COMP 478 - Image Processing

Assignment 01

Name	ID
Vaansh Vikas Lakhwara	40114764

Part I: Theoretical questions

1. a) In order to get a flat histogram, you would need an image with *L* groups of *L* pixels having the same intensity, which is usually not the case with an input image which is why usually no new intensity levels are created since the input (r_n) and output ($L-1 * p(s_n)$) intensities are going to be similar. The technique does flatten them a little, resulting in image with an enhanced contrast, but not completely for the aforementioned reasons.

b) The discrete histogram equalization technique produces a *uniformly distributed* image. Therefore, re-running this technique does not yield a flat histogram – the resulting image would be relatively unchanged.

2. No. H [af(x, y) + bg(x, y)] != aH [f(x, y)] + bH [g(x, y)] for an operator that computes the median of a set of pixels of a sub-image area.

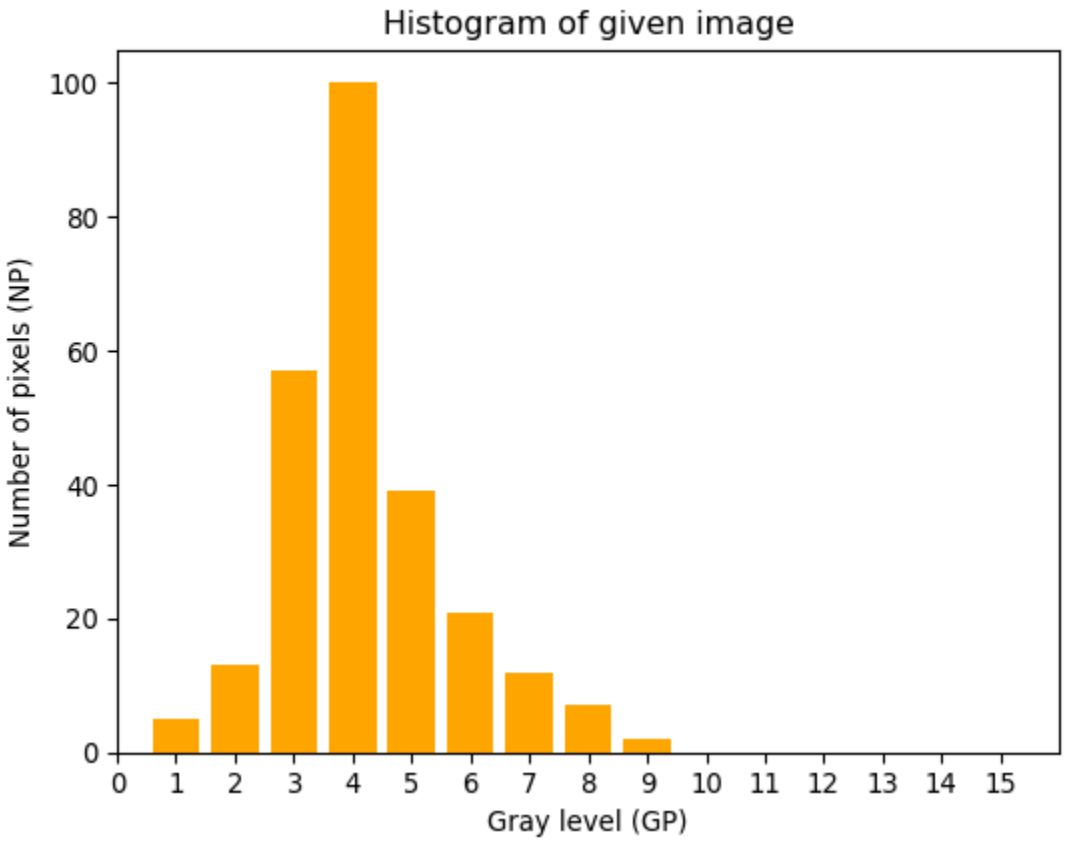
Consider an example where:

1. a = b = 1
2. f(x, y) = {1, 2, 3} => aH [f(x, y)] = median of {1, 2, 3} = 2
3. g(x, y) = {1, -2, 3} => bH [g(x, y)] = median of {1, -2, 3} = 1

Therefore,
LHS = H [af(x, y) + bg(x, y)] = H [{1, 2, 3} + {1, -2, 3}] = median of {2, 0, 6} = 2
RHS = aH [f(x, y)] + bH [g(x, y)] = 2 + 1 = 3

LHS != RHS => QED

3. a)



Code used to generate histogram:

```
def theory_function_3a():
    x = np.arange(16)
    y = np.array([0, 5, 13, 57, 100, 39, 21, 12, 7, 2, 0, 0, 0, 0, 0, 0])

    plt.bar(x, y, color="orange")

    plt.xlabel("Gray level (GP)")
    plt.ylabel("Number of pixels (NP)")

    plt.gca().set_xlim([0, 16])

    plt.title("Histogram of given image")
    plt.xticks(np.arange(0, 16, 1.0))

    plt.savefig("img/theory/Figure_3a.png", dpi=95)
    plt.show()
```

b) i) n = 256 since that is the sum of all n_k (image size)

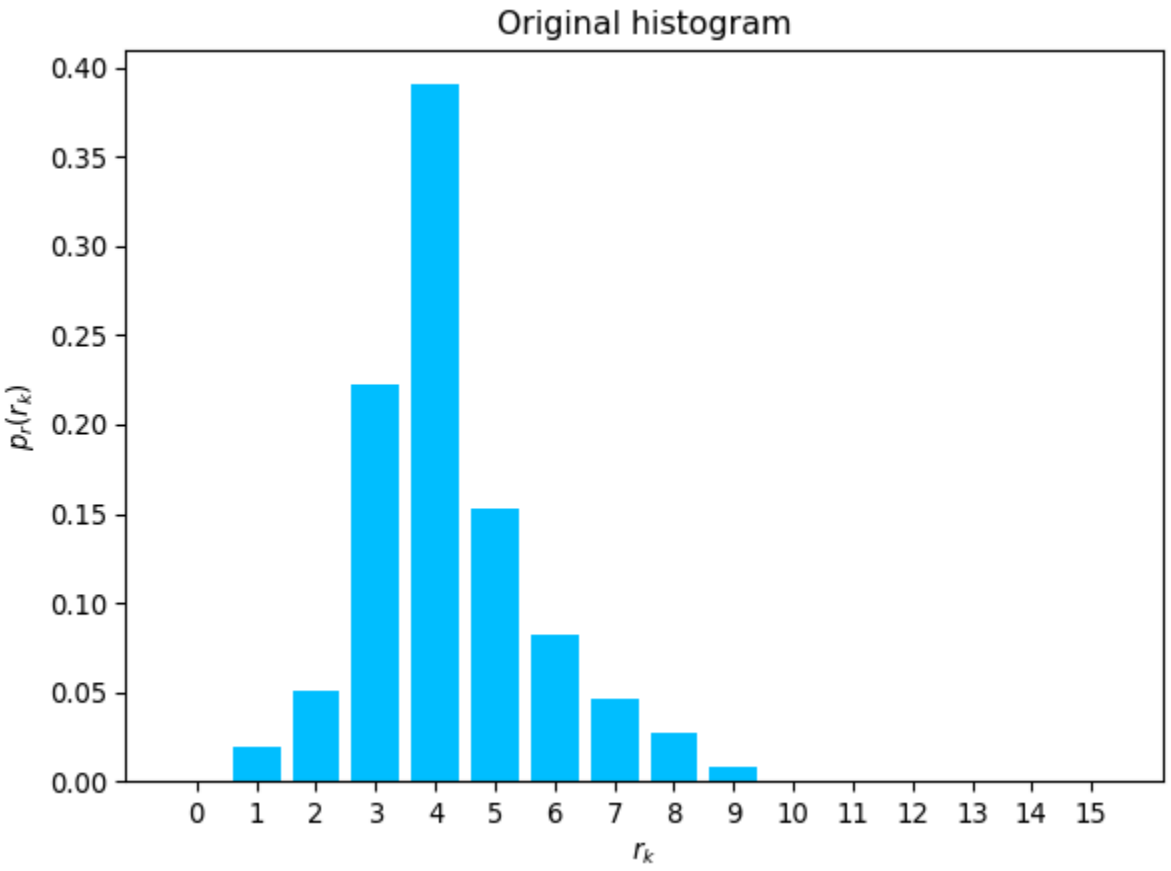
L = 16 since it is a 4 bit image (intensity)

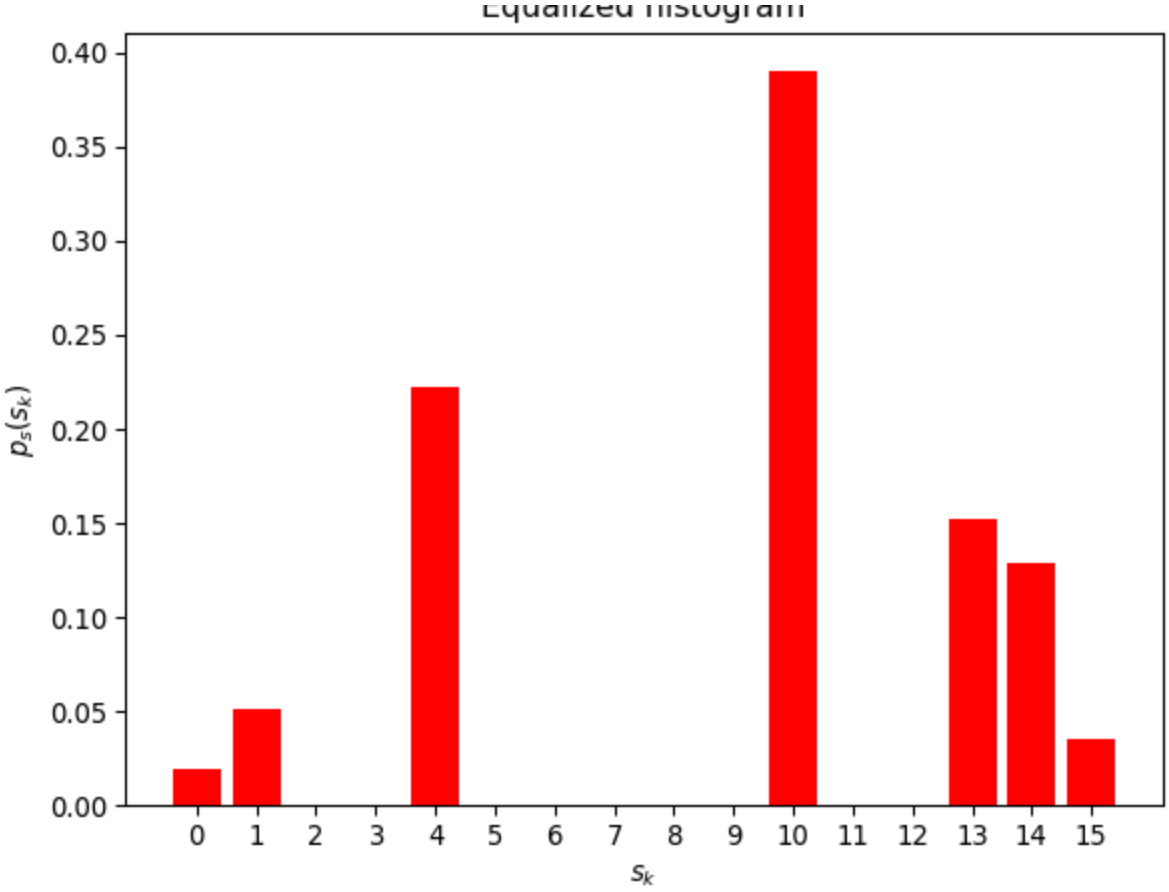
r_k	n_k	$p_r(r_k)$	s_k
0	0	0/256 = 0	0
1	5	5/256 = 0.01953125	0
2	13	13/256 = 0.05078125	1
3	57	57/256 = 0.22265625	4
4	100	100/256 = 0.390625	10
5	39	39/256 = 0.15234375	13
6	21	21/256 = 0.08203125	14
7	12	12/256 = 0.046875	14
8	7	7/256 = 0.02734375	15
9	2	2/256 = 0.0078125	15
10	0	0/256 = 0	15
11	0	0/256 = 0	15
12	0	0/256 = 0	15
13	0	0/256 = 0	15
14	0	0/256 = 0	15
15	0	0/256 = 0	15

ii)

s_k	n_k	$p_s(s_k)$
0	5	5/256 = 0.01953125
1	13	13/256 = 0.05078125

s_k	n_k	$p_s(s_k)$
4	57	57/256 = 0.22265625
10	100	100/256 = 0.390625
13	39	39/256 = 0.15234375
14	33	33/256 = 0.12890625
15	9	9/256 = 0.03515625





Code used to generate histograms:

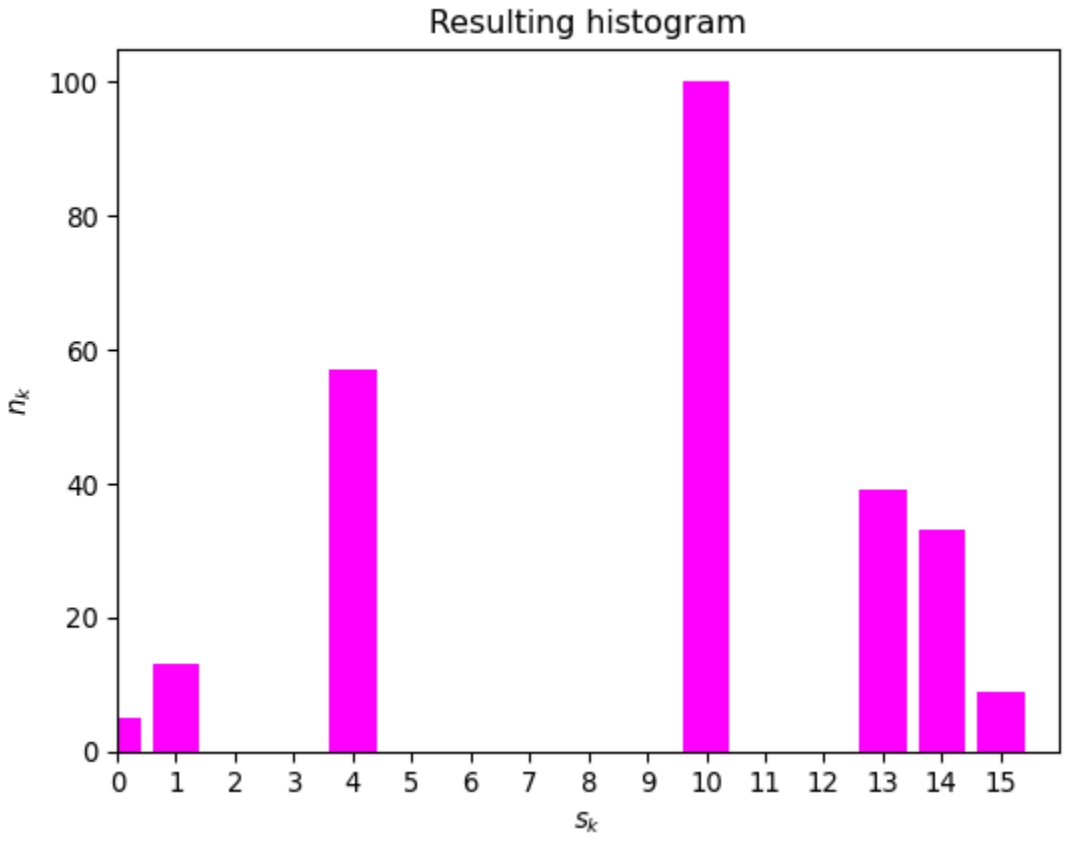
```
def theory_function_3bii():
    p_rk_x = np.arange(16)
    p_rk_y = np.array([
        0, 0.01953125, 0.05078125, 0.22265625, 0.390625, 0.15234375,
        0.08203125, 0.046875, 0.02734375, 0.0078125, 0, 0, 0, 0, 0, 0
    ])

    p_sk_x = np.array([0, 1, 4, 10, 13, 14, 15])
    p_sk_y = np.array([
        0.01953125, 0.05078125, 0.22265625, 0.390625, 0.15234375, 0.12890625,
        0.03515625
    ])
```

```
    plt.bar(p_rk_x, p_rk_y, color="deepskyblue")
    plt.xlabel("$r_k$")
    plt.ylabel("$p_r(r_k)$")
    plt.title("Original histogram")
    plt.xticks(np.arange(0, 16, 1.0))
    plt.tight_layout()
    plt.savefig("img/theory/Figure_3bii1.png", dpi=95)
    plt.show()

    plt.bar(p_sk_x, p_sk_y, color="red")
    plt.xlabel("$s_k$")
    plt.ylabel("$p_s(s_k)$")
    plt.xticks(np.arange(0, 16, 1.0))
    plt.tight_layout()
    plt.title("Equalized histogram")
    plt.savefig("img/theory/Figure_3bii2.png", dpi=95)
    plt.show()
```

c)



Code used to generate histogram:

```
def theory_function_3c():
    s_k_x = np.array([0, 1, 4, 10, 13, 14, 15])
    n_k_y = np.array([5, 13, 57, 100, 39, 33, 9])

    plt.bar(s_k_x, n_k_y, color="magenta")

    plt.xlabel("$s_k$")
    plt.ylabel("$n_k$")

    plt.gca().set_xlim([0, 16])

    plt.title("Resulting histogram")
    plt.xticks(np.arange(0, 16, 1.0))

    plt.savefig("img/theory/Figure_3c.png", dpi=95)
    plt.show()
```

4.

Let

$f(x, y) = r_k$

Given that

$g(x, y) = c$

Since the gray levels of the pixels of both images have positive values

1. For $f(x, y) + g(x, y)$, resulting image will:
    - have output intensity $r_k + c$ for each (x, y)
    - remain evenly spaced
2. For $f(x, y) * g(x, y)$, resulting image will:
    - have output intensity $r_k * c$ for each (x, y)
    - remain evenly spaced

Part II: Programming questions

1. 1)

```
from PIL import Image


def programming_function_1(
    path="img/HawkesBay.jpeg", save_dir="img/programming", fmt="JPEG"
):
    image = Image.open(path).convert("L")
    image.show()
    image.save(save_dir + "/Figure_1.jpeg", fmt)
```

Output:

2)

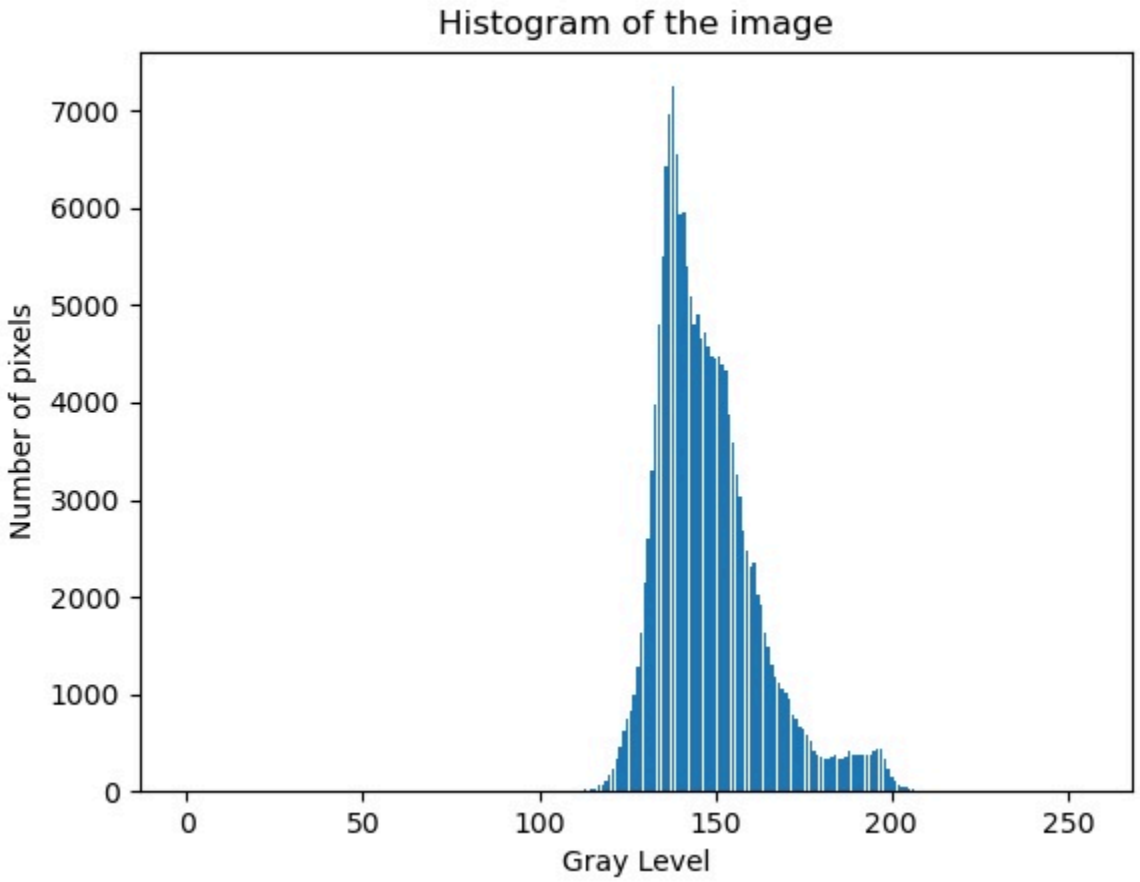
```
from PIL import Image
import matplotlib.pyplot as plt


def programming_function_2(path="img/HawkesBay.jpeg", save_dir="img/programming"):
    # creates 256 "buckets"
    grey_levels = list(range(256))

    # gets dimensions and loops over every pixel and
    # adds 1 otherwise 0 to each "bucket"
    image = Image.open(path)
    wid, hgt = image.size
    num_pixels = [
        sum(
            [
                1 if (image.getpixel((w, h)) == l) else 0
                for h in range(hgt)
                for w in range(wid)
            ]
        )
        for l in grey_levels
    ]

    plt.bar(grey_levels, num_pixels)
    plt.xlabel("Gray Level")
    plt.ylabel("Number of pixels")
    plt.title("Histogram of the image")
    plt.savefig(save_dir + "/Figure_2.jpeg")
    plt.show()
```

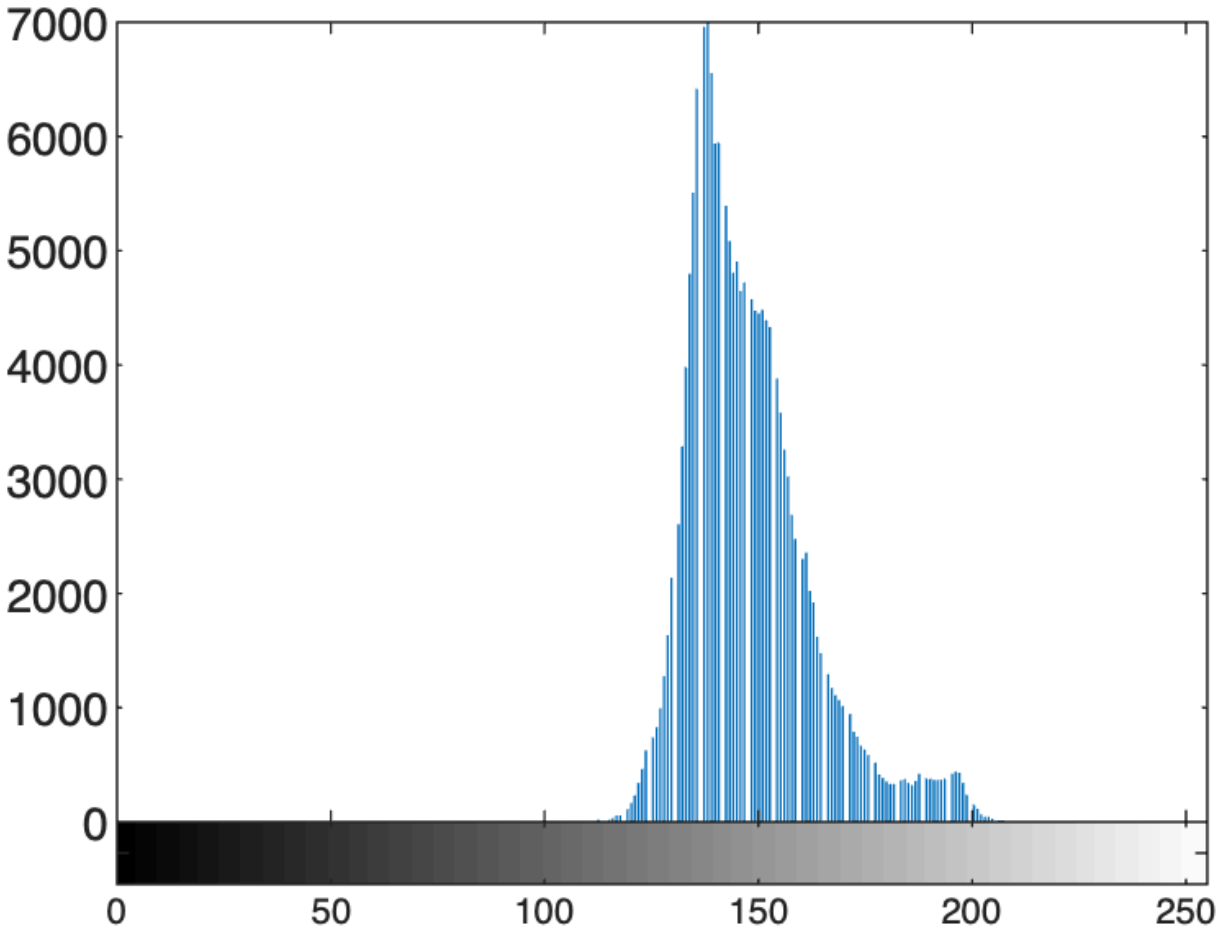
Output:



3)

Output: (MATLAB)

```
I = imread("img/HawkesBay.jpeg");
imhist(I)
ylim([0, 7000])
saveas(gcf, "img/programming/Figure_3.png")
```



4)

```
from itertools import accumulate
from PIL import Image
import matplotlib.pyplot as plt


def programming_function_4(path="img/HawkesBay.jpeg", save_dir="img/programming"):
    image = Image.open(path)
    grey_levels = list(range(256))
    wid, hgt = image.size
```

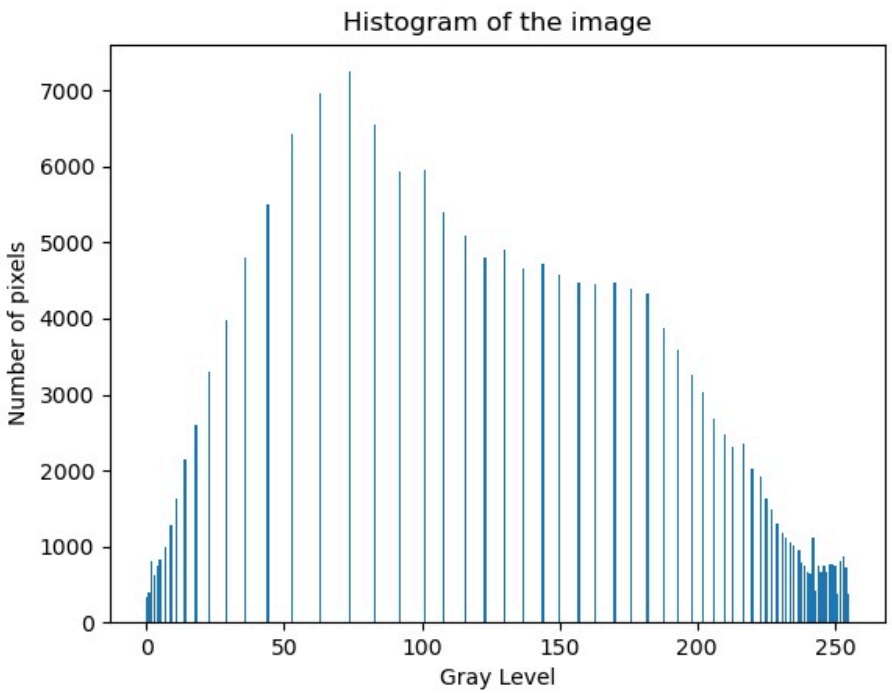
```
    n_k = [sum([1 if (image.getpixel((w, h)) == p) else 0
                for h in range(hgt)
                for w in range(wid)
            ])for p in grey_levels]

    L = wid * hgt
    pr_rk = [n / L for n in n_k]
    sk = [round(s * 255) for s in accumulate(pr_rk)]

    res_nk = [
        sum([n_k[i] if l == sk_r else 0 for i, sk_r in enumerate(sk)]) \
             for l in grey_levels
    ]

    plt.bar(grey_levels, res_nk)
    plt.xlabel("Gray Level")
    plt.ylabel("Number of pixels")
    plt.title("Histogram of the image")
    plt.savefig(save_dir + "/Figure_4.jpeg")
    plt.show()
```

Output:



5)

Output: (MATLAB)

```
I = imread("img/HawkesBay.jpeg");
J = histeq(I)
imhist(J)
ylim([0, 7500])
saveas(gcf, "img/programming/Figure_5.png")
```