COMP 335 Assignment 5

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Question 1.

$L_1 = \{a^{n^3+5} : n \geq 0\}$
Assume for contradiction that L_1 is regular and apply the Pumping Lemma.

Let m be the integer given by the pumping and choose $w = a^{m^3+5}$
Since $w \in L_1$ we can write $w = xyz$ such that $|xy| \leq m$ and $|y| \geq 1$, as well as $xy^iz \in L_1$ for all i =0,1,2,...
Let $|y| = k$, so $k \leq m$ and $xy^2z \in L_1$ where $xy^2z = a^{m^3+5+k}$

We know that, $m^3 + 5 < m^3 + 5 + k$ (Since $k \geq 1$)
$m < 3m < 3m^2 + 3m + 1$
$m^3 + k < m^3 + m < m^3 + 3m^2 + 3m + 1 < (m+1)^3$ (Since $|xy| \leq m \Rightarrow |y| \leq m$)
$m^3 + 5 < m^3 + 5 + k < m + 1^3 + 5$
$\therefore, a^{m^3+5} < a^{m^3+5+k} < a^{m+1^3+5}$
and, $a^{m^3+5+k} \notin L_1$

Therefore, the pumped string is of length that is strictly in between the two powers and cannot be in L_1.
Contradiction.

Question 2.

$L_2 = \{vuv^R : v, u \in \{a, b\}^*, |u| = 2\}$
Assume for contradiction that L_2 is regular and apply the Pumping Lemma.
Let m be the integer given by the pumping and choose $w = a^mb^mabb^ma^m$, where $v = a^mb^m, u = ab$ and $v^R = a^mb^m$
Since $w \in L_2$ we can write $w = xyz$ such that $|xy| \leq m$ and $|y| \geq 1$, as well as $xy^iz \in L_2$ for all i =0,1,2,...
Let $|y| = k$, then $xy^2z \in L_2$ where $xy^2z = a^{m+k}b^mabb^ma^m$

Here, $u = ab, v = a^{m+k}n^m$ and $v^R = b^ma^m$, however v^R is not the reverse of v, since $k \geq 1$
$\therefore, a^{m+k}b^mabb^ma^m \notin L_2$

Therefore, the pumped string produces a string that is not of the form vuv^R and cannot be in L_2.
Contradiction.

Question 3.

$L_3 = \{a^ib^jc^k : i \geq 2j + 3k\}$
Assume for contradiction that L_3 is regular and apply the Pumping Lemma.
Let m be the integer given by the pumping and choose $w = a^{5m}b^mc^m$
Since $w \in L_3$ we can write $w = xyz$ such that $|xy| \leq m$ and $|y| \geq 1$, as well as $xy^iz \in L_3$ for all i =0,1,2,...
Let $|y| = k$.
Pumping down, we get $a^{5m-k}b^mc^m$
Since $5m - k < 5m \Rightarrow$ the string is not of the form $a^ib^jc^k : i \geq 2j + 3k$
$\therefore, a^{5m-k}b^mc^m \notin L_3$

Therefore, the pumped down string cannot be in L_3.
Contradiction.