COMP 335 Assignment 7

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Question 1.

(a) Given Grammar:
S $\rightarrow$ ABaB|Bab
A $\rightarrow$ aA|bA|AC
B $\rightarrow$ ba|aB|λ
C $\rightarrow$ a|bC|λ

Step 1: Remove all λ productions
S $\rightarrow$ ABaB|ABa|AaB|Bab|Aa|ab
A $\rightarrow$ aA|bA|AC|A
B $\rightarrow$ ba|aB|a
C $\rightarrow$ a|bC|b

Step 2: Remove all unit productions
S $\rightarrow$ ABaB|ABa|AaB|Bab|Aa|ab
A $\rightarrow$ aA|bA|AC|aA|bA|AC
B $\rightarrow$ ba|aB|a
C $\rightarrow$ a|bC|b

Step 3: Remove all useless productions
S $\rightarrow$ ABaB|ABa|AaB|Bab|Aa|ab
A $\rightarrow$ aA|bA|AC
B $\rightarrow$ ba|aB|a
C $\rightarrow$ a|bC|b

Chomsky Normal Form:
S $\rightarrow V_0V_1|V_0T_a|V_2\text{B}|\text{B}V_3|\text{A}T_a|T_aT_b$
A $\rightarrow T_a\text{A}|T_b\text{A}|\text{AC}$
B $\rightarrow T_bT_a|T_a\text{B}|T_a$
C $\rightarrow T_a|T_b\text{C}|T_b$
$V_0 \rightarrow$ AB
$V_1 \rightarrow T_a$B
$V_2 \rightarrow \text{A}T_a$
$V_3 \rightarrow T_aT_b$
$T_a \rightarrow$ a
$T_b \rightarrow$ b

(b) Given Grammar:
S $\rightarrow$ 1P0Q
P $\rightarrow$ RQ|01|001
R $\rightarrow$ 0R|λ
Q $\rightarrow$ 1Q|λ

Step 1: Remove all λ productions
S $\rightarrow$ 1P0Q
P $\rightarrow$ RQ|R|Q|01|001
R $\rightarrow$ 0R|0
Q $\rightarrow$ 1Q|1

Step 2: Remove all unit productions
S $\rightarrow$ 1P0Q
P $\rightarrow$ RQ|0R|0|1Q|1|01|001
R $\rightarrow$ 0R|0
Q $\rightarrow$ 1Q|1

Step 3: Remove all useless productions
No useless productions

Chomsky Normal Form:
S $\rightarrow V_0$Q
P $\rightarrow$ RQ|T_0R|T_1Q|T_0V_2|V_2|T_1|T_0
R $\rightarrow T_0$R|T_0
Q $\rightarrow T_1$Q|T_1
$V_0 \rightarrow T_1$P
$V_1 \rightarrow T_0$Q
$V_2 \rightarrow T_0T_1$
$T_0 \rightarrow 0$
$T_1 \rightarrow 1$

Question 2.

Let grammar G be a grammar that does not have useless productions, such that when the procedure to remove unit productions is applied to G, we get a grammar G' that has useless productions.

Grammar G:
S $\rightarrow T_0V_0|T_1V_1|V_2$
$V_0 \rightarrow T_0$S$|T_0T_0$
$V_1 \rightarrow V_0|T_1$
$V_2 \rightarrow T_0T_1$
$T_0 \rightarrow 0$
$T_1 \rightarrow 1$

Removing unit productions we get:
Grammar G':
S $\rightarrow T_0V_0|T_1V_1|T_0T_0$
$V_0 \rightarrow T_0$S$|T_0T_0$
$V_1 \rightarrow T_0$S$|T_0T_0|T_1$
$V_2 \rightarrow T_0T_1$
$T_0 \rightarrow 0$
$T_1 \rightarrow 1$

Here (in G'), V_2 is a useless production.