COMP 335 Assignment 9

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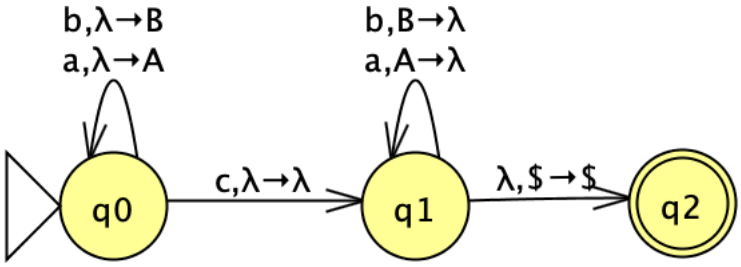
Question 1.

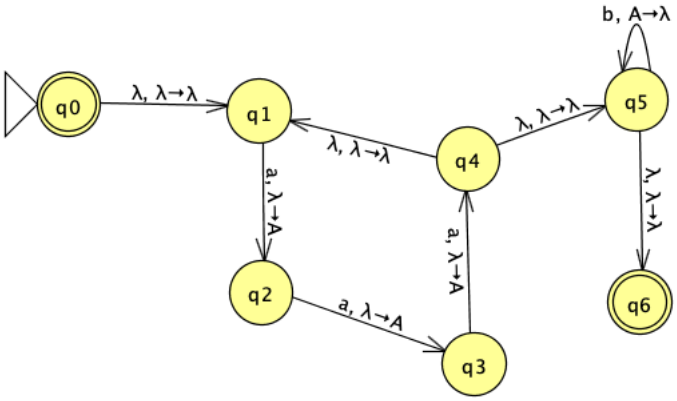
It is a **deterministic** CFL as the following DPDA accepts it.



Question 2.

(a)

The language $L_1 = \{w : w \text{ is of the form } a^n b^n \text{ and } |w| = 3k, n, k \geq 0\}$ can be represented as the following PDA:



Considering the PDA-CFL equivalence, we can say that the given language is a Context-free language.

(b)

Suppose $L_2 = \{a^k : k = n^2, n \geq 0\}$ is context-free.

Let m be the constant of the context-free pumping lemma for cfls. Choose $w = a^{m^2}$. Clearly w ∈ L and $|w| \geq m$. Let $w = uvxyz$ with $|vxy| \leq m$ and $|vy| \geq 1$. Then vy must be a^j for some j with $1 \leq j \leq m$

While uvxyz$\in L_2$,
uxz=$a^{k-j} \notin L_2$, (since $k-j$ is not necessarily $(n-1)^2$)

This is a **contradiction** to the Pumping Lemma.
∴,L_2 cannot be context-free.

(c)
Suppose $L_3 = \{a^i b^j c^k : k = ij\}$ is context-free.
Let m be the constant of the context-free pumping lemma for cfls. Choose $w = a^m b^m c^{m^2}$. Clearly w ∈ L and $|w| = m^2 + 2m > m$. Let $w = uvxyz$ with $|vxy| \leq m$ and $|vy| \geq 1$. Then one of the following cases holds:

Case 1: vy contains only a's.
Let $vy = a^j$ for some $1 \leq j \leq m$
Then $uxz = a^{m-j} b^m c^{m^2} \notin L$
Contradiction.

Case 2: vy contains only b's.
Let $vy = b^j$ for some $1 \leq j \leq m$
Then $uv^2xy^2z = a^m b^{m+j} c^{m^2} \notin L$
Contradiction.

Case 3: vy contains only c's.
Let $vy = c^j$ for some $1 \leq j \leq m$
Then $uxz = a^m b^m c^{m^2+j} \notin L$
Contradiction.

Case 4: vy contains both a's and b's.
Let $vy = a^j b^k$ for some $1 \leq j + k \leq m$
Then $uxz = a^{m-j} b^{m-k} c^{m^2} \notin L$
Contradiction.

Case 5: vy contains both b's and c's.
Let $vy = b^j c^k$ for some $1 \leq j + k \leq m$
Then $uxz = a^m b^{m-j} c^{m^2-k} \notin L$
Contradiction.

Case 6: vy contains both a's and c's.
Impossible since $|vxy| \leq m$
Contradiction.

In every case, we got a contradiction to the Pumping Lemma.
∴,L_3 cannot be context-free.