COMP 335 Assignment 1

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Question 1.

(a)
My example:
L = $\{(ab)^i, 3 \geq i \geq 1\}$

Reasoning:
L = {ab, abab, ababab}
|L| = 3
2|L|=6 L^R = {ba, baba, bababa}
L ∪ L^R = {ab, ba, abab, baba, ababab, bababa}
|L ∪ L^R| = 6
∴, |L ∪ L^R| = 2|L|

(b)
My example:
L = $\{(ab)^i, 3 \geq i \geq 0\}$

Reasoning:
L = {λ, ab, abab, ababab}
|L| = 4
2|L|=8
L^R = {λ, ba, baba, bababa}
L ∪ L^R = {λ, ab, ba, abab, baba, ababab, bababa}
|L ∪ L^R| = 7
∴, |L ∪ L^R| < 2|L|

(c)
Condition:
I think a general condition on a non-empty finite language L that is necessary and sufficient for |L ∪ L^R| = 2|L| is for L to not have any symmetric strings in it.

Reasoning:
If L has a symmetric string say "aaaa",
then L^R would also have the string "aaaa"
However, this would lead to their union to be less than twice the length of L
or, in other words |L ∪ L^R| < 2|L|

Example:
Let L = {ab, b, aaa, bbbb}
L^R = {ba, b, aaa, bbbb}
and, L ∪ L^R = {ab, ba}

$|\mathrm{L}| = 4 \Rightarrow 2|L| = 8$
$|L \cup \mathrm{L}^R| = 2$ ∴, $|\mathrm{L} \cup \mathrm{L}^R| \neq 2|\mathrm{L}|$

Question 2.

(a)
For $\mathbf{M_1}$:
Start state = q_0
For $\mathbf{M_2}$:
Start state = q_0

(b)
For $\mathbf{M_1}$:
Set of accept states = Q = $\{q_3\}$
For $\mathbf{M_2}$:
Set of accept states = Q = $\{q_0, q_1\}$

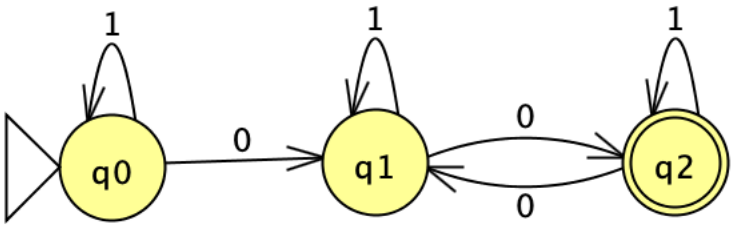
(c)
For $\mathbf{M_1}$:
Sequence of states: q_0, q_0, q_0, q_1, q_2, q_2
i.e, $q_0 \rightarrow q_0$; $q_0 \rightarrow q_0$; $q_0 \rightarrow q_1$; $q_1 \rightarrow q_2$; $q_2 \rightarrow q_2$
For $\mathbf{M_2}$:
Sequence of states: q_0, q_1, q_1, q_2, q_2, q_2
i.e, $q_0 \rightarrow q_1$; $q_1 \rightarrow q_1$; $q_1 \rightarrow q_2$; $q_2 \rightarrow q_2$; $q_2 \rightarrow q_2$

(d)
For $\mathbf{M_1}$:
No, it does not.
For $\mathbf{M_2}$:
No, it does not.

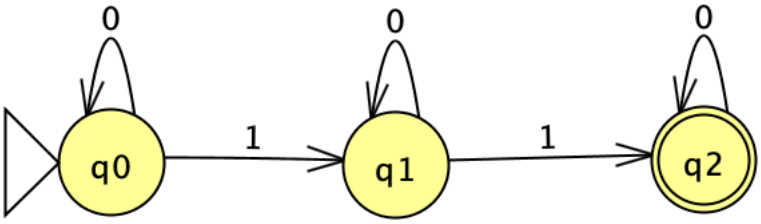
(e)
For both $\mathbf{M_1}$ and $\mathbf{M_2}$:
No, they do not since it is a DFA.

(f)
For $\mathbf{M_1}$:
$\{(\mathrm{a})^i \mathrm{b} (\mathrm{a})^j \mathrm{b} (\mathrm{a})^k \mathrm{b} (\mathrm{a})^l | i, j, k, l \geq 0\}$
For $\mathbf{M_2}$:
$\{(\mathrm{b}^i \mathrm{a}^j) | \mathrm{i,j} \geq 0\}$

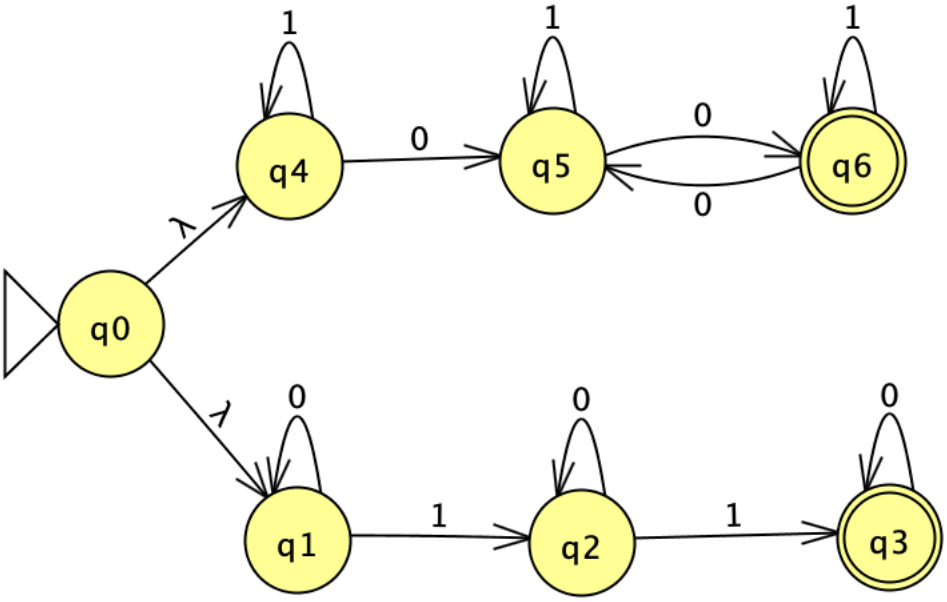
Question 3.



DFA - contains an even number of 0s

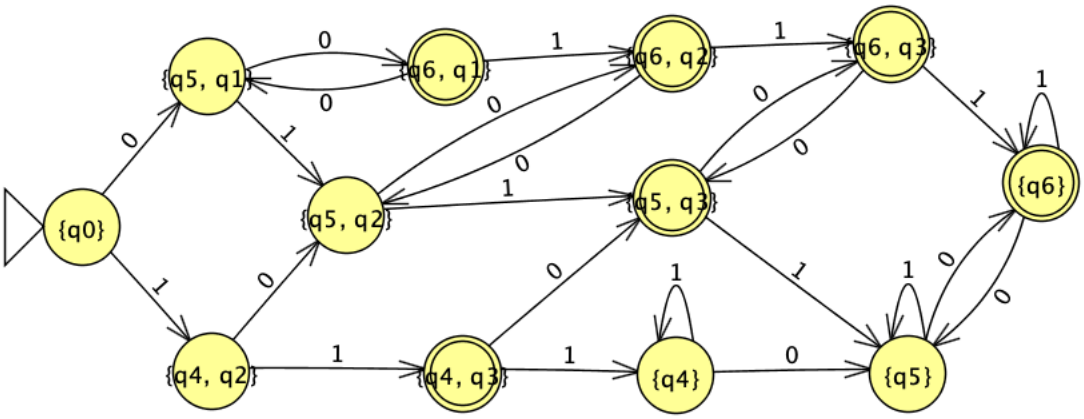


DFA - contains exactly two 1s



NFA by combining the two DFAs above

	0	1
{q0}	{q5, q1}	{q4, q2}
{q5, q1}	{q6, q1}	{q5, q2}
{q4, q2}	{q5, q2}	{q4, q3}
{q6, q1}	{q5, q1}	{q6, q2}
{q5, q2}	{q6, q2}	{q5, q3}
{q4, q3}	{q5, q3}	{q4}
{q6, q2}	{q5, q2}	{q6, q3}
{q5, q3}	{q6, q3}	{q5}
{q4}	{q5}	{q4}
{q6, q3}	{q5, q3}	{q6}
{q5}	{q6}	{q5}
{q6}	{q5}	{q6}



NFA converted to DFA, conversion table on the top of this page

A simplified version of this diagram would be: