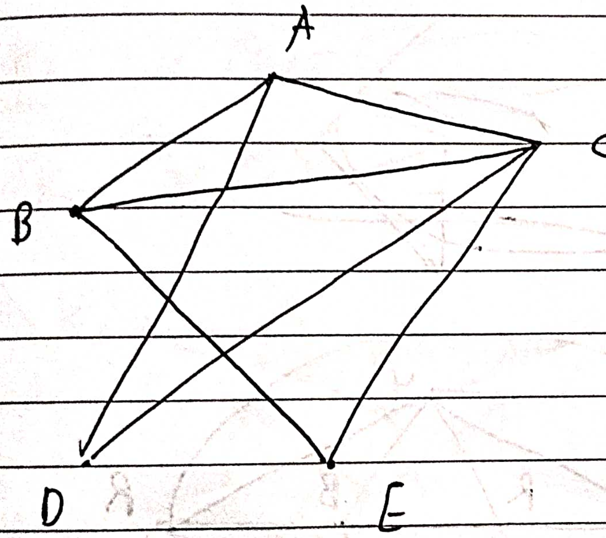


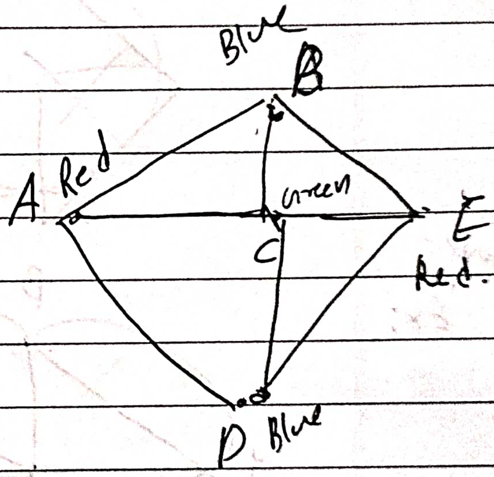
* Graph coloring

Map coloring

Ex:







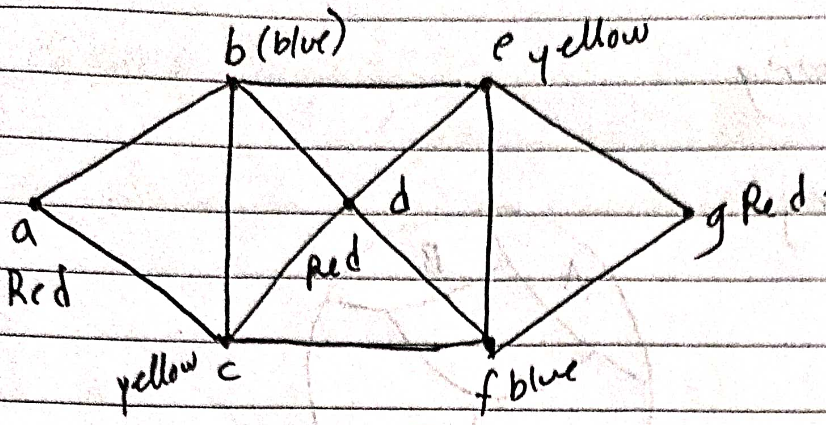
* Coloring of graph

coloring of simple graph is the assignment of color to each vertex of graph so that no two adjacent vertices are assign the same color.

* Chromatic number of graph $\chi(G)$.

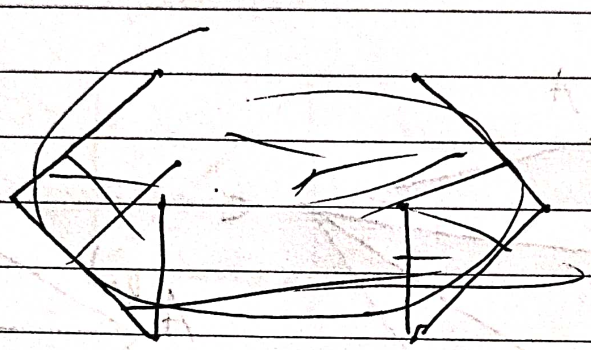
Minimum number of color required to graph.

Ex:

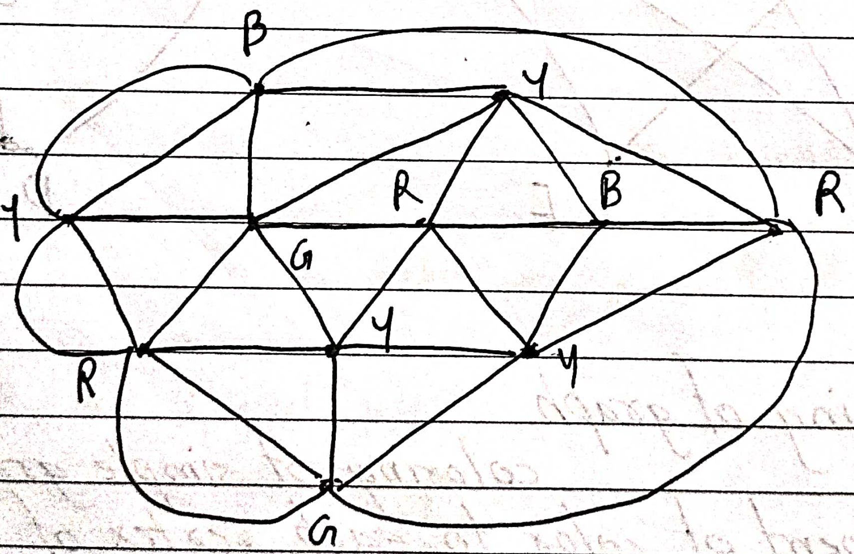


$\chi(G) = 3$

* Ex



Imp:
Ex:



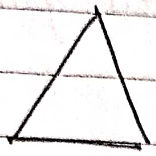
* chromatic number of planer graph is less than or equal to 4

$\chi(G) \leq 4$

Ex: 1) K_2

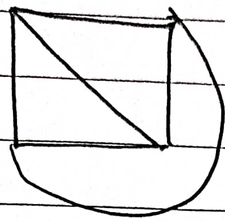
$\chi(G) = 2$

2) K_3



$\chi(G) = 3$

3) K_4

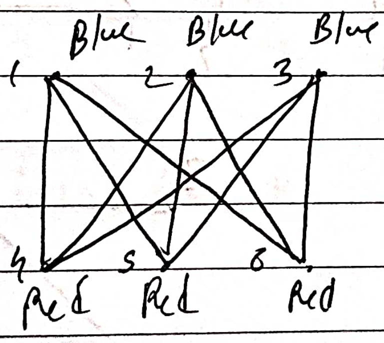


$\chi(G) = 4$

4) K_5 $\chi(G) = 5$

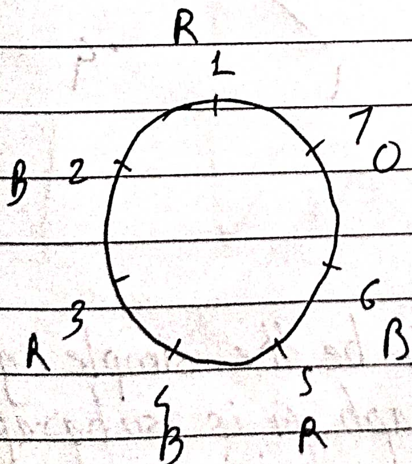
5) K_n $\chi(G) = n$

6) $K_{3,3}$



$\chi(G) = 2$.

7)



even no of vertices
$\chi(G) = 2$

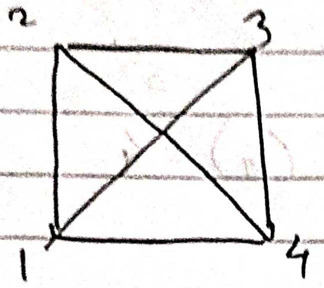
odd no of vertices
$\chi(G) = 3$.

Date 30/11/21

Page

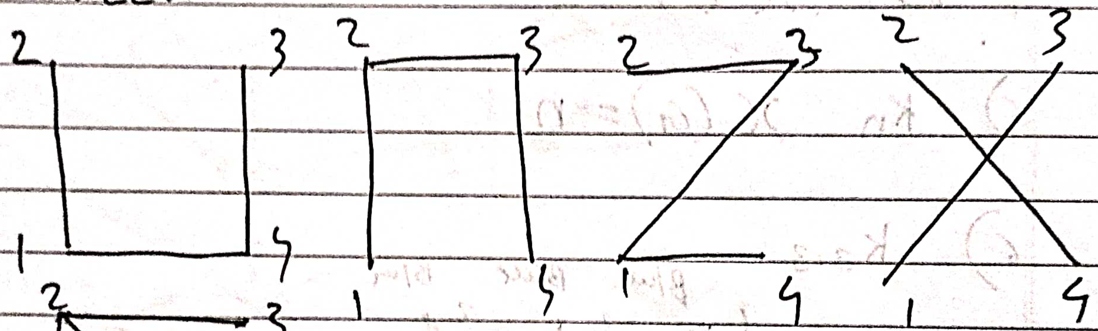
Tree :-

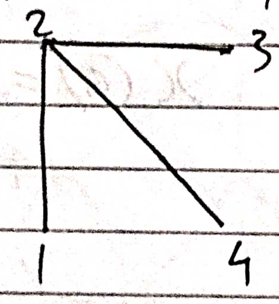
Connected, undirected graph with no circuits. & no cycle.

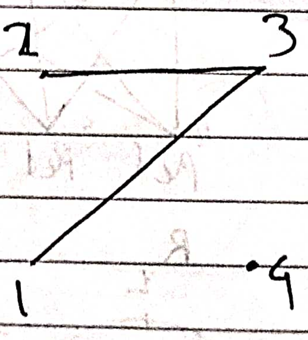


Graph

Trees







not connecte

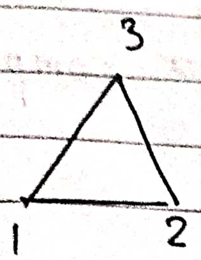
so

it is not a tree.

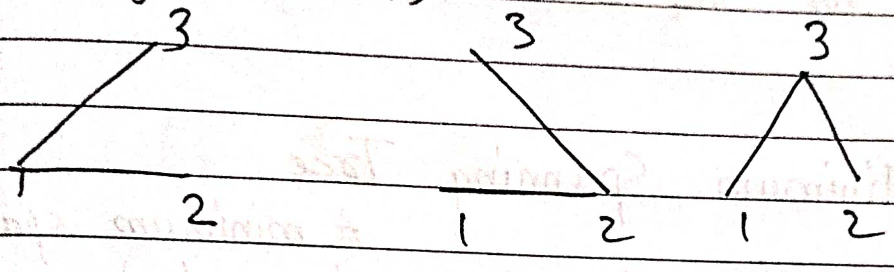
Spanning Tree

Let G be the simple graph A spanning tree of graph G is subgraph of G, that is a tree containing every vertices of G.

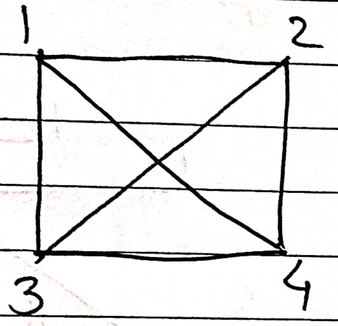
Ex: K_3



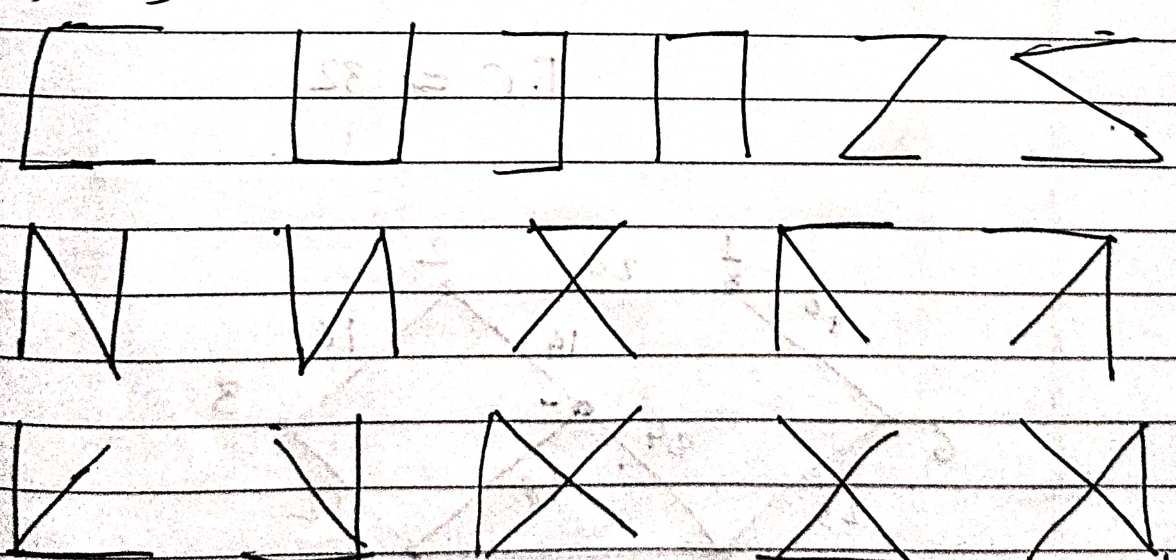
spanning tree of K_3



2) K_4



Spaning tree for K_4



No of spanning tree for complete graph

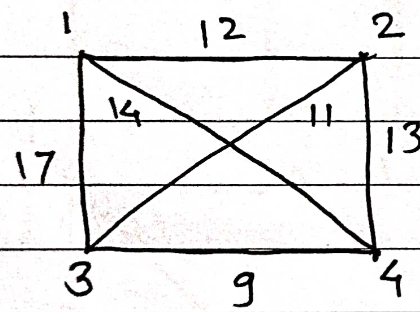
$= n^{n-2}$ 5^3

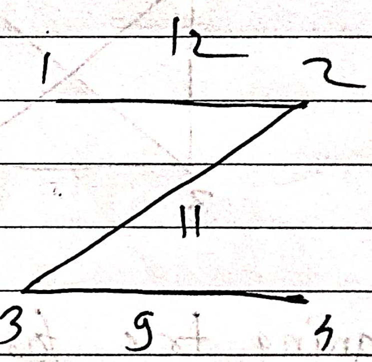
for $K_4 = 4^{4-2} = 4^2 = 16$

* Minimum Spanning Tree

A minimum spanning tree in connected weighted graph is spanning tree that has the smallest possible ~~so~~ sum of weights of it's edges.

Ex:





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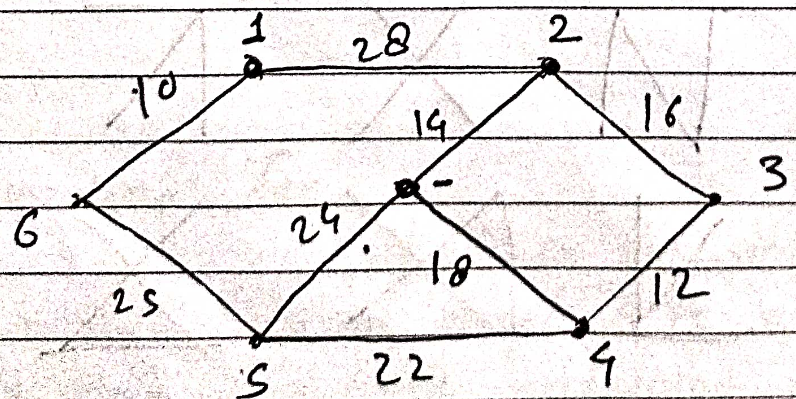

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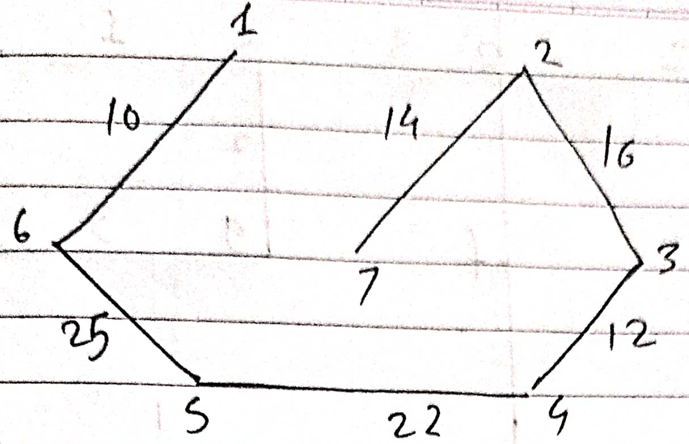
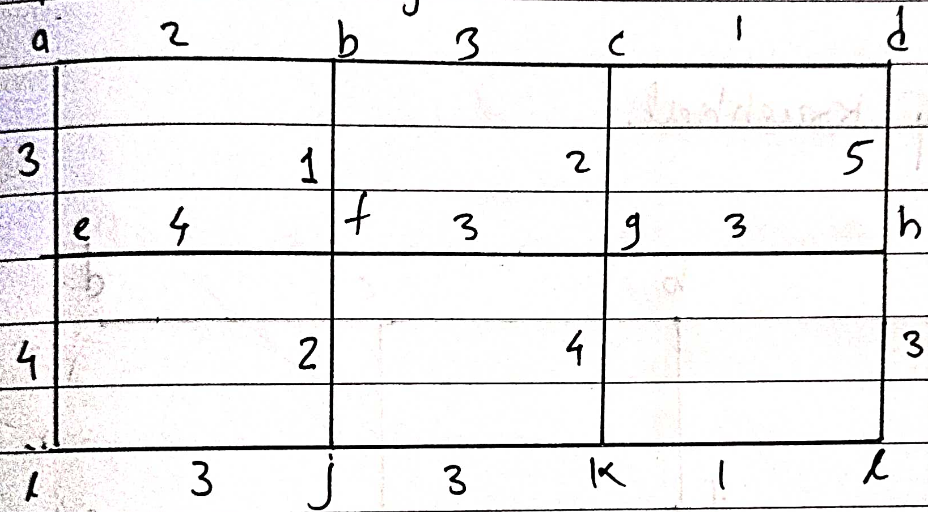
Page

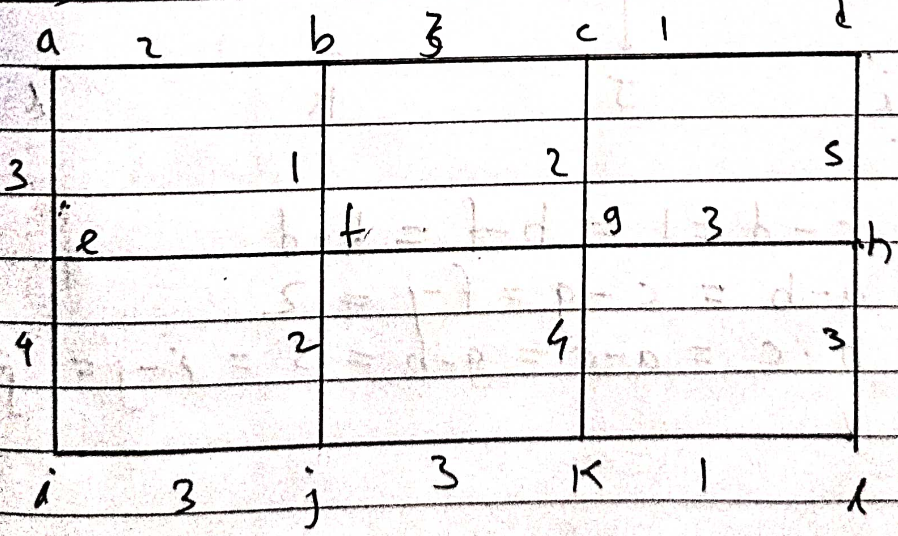


Kruskal's Algorithm.



⇒ prim's

Q.



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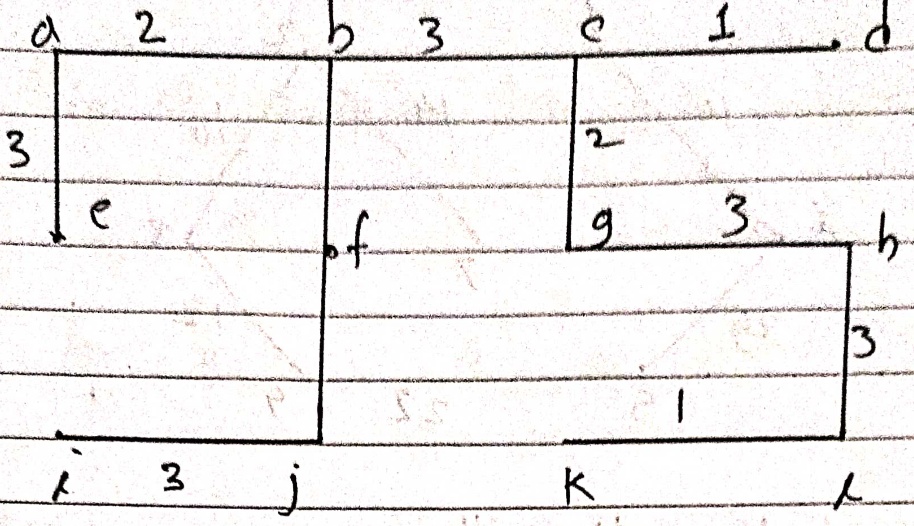
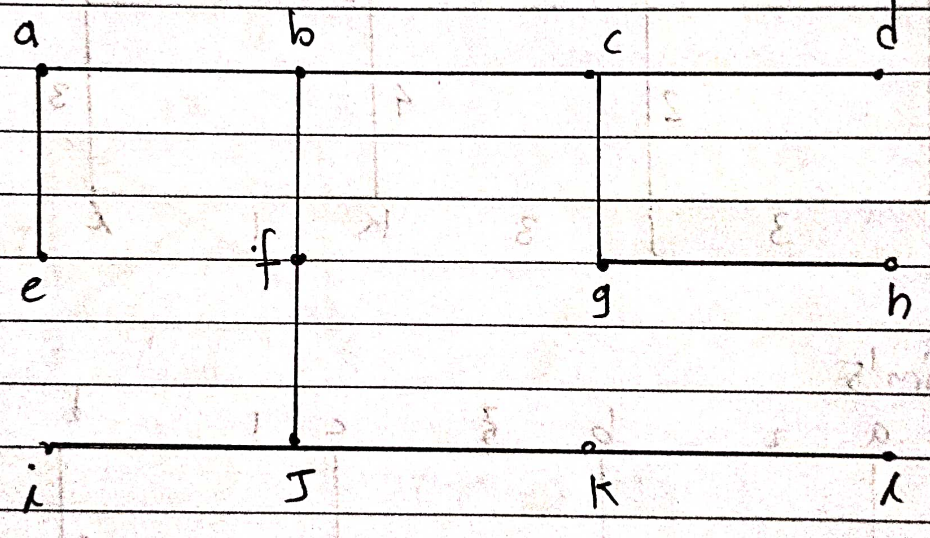


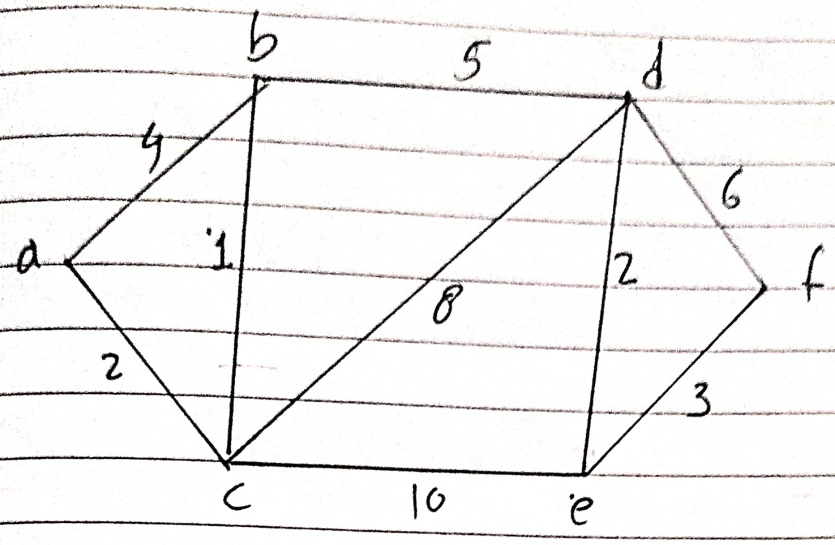
24

⇒ By Krushkall.

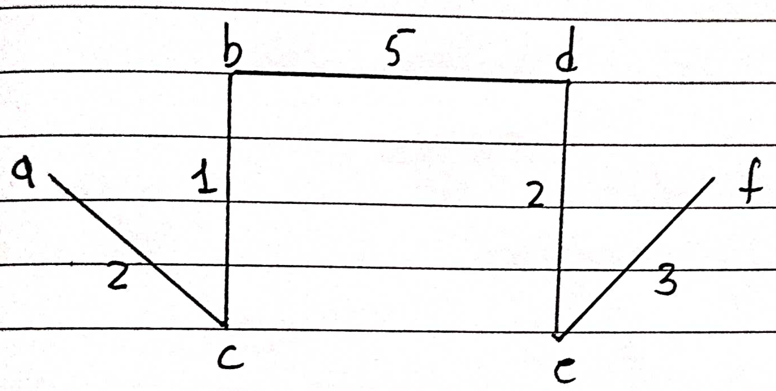


1) $c-d = 1 = b-f = k-d$
2) $a-b = c-g = f-j = 2$
3) $b-c = a-c = g-h = 3 = i-j = j-k$

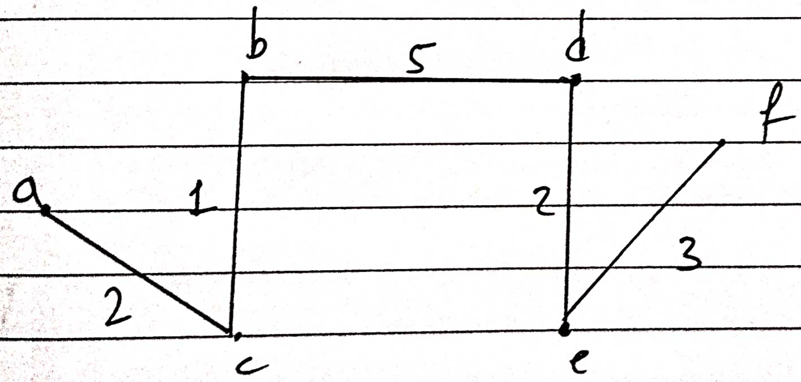
Ex:



shortest path.



prims algo



krushkal algo.