COMP335, Pumping Lemma

October 27, 2018

Prove that the following languages are not regular:

1. $L = \{a^n b^l a^k : k \geq n + l\}$
2. $L = \{a^n :$ n is prime number$\}$
3. $L = \{a^n : n = k^3$ for some $k \geq 0\}$
4. $L = \{a^n : n = 2^k$ for some $k \geq 0\}$
5. $L = \{a^n :$ n is the product of two prime numbers$\}$
6. $L = \{a^n b^l : |n - l| = 2\}$
7. $L = \{ww : w \in \{0, 1\}^*\}$
   (a) $L_1 = \{w_1 w_2 : w_1, w_2 \in \{0, 1\}^*, |w_1| = |w_2|\}$ (regular?)
8. $L = \{ww^R : w \in \{0, 1\}^*\}$
9. $L = \{a^{n!} : n \geq 0\}$
10. $L = \{w :$ w has different number of 0s and 1s$\}$

1 Solutions

Assuming m is pumping length:

1. $w = a^m b a^{m+1}$ and you get a contradiction for xy^2z
2. $w = a^p$ s.t. p is a prime number and $p \geq m$ and you get a contradiction for $xy^{p+1}z$

3. $w = a^{m^3}$ and you get a contradiction for xy^2z
   hint1: $m < 3m < 3m^2 + 3m + 1$
   hint2: $m^3 + r < m^3 + 3m^2 + 3m + 1 < (m + 1)^3$

4. $w = a^{2^m}$ (m is the power of 2, don't confuse it with a^{2m})and you get a contradiction for xy^2z
   hint: $m < 2^m$

5. $w = a^{pq}$ s.t. p and q are prime numbers and $p, q \geq m$ and you get a contradiction for $xy^{pq+1}z$

6. $w = a^m b^{m-2}$ and you get a contradiction for xy^2z

7. $w = 0^m 1 0^m 1$ and you get a contradiction for xy^2z

   (a) L_1 is regular

8. $w = 0^m 1 1 0^m$ and you get a contradiction for xy^2z

9. $w = a^{m!}$ and you get a contradiction for xy^2z
   hint: get to the contradiction for both cases (a) $m = 1$, and (b) $m \geq 2$

10. prove that $\overline{L}$ is not regular.