The Impact of Trade Activities on Exchange Rates: A Multivariate Time Series Approach

for the Bachelor of Science (special) Degree
Time Series Analysis(MSP4144)
By

O.G.V.V Bandara
SC/2019/10723

Supervisor:
Dr H.L Jayathilake

Department of Mathematics
Faculty of Science
University of Ruhuna
Matara
Sri Lanka

2024

Abstract

The exchange rate is a important metric in evaluating the relationship between internal economy and international economy. The objective was to assess the causality between each of the variables that affect the exchange rate of Sri Lanka. Also to build a model that would forecast exchange rate of Sri Lanka accurately. The objectives have been achieved with satisfactory accuracy. The dataset was obtained from the official Central Bank of Srilanka website's economic data library. The dataset consisted of import and export monthly expenditure of various commodities as well as the variation of monthly average exchange rate. Due to the nature of the time series variations of the variables, the Vector AutoRegression model VAR(P) with lag order p was chosen for forecasting. The VAR model was fit for A lag value of 5 and accuracy assessed with various metrics. The optimal lag for the VAR model was determined to be 5 by the AIC, SBIC, HQIC, FPE criterions. It was determined through the portmanteau test that there is no residual auto-correlation. By the use of the Jarque-Bera Test (JB-Test), Skewness Test and Kurtosis test the residuals were determined to be not normally distributed, which was assumed to have no substantial effect on the model. Analysing the OLS-CUSUM plots gave insight that there are no structural breaks in any variables. Through Granger casuality analysis of each variable it was determined that all variables have Granger-cause. The effects of shocks of each variable were observed and the proportion of each shock were analyzed through IRF and FEVD. It was concluded that within a 95 percent confidence interval, the VAR(5) model prediction depicts a rapid increase in the monthly average exchange rate. The forecasts aligns with the real world scenario since the exchange rate of Sri Lanka was subjected to rapid increase within the forecasted time period. We can conclude that the underlying reason for the rapid increase in the exchange rate of Sri Lanka was a result of long term inefficient policy measures and mismanagement of imports and exports within the analyzed time period.

Acknowledgment

I would like to express my deepest gratitude to my supervisor, Dr H.L Jayathilake, for their invaluable guidance, support, and encouragement throughout the course of this research project. Their expertise and insightful feedback were instrumental in shaping this work.

I am also immensely grateful to my teammates, W.K.T.H Madhumal, J.C.N Jayakody, for their collaboration, hard work, and dedication. Their contributions and unwavering support made this project a success.

Contents

Chapter 1

Introduction

1.1 Background of the study

For a substantial period the governments of developing countries have dismissed the importance of exchange rates as a facet of development policy. The metric was assumed to automatically adjust with industrialization. The exchange rate is a decisive link between the internal economy and international economy. With the increase in the exchange rate the price of imports tend to increase and if the exports are not maintained in a equivalent amount, the price of good imported would increase substantially. The nominal exchange rate is a key term in determining the exchange rate in short and medium terms. But the exchange rates are a key term in determining the macroeconomic stability. And also the incentive to engage in trade.

1.1.1 Modern exchange rate economics

The target of exchange rate economics is to primarily understanding the determining factors in a floating exchange rate. A floating exchange rate primarily depends on what is about to happen in the future rather than the occurrences in the present. The exchange rate is a forward looking asset price. Both income flows and exchange rates are determined simultaneously in a general equilibrium setting. A government has to keep the exchange rate stable and unchanged to stabilize the markets to reduce market pressure. It could be assumed that that there are two stages when determining the level of a floating exchange rate.

- Determining the steady-state level of real (price-deflated) exchange rate: current account exercises the it's main influence by pinning down the value of real exchange rate. The equilibrium real exchange rate also depends on variables like the stock of net foreign assets.

- Adjustment of exchange rate towards it's steady state level: The standard model attributes this to the rational expectation of what will happen in the interval between now and in the long run.standard exchange rate economics is typical of pos-Lucas macroeconomics.

1.1.2 The standard model

The standard modelling to explain the exchange rate can be subdivided into two as the explanation of long run equilibrium rate and short run process of transition to equilibrium.

- Long run equilibrium: characterized by a current account imbalance. To keep the debt to GDP ratio constant where debt also includes ownership of equity positions as well as debt. The larger the discrepancy between a countries discount rate and the world interest rate, the larger the absolute value is the steady state ratio of NIIP/GDP. A debtor country has to run a larger surplus in steady state equilibrium if it's debt level is higher.

- short run equilibrium: The famous class of models here assumes perfect capital mobility. Which means that the market must expect equalization of the rate of return from holding different currencies. The interest differential must be equal to the expected rate of change of one currency in terms of the other.The forward exchange rate must be equal to the expected future spot exchange rate. The forward interest or premium must be equal to the interest differential. Thus the interest differential will trace out the expected path of adjustment to the equilibrium. With the introduction of generalized floating in 1973, the exchange rate has to be viewed as an asset price, whose value is determined at an asset market level.

1.1.3 Richard Meeese-Kenneth Rogoff model

This is a famous dismissal of the empirical relevance of short run of all existing models. The following gives the quasi-reduced form specifications of all models they examined.

$$s = a_0+a_1(m-m^*)+a_2(y-y^*)+a_3(r_s-r_s^*)+a_4(\pi^c-\pi^{c*})+a_3\sum TB+a_6\sum TB^*+u \tag{1.1}$$

- s=logarithm of dollar price of foreign currency
- $m - m^*$=log of ratio of US money supply to foreign money supply
- $y - y^*$=Log of ratio of US to foreign real income
- $r_s - r_s^*$=short-term interest differential
- $\pi^c - \pi^{c*}$=expected long run inflation differential
- $\sum TB$=cumulative US trade balance
- $\sum TB^*$=cumulative foreign trade balance
- u=disturbance term

All the models they consider , aim to explain the bilateral exchange rate between USA and a single other country which implicitly must strictly cover teh rest of the world.

1.1.4 Advancements in exchange rate economics

In the 1970's there was a big leap regarding exchange rate economics when the "flow models" of the foreign exchange market were replaced by the models that recognized that the exchange rate is a asset price that depends on expectations of the future. Certain models are assumed to be accepted by the economic community to have perfect understanding of the exchange rate dynamics such as the "REEM" model. And also the "De Grauwe and Grimalda's" behavioral finance model. Those models appear to be consistent with the main facts about the foreign exchange markets.

1.1.5 Multivariate time series

Multivariate time series forecasting is a versatile area of time series forecasting. Multivariate time series forecasting is used for applications such as traffic flow prediction, commodity demand forecasting ,financial forecasting etc. Multivariate time series forecasting plays a crucial role in data science and machine learning. Also instances such as spatiotemporal traffic volume forecasting involves the time series of traffic volumes on multiple intersections. Multivariate time series have and upper limit of predictability such as the multivariate sample entropy(MsE). There is also a relationship between multivariate predictability and prediction accuracy.Various statistical models are used for time series forecasting. AR and ARIMA are two widely used models.
Many time series arising in practice are best considered as components of some vector valued time series. Having not only serial dependence within the series but interdependence within the different components of the series.

m-variate weakly stationary

A multivariate stationary time series is defined as for a m-variate series X_t is weakly stationary if $\mu_x(t)$ is independent of t and $\tau_x(t+h,t)$ is independent of t for each h.

white noise with mean 0 and covariance Σ

Also and m-variate series Z_t is white noise if mean 0 and covariance matrix Σ where

$$Z_t \sim WN(0,\Sigma) \tag{1.2}$$

m-variate Linear process

An m-variate series X_t is a linear process if it has the representation

$$X_t = \sum_{j=-\infty}^{\infty} C_j Z_{t-j} \tag{1.3}$$

where $Z_t \sim WN(0,\Sigma)$.

MA(∞) process

is a linear process with $C_j = 0$ for $j < 0$ .Thus $MA(\infty)$ process if and only if there exists white noise Z_t and a sequence of matrices C_j with absolutely summable components such that

$$X_t = \sum_{j=1}^{\infty} C_j Z_{t-j} \tag{1.4}$$

ARMA(p, q) process

Any causal ARMA(p, q) process can be expressed as an MA(∞) process, while any invertible ARMA(p, q) process can be expressed as and AR(∞) process satisfying equations of the form

$$X_t + \sum_{j=1}^{\infty} A_j X_{t-j} = Z_t \tag{1.5}$$

1.2 Objectives

- Forecasting the future exchange rate of Sri Lanka using time series approach
- Evaluation of the effect of imports and exports volumes on the fluctuation of exchange rates
- Assessing the prediction accuracy of exchange rates using multivariate time series approach
- Alleviate the requirement for empirical calculations in assessing the trend of the exchange rate
- A novel approach to forecasting the exchange rate of Sri Lanka
- Assessing the Cointegration of imports and exports towards the exchange rate of Sri Lanka
- Arriving at conclusions about exchange rate dynamics
- Evaluating the relationship between import and exports towards the exchange rate in the long run and short term

Chapter 2

Problem Statement

2.1 Cointegration between Exchange rate and Imports,Exports

Aspects of exchange rate dynamics are studied in a variety of models. Some of which are based on postulated supply and demand functions for assets and goods. Some of which are based on explicit utility maximizing functions.Private wealth is an explicit determinant for both money and goods. Stocks of external assets and domestic capital are predetermined variables that influence the rate at which new wealth is accumulated through current account surplus and investment. Changes in wealth in turn moves the economy's short run equilibrium over time. The overall dynamics of the system will result from foreseen changes in exogenous variables as well as from adjustment of external claims and capital to the long-run levels desired by firms and individuals. The project intends to assess the relationship between endogenous variables effecting exchange rates and the nature of the relationships between them through time series forecasting. Also to assess the nature of cointegration between variables.

2.2 Adequate model for Exchange rate forecasting

Understanding the long term equilibrium relationship between endogenous variables and build a better forecasting model that has good cointegration. Through statistical assessment an adequate model for time series forecasting and VAR model must be achieved. Exchange rates are very dynamic and hard to predict, but neglecting sudden changes such as political turmoil or natural occurrences , the model should be capable of capturing long term equilibrium relationships.

Chapter 3

Methodology

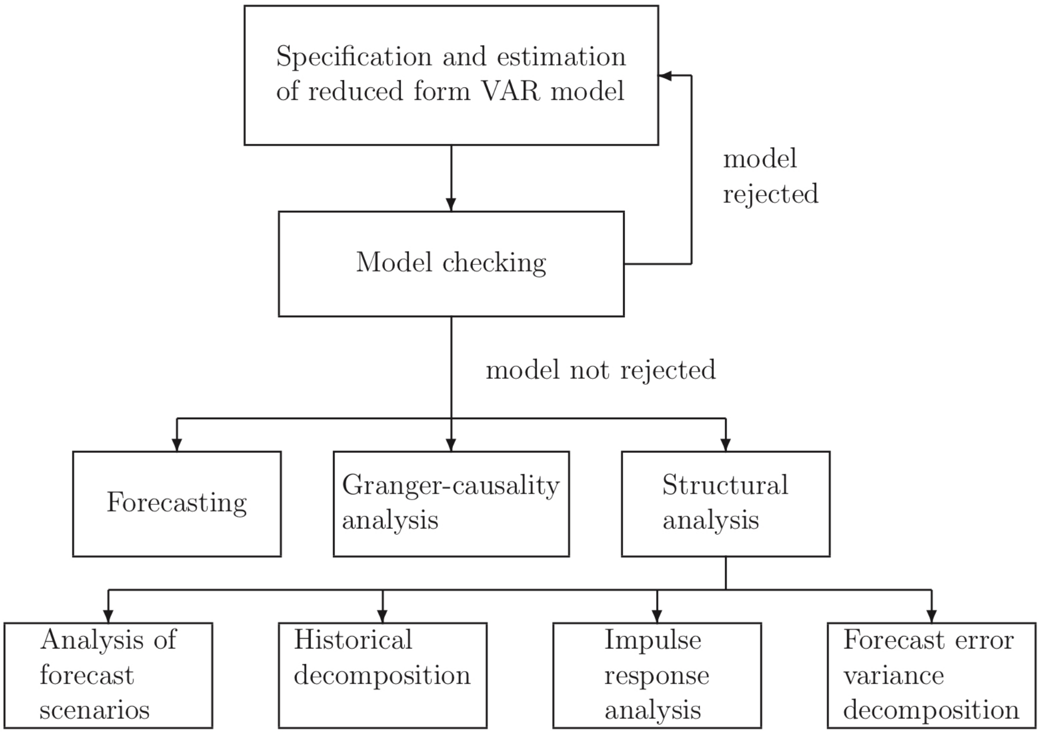
3.1 Vector Autoregression

3.1.1 Introduction

Multivariate simultaneous equation model have been used widely for macroeconomic analysis. The VAR models on the other hand treat all variables as A priori endogenous. Endogenous variables are effected by the variables present within the model. On the other hand exogenous variables outside the system are not influenced by variables inside the system. A priori data are trends, seasonality or patterns derived from domain knowledge or prior information. But exogenity of some of the variables may be imposed in VAR models based on statistical procedures. The setup of VAR models involves explaining current values of a set of variables based on past values of the of the variables involved. Traditionally VAR models are designed for stationary variables without time trends. But trend behavior can be captured by including "deterministic polynomial" terms. After the discover of "cointegration" it can also be stated that stochastic trends can also be captured by VAR models.

3.1.2 VAR model analysis procedure

The general steps for VAR model forecasting can be shown by following figure. VAR analysis proceeds by first specifying and estimating a reduced form model. And then checking it's adequacy. Model deficiencies detected at the latter stage are resolved by modifying the model. If the reduced model passes the checking stage then it is used for forecasting, granger-causality or structural analysis.



3.2 VAR processes

An investigator is interested in K related time series variables collected in $y_t = (y_{1t}, ..., y_{kt})'$. It is important to distinguish between stochastic and deterministic components of the DGP's of economic variables. It is convenient to separate the two components by assuming that

$$y_t = \mu_t + x_t \tag{3.1}$$

μ_t

μ_t is the deterministic part and x_t is a purely stochastic process with zero mean. The deterministic term μ_t may be linear, zero or just a constant. determinsitic terms have implausible implications in the context of forecasting. The purely stochatic part x_t may be I(1) and hence may include stochastic trends and cointegration realtions.

x_t

x_t is the purely stochastic part and may be $I(1)$. Hnece may include stochastic trends and cointegration relations. It has mean zero and a VAR representation.

y_t

The properties of the observable process y_t are determined by those of x_t and μ_t. The order of integration and cointegration relations are determined by x_t. suppose the stochastic part x_t is a VAR process of order p, VAR(P) of the form

$$x_t = A_1 x_{t-1} + ... + A_p x_{t-p} + \mu_t \tag{3.2}$$

where the A_i are $(K \times K)$ paramter matrices and the error process $\mu_t = (u_{1t}, ..., u_{kt})'$ is a K dimensional zero mean white noise process with covariance matrix $E(\mu_t \mu_t') = \sum_u$.

$$u_t \sim (0, \Sigma_u) \tag{3.3}$$

using the lag operator and defining the matrix polynomial in the lag operator $A(L)$ as

$$A(L) = I_K - A_1 L - ...... - A_p L^p \tag{3.4}$$

can be equivalently written as

$$A(L) x_t = \mu_t \tag{3.5}$$

for x_t to be stable $detA(z) = det(I_k - A_1 z - ..... - A_p z^p$ for $|z| \leq 1$. To put it into different terms x_t is stable if all roots of the determinantal polynomial are outside the complex unit circle. In this case x_t is $I(0)$. Under the usual assumptions a stable process x_t has invariant means, variances and covariance structure, hence stationary.
But if $detA(z) = 0$ for $z = 1$ and all other roots of the determinantal polynomial are outside the complex unit circle, then some or all of the variables are integrated, which means the process is non stationary and the variables are cointegrated.
When x_t is a stochastic part and y_t is a vector of observed variables , considering

$$A(L) y_t = A(L) \mu_t + u_t \tag{3.6}$$

shows that y_t inherits the $VAR(P)$ representation from x_t.

$$\mu_t = \mu_0 + \mu_1 t \tag{3.7}$$

$$A(L) y_t = \theta_0 + \theta_1 t + A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \tag{3.8}$$

where

$$\theta_0 = (I_k - \sum_{j=1}^{p} A_j)\mu_0 + (\sum_{j=1}^{p} j A_j)\mu_1 \tag{3.9}$$

and

$$\theta_1 = (I_k - \sum_{j=1}^{p} A_j)\mu_1 \tag{3.10}$$

since all the variables appear in levels ,this form is known as the levels form of the VAR process.

3.3 Structural forms

In structural form models contemporaneous variables might appear as explanatory variables.

$$A y_t = \theta_0^* + \theta_1^* + A_1^* y_{t-1} + ... + A_p^* y_{t-p} + \theta_t \tag{3.11}$$

The $(K \times K)$ matrix A reflects the instantaneous relations $\theta_i^* = A\theta_i$ and $A_j^* = AA_j$. The structural form error term $\theta_t = Au_t$ is iid white noise with covariance matrix $\Sigma_v = A\Sigma_u A'$.

3.4 Model specification

3.4.1 Test for stationary

The Philips-perron test used for evaluating the stationary or non stationary nature of the time series. Philips-perron tests that a variable has a unit root.

- H0: variable contains a unit root
- H1: variable was generated by a stationary process

By analyzing the p-value, we can determine whether to reject or accept the null hypothesis. In order for the time series to be stationary the null hypothesis needs to be rejected. Hence must be no unit root which in terms implies that the time series is stationary.

3.4.2 Lag order

Model specification in the present context involves electing the VAR order and possibly imposing restrictions on VAR parameters. The number of parameters of the VAR model increases with VAR order. VAR order is chosen by sequential testing procedures and model selection criteria.
Sequential testing procedure occurs by specifying a maximum reasonable lag order , say p_{max} and testing the following sequence of null hypothesis.

$$H_0 : A_{p_{max}} = 0 \tag{3.12}$$

$$H_1 : A_{p_{max}-1} = 0 \tag{3.13}$$

The procedure halts when the null hypothesis is rejected for the first time. The order is then chosen accordingly. For stationary processes the usual Wald or LR χ^2 tests for parameter restrictions can be used.

3.4.3 Model selection criteria

An alternative approach is the 'model selection criteria'. which has the general form

$$C(m) = logdet(\hat{\Sigma_m}) + c_T\varphi(m) \tag{3.14}$$

where $\hat{\sum}_m = T^{-1}\sum_{t=1}^{T} \hat{u}_t\hat{u_t'}$ is the OLS residual covariance matrix estimator for a reduced for VAR model of order m.

Akaike's information criteria

$$AIC(m) = logdet(\hat{\Sigma_m}) + \frac{2}{T}mK^2 \tag{3.15}$$

where $c_T = \frac{2}{T}$

Hannan-Quinn criterion

$$HQ(m) = logdet(\hat{\Sigma_m}) + 2log\frac{logT}{T}mK^2 \tag{3.16}$$

where $c_T = 2log\frac{logT}{T}$

Schwartz criterion

$$SC(m) = logdet(\hat{\Sigma_m}) + \frac{logT}{T} mK^2 \tag{3.17}$$

where $c_T = \frac{logT}{T}$
The VAR order is chosen such that the respective criterion is minimized over the possible orders $m = 0, ..., p_{max}$. Among these 3 criteria, AIC always suggests the largest order, SC chooses the smallest order and HQ is between. Also the criterias may all suggest the same lag order. The HQ and SC are both consistent. AIC tends to overestimate the order asymptotically with a samll probability.
The lag order obtained with sequential testing or model selection criteria depends on the choice of p_{max}. choosing a small p_{max} may result in an inappropriate model. choosing a large p_{max} may result in a large and spurious value. choosing a moderate p_{max} is a sensible strategy.

3.5 Model checking

The process of checking whether VAR models represents the data generating properties of the variables adequately range form formal tests of underlying assumptions to informal procedures such as inspecting plots of residuals and autocorrelations. Since a reduced form is underlying every structural form , model checking focuses on reduced form models.

3.5.1 Tests for residual autocorrelation

Portmanteau and Breusch-Godfrey-LM tests are standard tools for checking residual autocorrelation in VAR models.

Portmanteau test

$$H_0 : E(u_t u'_{t-i}) = 0 \tag{3.18}$$
$$H_0 : \text{ at least one autocorrelation is non zero}$$

And the test statistic is based on the residual covariances

$$\hat{C}_j = T^{-1} \sum_{t=j+1}^{T} \hat{u}_t \hat{u_{t-j}}' \tag{3.19}$$

where the $\hat{u_t}'$ are the mean adjusted estimated residuals. The test statistic is of the form

$$Q_h = T \sum_{j=1}^{h} tr(\hat{C_j}' \hat{C_0}^{-1} \hat{C_j} \hat{C_0}^{-1}) \tag{3.20}$$

or the modified version

$$Q_h^* = T^2 \sum_{j=1}^{h} \frac{1}{T-j} tr(\hat{C_j}' \hat{C_0}^{-1} \hat{C_j} \hat{C_0}^{-1}) \tag{3.21}$$

for an unrestricted stationary process VAR(p) their null distributions can be approximated by $\chi^2(K^2(h-p))$. Portmanteau test is usually used for aurtocorrelation of high order.

Breusch-Godfrey-LM tests

used for low order autocorrelation. May be viewed as a test for zero coefficient matrices in a VAR model for residuals

$$u_t = B_1 u_{t-1} + ... + B_h u_{t-h} + e_t \tag{3.22}$$

where e_t denotes a white noise error term. Thus the hypothesis test is applied

$$H_0 : B_1 = ... = B_h = 0 \tag{3.23}$$

$$H_1 : B_i \neq 0, i \in 1, ..., h \tag{3.24}$$

3.6 Policy Simulations

3.6.1 Granger causality analysis

A Granger causality test specifically measures a time series prediction ability of future values using prior values of another time series. To be precise, Granger causality tests for temporal relations in the context of whether one variable forecasts another. Since the VAR models describe the joint generation process of variables, they can be used for investigating relations between variables. A specific type of relation proposed by Granger as Granger-causality. Granger called a variable y_{2t} causal for variable y_{1t} if the past and present values of y_{2t} are helpful in improving the forecasts of y_{1t}.

3.6.2 Impulse response function

An impulse response can be defined as the reaction of a system, in response to an external change. In a multivariate autoregressive model the external change is referred to as an exogenous shock. All variables in the VAR model depend on each other. Visualizing the Impulse response gives an idea of the dynamical behaviour of the VAR model. The IRF allows us to trace the transmission of a single shock within a system of equations.

3.6.3 Forecast Error Variance Decomposition

Forecast error variance decomposition (FEVD) is a part of structural analysis which "decomposes" the variance of the forecast error into the contributions from specific exogenous shocks. Each histogram (or bar plot) represents the proportion of the forecast error variance of a variable that is attributed to various shocks over different forecast horizons. The percentages denote how much of the forecast error variance of the variable is explained by each of the shocks.

3.7 Forecasting

The reduced form VAR models represent conitional mean of a stochastic process. They are suitable for forecasting.

3.7.1 Forecasting known VAR process

If y_t is generated by a VAR(p) process, The conditional expectation of y_{t+h} given y_t,$t \leq T$ is

$$y_{T+h|T} = E(y_{T+h}|y_T, y_{T-1}, ...) \tag{3.25}$$

where $y_{T+j|T} = y_{T+j}$ for $j \leq 0$. For the optimal MSE h-step ahead forecast u_t is required to be a white noise iid. The forecasts can be easily computed for $h = 1, 2, ...$ The forecast error associated with h-step forecast is

$$y_{T+h} - y_{T+h|T} = u_{T+h} + \Phi_1 u_{T+h-1} + ... + \Phi_{h-1} u_{T+1} \tag{3.26}$$

where the Φ_i matrices may be obtained recursively as

$$\Phi_i = \sum_{j=1}^{i} \Phi_{i-j} A_j \tag{3.27}$$

The forecast error covariance matrix is

$$\sum_y(h) = E[(y_{T+h} - y_{T+h|T})(y_{T+h} - y_{T+h|T})'] \tag{3.28}$$

$$\sum_y(h) = \sum_{j=0}^{h-1} \Phi_j \Sigma_u \Phi_j' \tag{3.29}$$

that is $y_{T+h} - y_{T+h|T} \sim (0, \Sigma_y(h))$. For a gaussian VAR process y_t with $u_t \sim N(0, \Sigma_u)$, the forecast errors are also multivariate normal $y_{T+h} - y_{T+h|T} \sim N(0, \Sigma_y(h))$.For a non gaussian process y_t with unknown distribution other methods for setting up forecast intervals are called for bootstrap methods may be considered.

Forecasting estimated VAR processes

If the data generating properties are unknown and hence the VAR model only approximates true Data Generating properties the previously discussed forecasts will not be available. If $y_{T\hat{+}h|T}$ denotes the forecast based on VAR model. The forecast error is

$$y_{T+h} - y_{T\hat{+}h|T} = (y_{T+h} - y_{T+h|T}) + (y_{T+h|T} - y_{T\hat{+}h|T}) \tag{3.30}$$

The MSE matrix has the form

$$\Sigma_{\hat{y}}(h) = E[(y_{t+h} - y_{T\hat{+}h|T})(y_{t+h} - y_{T\hat{+}h|T})'] \tag{3.31}$$

$$\Sigma_{\hat{y}}(h) = \Sigma_y(h) + MSE(y_{T+h|T} - T \hat{+} h|T) \tag{3.32}$$

Chapter 4

Discussion

4.1 Dataset

The dataset selection was based on the necessity to assess the exchange rate variation of Srilanka and due to the economic crisis. There have been a number of hypothesis about the reason behind the issue. By the use of multivariate time series analysis it is possible to evaluate the effect of long term variations of imports and export expenditure volumes on exchange rates since exchange rates are a major value effected during international trade. The VAR model forecasting was performed for data obtained from the central bank of Srilanka official website economic data library
`https://www.cbsl.lk/eresearch/`.
The dataset consists monthly data from 2015-January to 2020-December for each variable. Where each variable depicts the monthly expenditure volumes for various import and exports. Also the monthly average exchange rate which is the variable that is of our main concern.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Item Name	Unit	Scale	2015-Jan	2015-Feb	2015-Mai	2015-Apr	2015-Ma	2015-Jun	2015-Jul	2015-Auç	2015-Sep	2015-Oct	2015-No
2		Imports and Exports-Imports (in USD terms)													
3	1	Merchandise Imports - Total	US Dollai	Millions	1682	1530	1581	1490	1585	1679	1534	1523	1583	1638	1465
4	2	Consumer Goods Imports	USD	Millions	397.34	352.45	441.56	375.99	348.76	391.33	414.69	420.3	392.36	403.9	390.97
5	3	Intermediate Goods Imports	USD	Millions	841.63	814.32	741	756.07	889.23	909.68	751.97	738.75	829	820.57	690.55
6	4	Investment Goods Imports	USD	Millions	441.84	362.31	397.53	353.18	345.81	377.1	365.83	363.8	359.6	413.07	382.58
7		Imports and Exports-Exports (in USD terms)													
8	5	Merchandise Exports - Total	US Dollai	Millions	920	907	1068	717	886	946	937	802	853	851	838
9	6	Agricultural Exports	USD	Millions	201.97	202.87	214.1	182.1	216.95	226.73	245.35	177.76	205.63	217.51	199.37
10	7	Industrial Exports - Total	USD	Millions	714.43	699.31	849.13	531.42	664.81	715.28	687.39	621.44	640	630.35	636.46
11	8	Mineral Exports	USD	Millions	2.84	3.83	3.32	2.21	2.89	2.35	3.16	1.8	1.7	1.68	1.48
12		Imports and Exports-Commodity Prices (in USD terms)													
13	9	Rice Imports	USD	Per MT	459.48	474	415.52	487.61	499.73	535.7	530.34	427.23	626.93	582.87	399.83
14	10	Sugar Imports	USD	Per MT	458.47	450.5	433.04	409.6	394.81	401.16	381.93	367.15	384.33	364.26	396.06
15	11	Wheat Imports	USD	Per MT	0	0	0	0	0	0	0	0	0	0	0
16	12	Crude Oil Imports	USD	Per Barre	55.47	69.52	0	0	63.09	66.17	60.49	54.92	48.65	49.26	45.46
17		Tourism-Tourist Earnings													
18	13	Tourist Earnings	USD	Millions	248.7	263.5	250	194.6	180.7	183.8	279.9	265.2	228.2	210.6	229.5
19		Exchange Rates-Monthly Exchange Rates													
20	14	Month End a Exchange Rates			132.2	132.9	132.9	132.9	133.9	133.7	133.6	134.3	141.2	140.9	143.2
21	15	Monthly Average Exchange Rates			131.6	132.7	132.9	132.9	133.5	133.9	133.7	133.9	138.9	140.9	142

Figure 4.1: Dataset

The dataset consists two exchange rates. The month end exchange rate and the monthly average exchange rate. It was decided that the monthly average exchange rate is more suitable since it represents an average of the monthly exchange rate variation. And since the analysis involves time series with monthly frequency.

4.2 Visualizing each time series

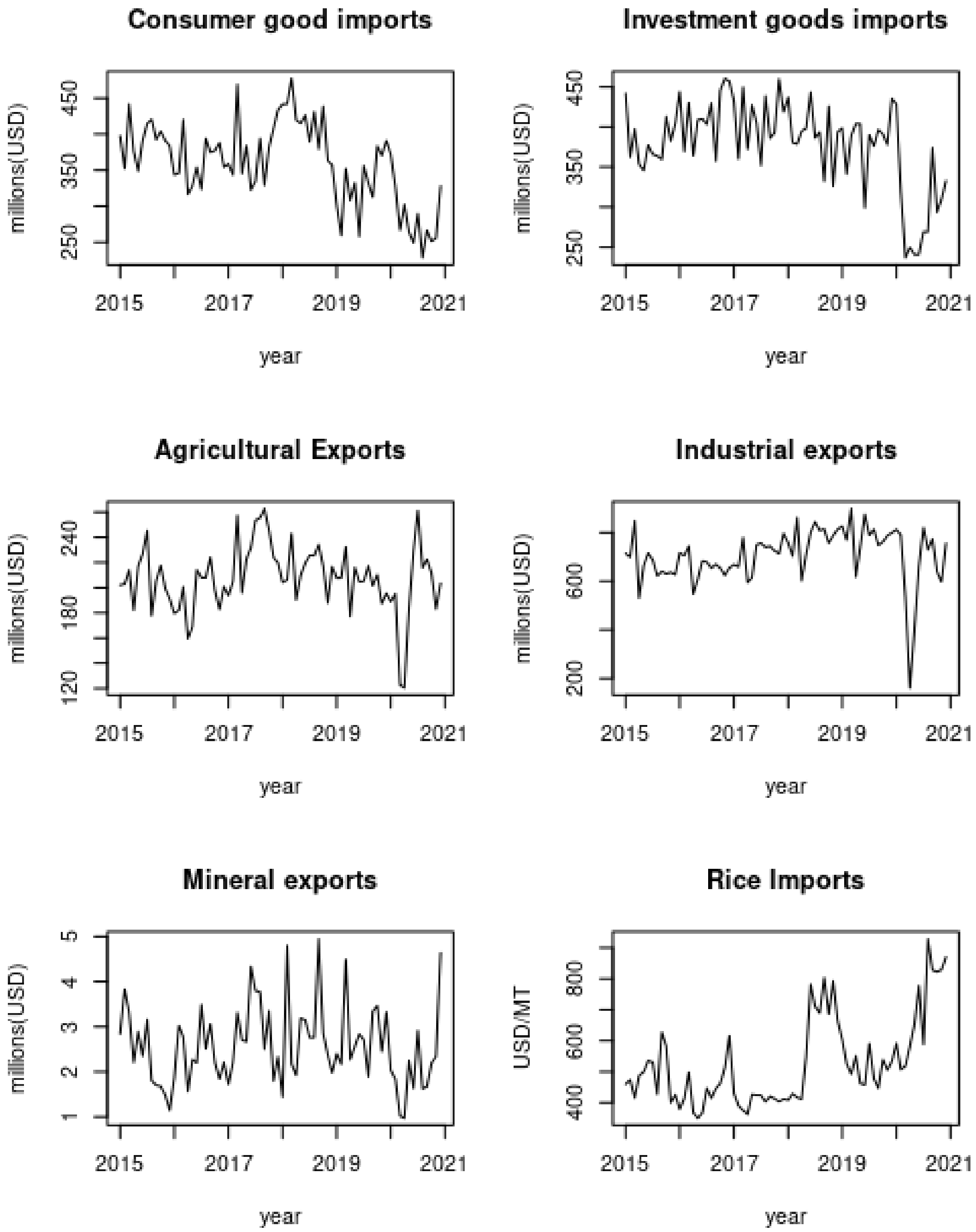


Figure 4.2: Time series visualization

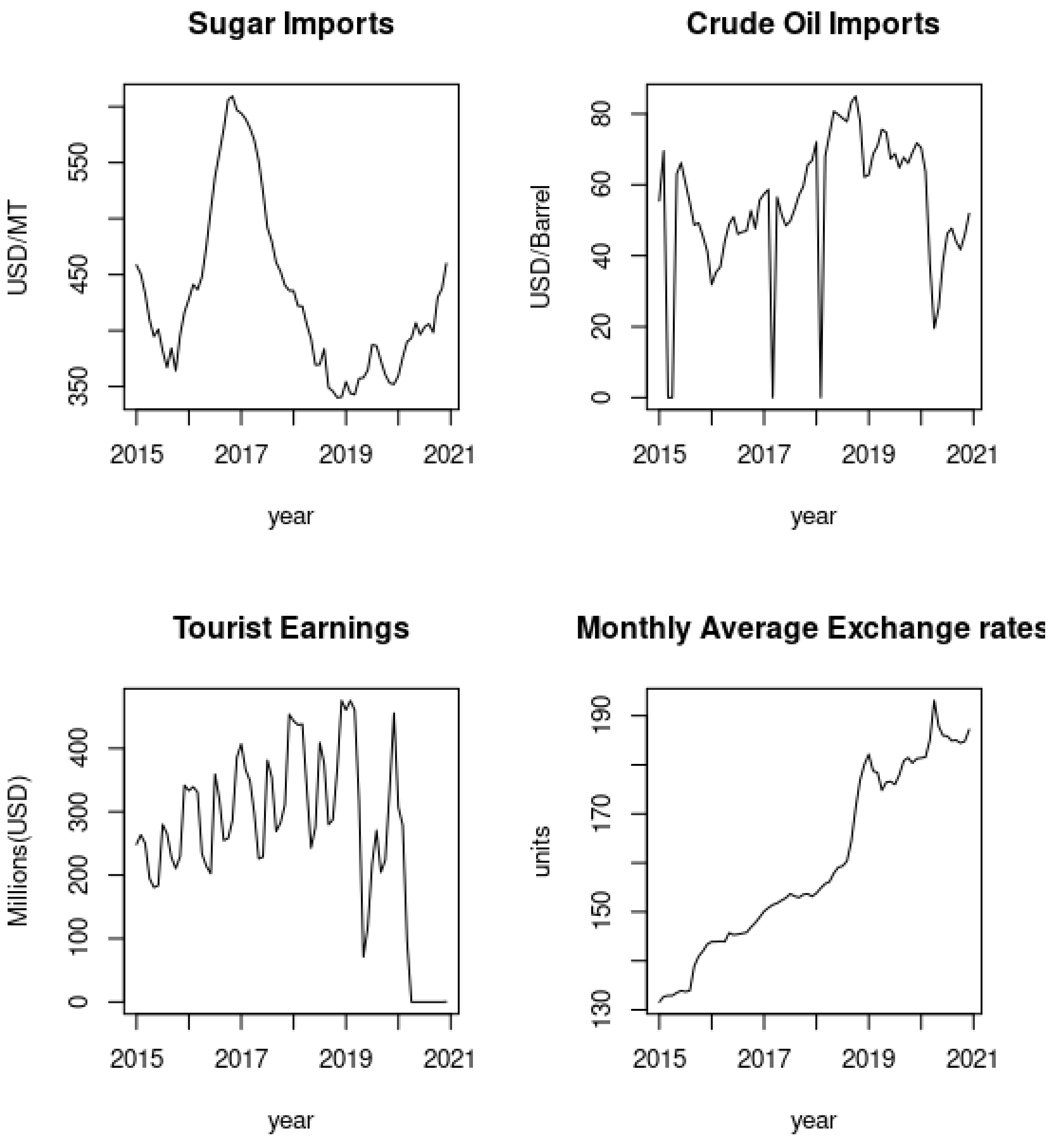


Figure 4.3: Time series visualization

All time series except Rice Imports, Sugar Imports, Tourist Earnings, monthly average exchange rates time series depict a stationary nature. The Monthly average exchange rates depicts a trend pattern.

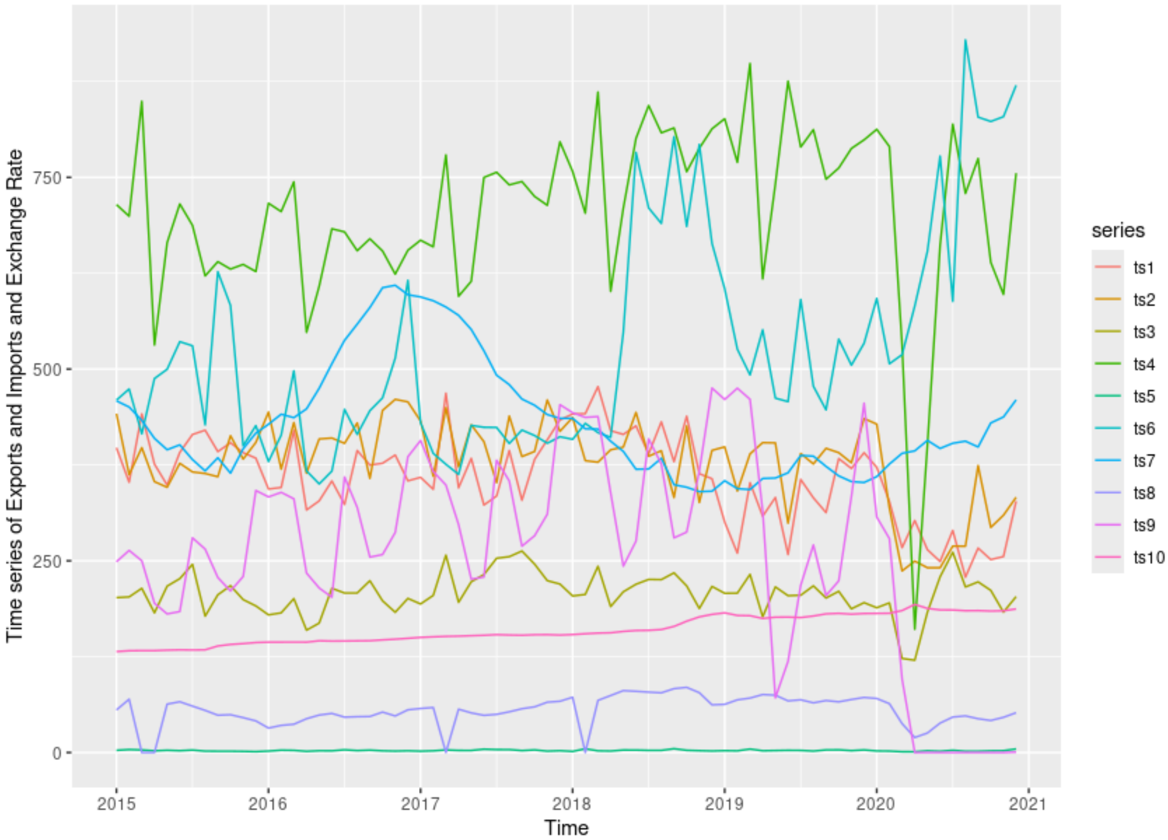


Figure 4.4: simultaneous visualization of all time series

- ts1: consumer good imports time series
- ts2: investment goods imports time series
- ts3: agricultural exports time series
- ts4: industrial exports time series
- ts5: mineral exports time series
- ts6: Rice Imports time series
- ts7: Sugar Imports time series
- ts8: Crude Oil Imports time series
- ts9: Tourist Earnings time series
- ts10: monthly average exchange rates time series

4.3 Test for Structural Breaks

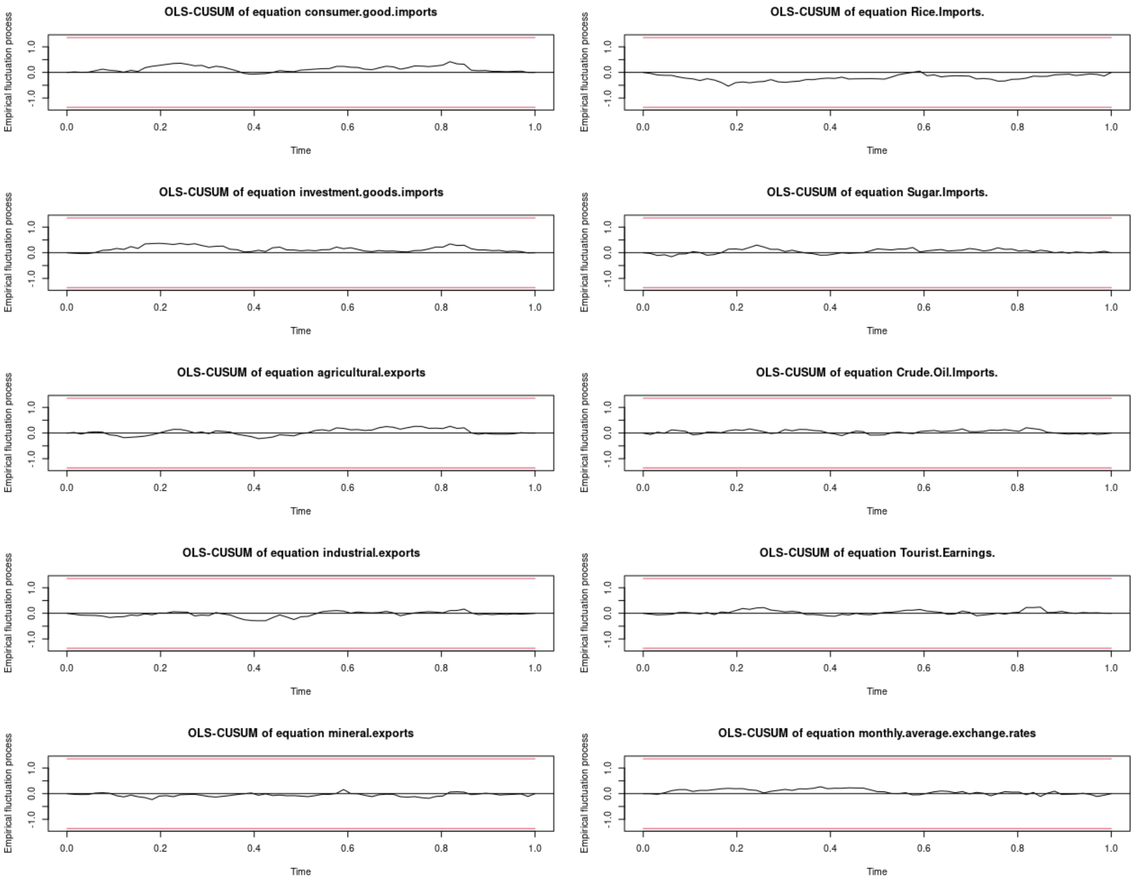


Figure 4.5: OLS-CUSUM plot

The plot typically shows the cumulative sum of the residuals of the model over time. This line can help identify periods when the model's residuals significantly deviate from zero, indicating potential structural changes. The plot often includes confidence bands (usually at 95 percent confidence level). If the CUSUM line crosses these bands, it suggests a structural break or instability in the model. Through observation of each plot, we can conclude that there are no structural breaks in any of the variables since none of the points in the plot pass the red line.

4.4 Forecast Error Variance Decomposition

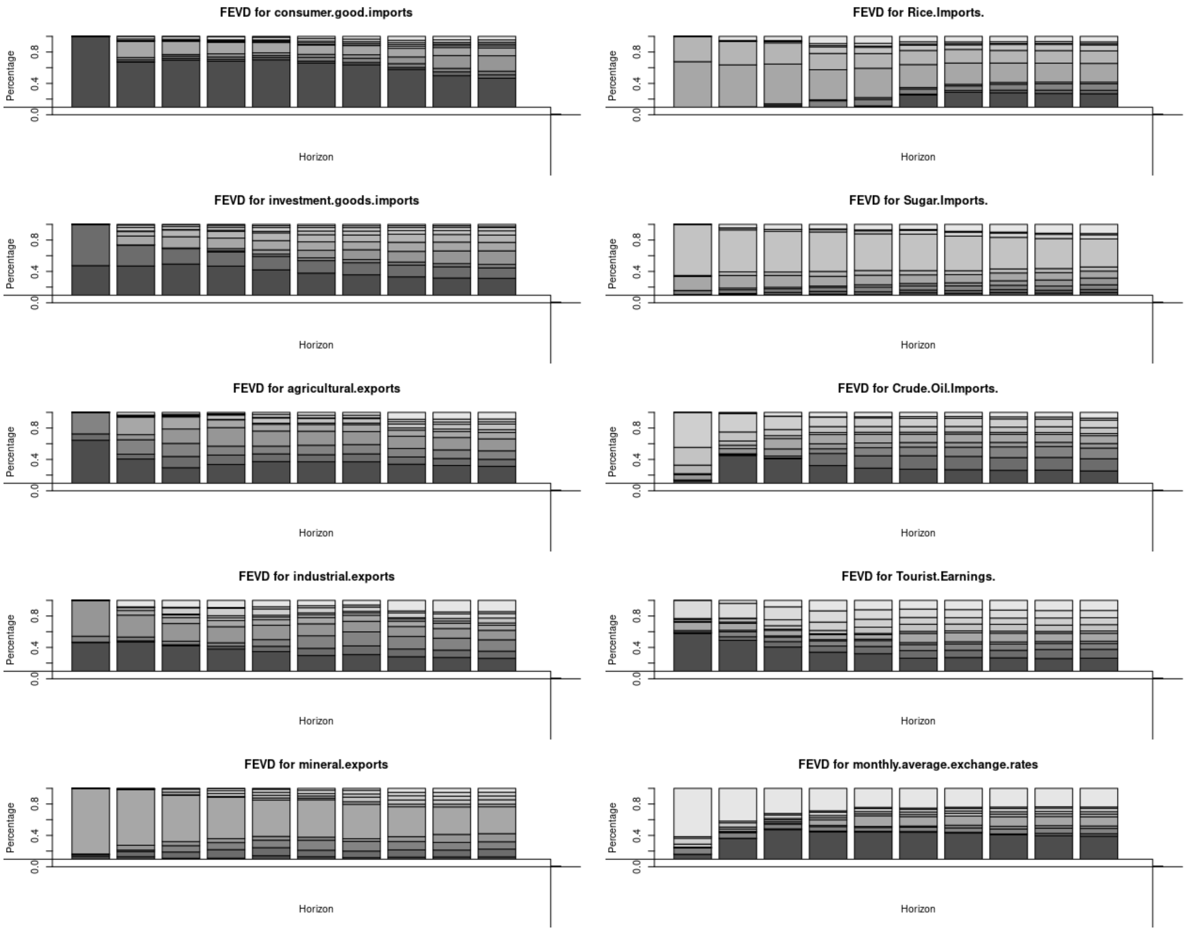
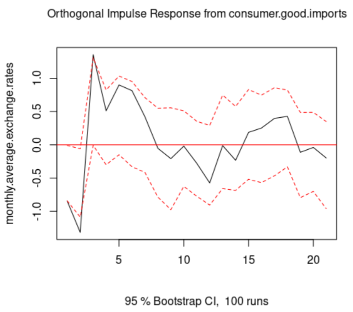


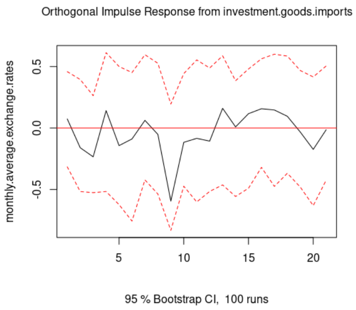
Figure 4.6: Forecast error variance decomposition

This plot is useful in assessing the development in shocks of the variables. Through examination of each plot we can observe that to every variable there is a significant contribution from other variables to the forecast error variance. We can observe shocks in different proportions to each variable using the bar plots.

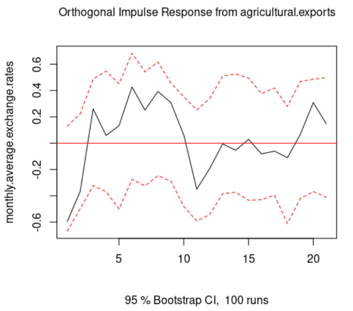
4.5 IRF plots depicting the response of monthly average exchange rate by an Impulse of other variables



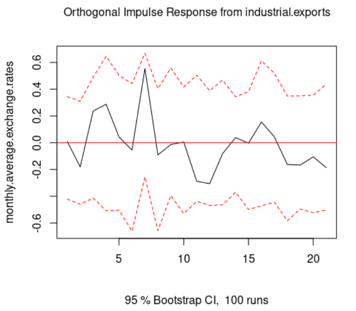
(a) Impulse of consumer good imports



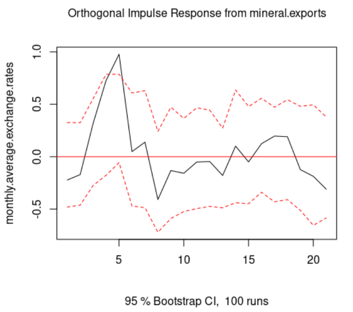
(b) Impulse of investment goods imports



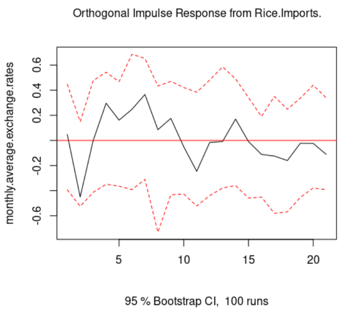
(c) Impulse of agricultural exports



(d) Impulse of industrial exports

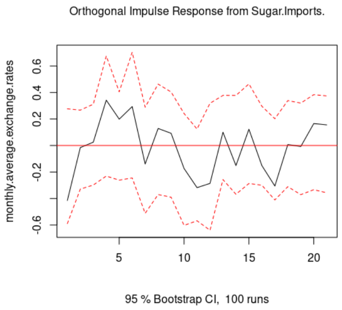


(e) Impulse of mineral exports

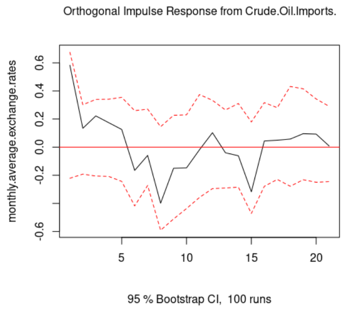


(f) Impulse of rice imports

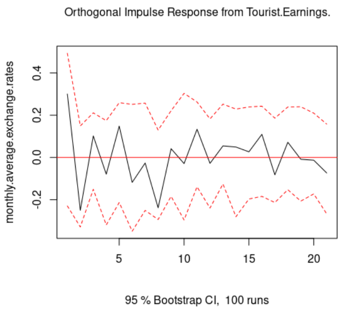
Figure 4.7: Impulse response functions plot



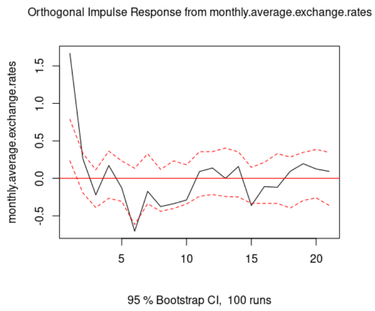
(a) Impulse of sugar imports



(b) Impulse of crude oil imports



(c) Impulse of tourist earnings



(d) Impulse of monthly average exchange rates

Figure 4.8: Impulse response functions plot

Majority of the plots depict temporary shocks. A few depict a statistically insignificant nature. The nature of shock on exchange rate by each of the various can be observed from the plots. The shocks by consumer goods imports, rice imports, tourist earning and also by monthly avergae exchnage rates on itself seem to dampen over time.

4.6 Forecasting

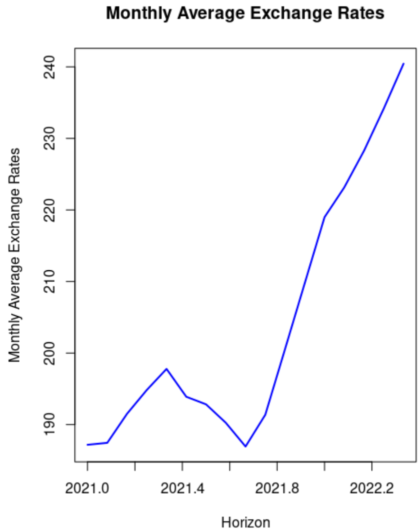


Figure 4.9: Forecasting Monthly Average Exchange Rates

The forecasts were obtained from 2021-January to 2022-may. The forecasted values are

year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2021	187.18	187.45	191.51	194.84	197.78	193.89	192.83	190.26	186.94	191.37	200.56	209.78
2022	218.99	223.12	228.23	234.17	240.44	-	-	-	-	-	-	-

The actual time series variation in the real world scenario has a similar trend pattern to the forecast. We can capture the trend of the monthly average exchange rate to a satisfactory accuracy.

Chapter 5

Conclusion

The VAR(P) model of lag order p=5 has been fitted with satisfactory accuracy. The optimal lag for the VAR model was determined to be 5 by the AIC= 'akike information criterion', SBIC='swartz information criterion' or 'bayesian information criterion', HQIC='Hannan-Quinn Information Criterion', FPE='Final Prediction Error'. It was determined through the portmanteau test that there is no residual autocorrelation. By the use of the Jarque-Bera Test (JB-Test), Skewness Test and Kurtosis Test the residuals were determined to be not normally distributed. Analysing the OLS-CUSUM plot it was concluded that there are no structural breaks in any variables. Through Granger casuality analysis of each variable it was determined that all variables have Granger-cause. The effects of shocks of each variable were observed and the proportion of each shock were analyzed through the impulse response functions and the forecast error variance decomposition functions. It was concluded that within a 95 percent confidence interval, the VAR(5) model predicts the monthly average exchange rate to increase rapidly in the future. The forecasts allign with the real world scenario since the exchange rate of Sri Lanka was subjected to rapid increase within the forecasted time period. We can conclude that the underlying reason for the rapid increase in the exchange rate of Sri Lanka was a result of long term inefficient policy measures and mismanagement of imports and exports within the analyzed time period.

Bibliography

[1] Helmut Lütkepohl. *New introduction to multiple time series analysis*. Springer Science & Business Media, 2005.

[2] Helmut Lütkepohl. Vector autoregressive models. In *Handbook of research methods and applications in empirical macroeconomics*, pages 139–164. Edward Elgar Publishing, 2013.

[3] Maurice Obstfeld and Alan C Stockman. Exchange-rate dynamics. *Handbook of international economics*, 2:917–977, 1985.

[4] Richard A.Davis Peter J.Brockwell. *Introduction to Time Series and Forecasting Second Edition*. Sringer.

[5] John Williamson. Exchange rate economics. *Open Economies Review*, 20:123–146, 2009.

[6] Qinpei Zhao, Guangda Yang, Kai Zhao, Jiaming Yin, Weixiong Rao, and Lei Chen. Multivariate time-series forecasting model: Predictability analysis and empirical study. *IEEE Transactions on Big Data*, 2023.

[5] [2] [6] [4] [3] [1]

Appendix

Maxima Codes

```
############Installing furthur required packages ##############
update.packages(ask = FALSE, checkBuilt = TRUE)#updating available
   packages prior to running code
install.packages("vars")
install.packages("mFilter")
install.packages("tseries")
install.packages("TSstudio")
install.packages("forecast")
install.packages("ragg")
install.packages("tidyverse")
install.packages("ggplot2")
#####importing the csv file ########
df <- read.csv("/home/vaasala/Desktop/extra_coding/time_series/import
   _export_exchangerate.csv")
head(df)#visualizing the first 6 rows of the dataframe

############# Data preprocessing ##########################

df_trans<-t(df)#obtaining the transpose of the original dataframe
print(df_trans)#printing the transposed dataframe
df_trans <- df_trans[-(1:4), ]#removing first four rows from the
   transposed dataframe
print(df_trans)#printing the transposed dataframe
data <- df_trans[, -c(1,6,11,16,18)]#removing unwanted columns from
   the dataframe
print(data)#printing the current dataframe

#removing unwanted variables
summary(data)#displays the five number summary
#the data is observed to be in the character class
data<-subset(data,select = -c(11,14))#removing the column 11 and 14
   of the dataframe due to many zero values
print(data)#printing the dataframe

#converting characters to numeric
data<-apply(data,2,as.numeric)#converting all columns of the
   dataframe from chatacters to numeric values
summary(data)#dsiplays the five number summary
data<-data[,1:13]#isolating the first 13 columns of the dataframe by
   removing other columns
summary(data)#observing give number summary

#removing variables with missing values
```

```
data<-subset(data,select = -c(1,3,5))#removing rows 1,3,5 dues to
    presence of 'NA' values
print(data)#printing dataframe
summary(data)#analysing five number summary


#############Visualizing each time series ################

par(mfrow=c(2,1))#chart of 2 rows and 1 column
#consumer good imports time series
ts1<-ts(data[,1],start = c(2015,1),frequency=12)
print(ts1)
plot.ts(ts1,xlab="year",ylab="millions(USD)")
#investment goods imports time series
ts2<-ts(data[,2],start = c(2015,1),frequency=12)
print(ts2)
plot.ts(ts2,xlab="year",ylab="millions(USD)")

par(mfrow=c(2,1))#chart of 2 rows and 1 column
#agricultural exports time series
ts3<-ts(data[,3],start = c(2015,1),frequency=12)
print(ts3)
plot.ts(ts3,xlab="year",ylab="millions(USD)")
#industrial exports time series
ts4<-ts(data[,4],start = c(2015,1),frequency=12)
print(ts4)
plot.ts(ts4,xlab="year",ylab="millions(USD)")

par(mfrow=c(2,1))#chart of 2 rows and 1 column
#mineral exports time series
ts5<-ts(data[,5],start = c(2015,1),frequency=12)
print(ts5)
plot.ts(ts5,xlab="year",ylab="millions(USD)")
#Rice Imports time series
ts6<-ts(data[,6],start = c(2015,1),frequency=12)
print(ts6)
plot.ts(ts6,xlab="year",ylab="USD/MT")

par(mfrow=c(2,1))#chart of 2 rows and 1 column
#Sugar Imports time series
ts7<-ts(data[,7],start = c(2015,1),frequency=12)
print(ts7)
plot.ts(ts7,xlab="year",ylab="USD/MT")
#Crude Oil Imports time series
ts8<-ts(data[,8],start = c(2015,1),frequency=12)
print(ts8)
plot.ts(ts8,xlab="year",ylab="USD/Barrel")

par(mfrow=c(2,1))#chart of 2 rows and 1 column
#Tourist Earnings time series
ts9<-ts(data[,9],start = c(2015,1),frequency=12)
print(ts9)
plot.ts(ts9,xlab="year",ylab="Millions(USD)")
#monthly average exchange rates time series
ts10<-ts(data[,10],start = c(2015,1),frequency=12)
print(ts10)
plot.ts(ts10,xlab="year",ylab="units")
```

```
#plotting all time series simultaneously
library(forecast)#for time series forecasting
autoplot(cbind(ts1,ts2,ts3,ts4,ts5,ts6,ts7,ts8,ts9,ts10),ylab = "Time
     series of Exports and Imports and Exchange Rate")
######Importing further required packages #####################
library(vars)#modelling and analyzing vector autoregressive models
library(mFilter)#filters and decomposing time series
library(tseries)#time series analysis and computational finance
library(TSstudio)#descriptive and predictive analysis of time series
library(tidyverse)#collection of packages designed for data science

#######philips perron test for stationarity #################
#null hypothesis: non stationary
#alternate hypothesis: stationary
pp.test(ts1)
#p value=0.01<0.05. therefore we reject the null hypothesis and
   accept the alternate hypothesis to be stationary
pp.test(ts2)
#p value=0.01<0.05. therefore we reject the null hypothesis and
   accept the alternate hypothesis to be stationary
pp.test(ts3)
#p value=0.01<0.05. therefore we reject the null hypothesis and
   accept the alternate hypothesis to be stationary
pp.test(ts4)
#p value=0.01<0.05. therefore we reject the null hypothesis and
   accept the alternate hypothesis to be stationary
pp.test(ts5)
#p value=0.01<0.05. therefore we reject the null hypothesis and
   accept the alternate hypothesis to be stationary
pp.test(ts6)
#p-value = 0.05421>0.05 suggests that the time series is non
   stationary
diff_ts6<-diff(ts6)
pp.test(diff_ts6)
plot.ts(diff_ts6)
#pvalue=0.01<0.05 suggest that the ts6 time series is stationary for
   1st difference
pp.test(ts7)
#p-value = 0.9023>0.05 suggests non stationarity
diff_ts7<-diff(ts7)
pp.test(diff_ts7)
plot.ts(diff_ts7)
# p-value = 0.01<0.05 implies that it is stationary for 1st
   difference
pp.test(ts8)
# p-value = 0.01<0.05 therefore we reject the null hypothesis and
   accept the alternate hypothesis to be stationary
pp.test(ts9)
#p-value = 0.3563>0.05 suggests non stationarity
diff_ts9<-diff(ts9)
pp.test(diff_ts9)
#p-value = 0.01<0.05 suggests stationarity
plot.ts(diff_ts9)
pp.test(ts10)
#p-value = 0.3756>0.05, suggests non stationarity
diff_ts10<-diff(ts10)
pp.test(diff_ts10)
#p-value = 0.01<0.05 implies stationarity
```

```
######################## model  ##########################


#combining tine series into one dataframe
v1 <- cbind(ts1, ts2, ts3, ts4, ts5, diff_ts6, diff_ts7, ts8, diff_
   ts9, diff_ts10)#combining R objects as columns
print(v1)
#assigning column names to the new object
colnames(v1) <- cbind("consumer good imports","investment goods
   imports","agricultural exports", "industrial exports",
                      "mineral exports","Rice Imports ","Sugar
                         Imports ","Crude Oil Imports ","Tourist
                         Earnings ", "monthly average exchange rates"
                         )
print(v1)

#Removing rows with 'NA' values
v1<-na.omit(v1)
print(v1)
################# Model selection criteria ########################

#we will select the lag order by using the command VARselect() .The
   command will automatically generate lag order based on
#multivariate iterations of the AIC, SBIC, HQIC and the FPE
#AIC=akike information criterion
#SBIC=swartz information criterion or bayesian information criterion
#HQIC=Hannan-Quinn Information Criterion
#FPE=Final Prediction Error
lagselect <- VARselect(v1, lag.max = 20, type = "const")#the maximum
   lag is assigned as 20,constant intercept is optimal
lagselect$selection
#most criterion suggest the optimal lag length to be 5. Therefore we
   select the optimal lag length as 5.

####################### model fitting #########################
Model1 <- VAR(v1, p = 5, type = "const", season = NULL, exog = NULL)
#lag length of 5. There are no observed seasonal patterns in the time
    series. Also it is assumed to not consist exogenous variables
summary(Model1)

######################## model diagnostics #####################
#tests for residual autocorrelation

#portmantau test
#null hypothesis: there is no residual autocorrelation
#alternate hypothesis: there is residual autocorrelation
Serial1 <- serial.test(Model1, lags.pt = 5, type = "PT.asymptotic")
print(Serial1)
#since thep-value < 2.2e-16 , we fail to reject the null hypothesis.
   There is no residual autocorrelation


#tests for normality of residuals
Norm1 <- normality.test(Model1, multivariate.only = TRUE)
print(Norm1)
#Jarque-Bera Test (JB-Test)
#H0: residuals are normally distributed
```

```
#H1: residuals are not normally dsitributed
#since p-value: 0.04137<0.05, we reject the null hypothesis and say
   that the residuals are not normally distributed

#Skewness Test
#H0: residuals are symmetric
#H1: residuals have skewness
#p-value: 0.4611>0.05, there we reject the null hypothesis and assume
    residuals have skewness

#Kurtosis Test
#H0: normally distributed residuals
#H1: residuals are not normally distributed
#p-value: 0.01312<0.05, therefore there is significant kurtosis in
   residuals

#The final conclusion is that the residuals are not normally
   distributed. Which can have issues on confidence intervals.
#But is a issue that can be neglected

#stability test(assessing presence of structural breaks)
Stability1 <- stability(Model1, type = "OLS-CUSUM")
plot(Stability1)
#since none of the points in the graph pass the red critical region,
   no structural brakes can be seen

################# Policy simulations #####################

#Granger Causality
#null hypothesis: does not granger-cause other variables
#alternate hypothesis:granger -cause other variables

#f-test: long term causality. indicates the ability of one variable
   to predict the other
#chi-squared: instantaneous causality, or causlity within same time
   frame

Grangerts1<- causality(Model1, cause = "consumer.good.imports")
Grangerts1
#p-value = 7.438e-14<0.05, there is  granger-cause

Grangerts2<- causality(Model1, cause = "investment.goods.imports")
Grangerts2
#p-value = 1.064e-11<0.05,therefore there is granger-cause

Grangerts3<- causality(Model1, cause = "agricultural.exports")
Grangerts3
#p-value = 8.882e-16<0.05,therefore there is  granger cause

Grangerts4<- causality(Model1, cause = "industrial.exports")
Grangerts4
#p-value = 9.723e-13<0.05, therefore there is granger-cause
Grangerts5<- causality(Model1, cause = "mineral.exports")
Grangerts5
#p-value = 2.533e-08<0.05,therefore there is granger cause
Grangerts6<- causality(Model1, cause = "Rice.Imports.")
Grangerts6
#p-value = 0.001726<0.05,therefore there is granger cause
```

```
Grangerts7<- causality(Model1, cause = "Sugar.Imports.")
Grangerts7
#p-value = 5.149e-07<0.05,therefore there is granger cause
Grangerts8<- causality(Model1, cause = "Crude.Oil.Imports.")
Grangerts8
#p-value = 0.000331<0.05,therefore there is granger cause
Grangerts9<- causality(Model1, cause = "Tourist.Earnings.")
Grangerts9
#p-value = 6.661e-16<0.05, therefore there is granger-cause
Grangerts10<- causality(Model1, cause = "monthly.average.exchange.
   rates")
Grangerts10
#p-value = 1.457e-10<0.05, therefore there is granger cause


#Forecast Error Variance Decomposition

#we can trace the development of shocks in our system to explaining
   the forecast error variances of all the variables in the system
FEVD1 <- fevd(Model1, n.ahead = 10)
FEVD1
plot(FEVD1)


#Impulse Response Functions
ts1irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "consumer.good.imports", n.ahead = 20, boot = TRUE)
plot(ts1irf)
#The shocks are temporary and dampen over time
ts2irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "investment.goods.imports", n.ahead = 20, boot = TRUE)
plot(ts2irf)
#The shocks are temporary and will stabilize
ts3irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "agricultural.exports", n.ahead = 20, boot = TRUE)
plot(ts3irf)
#The shocks are temporary and dampen over time
ts4irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "industrial.exports", n.ahead = 20, boot = TRUE)
plot(ts4irf)
#The shock is not statistically significant
ts5irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "mineral.exports", n.ahead = 20, boot = TRUE)
plot(ts5irf)
#shocks are temporary
ts6irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "Rice.Imports.", n.ahead = 20, boot = TRUE)
plot(ts6irf)
#Shocks are temporary and dampen over time
ts7irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "Sugar.Imports.", n.ahead = 20, boot = TRUE)
plot(ts7irf)
#The shock is not statistically significant
ts8irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "Crude.Oil.Imports.", n.ahead = 20, boot = TRUE)
plot(ts8irf)
#The shocks are temprorary and damp over time. Variable shows mean
   reversion
```

```
ts9irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "Tourist.Earnings.", n.ahead = 20, boot = TRUE)
plot(ts9irf)
#shocks are not statistically significant
ts10irf <- irf(Model1, response = "monthly.average.exchange.rates",
   impulse = "monthly.average.exchange.rates", n.ahead = 20, boot =
   TRUE)
plot(ts10irf)
#shocks are temporary

################### forecasting  model ################

forecast <- predict(Model1, n.ahead = 4, ci = 0.95)#predictions 2
   years ahead
fanchart(forecast, names = "consumer.good.imports", main = "Fanchart
   for consumer.good.imports", xlab = "Horizon", ylab = "consumer.
   good.imports")
fanchart(forecast, names = "investment.goods.imports", main = "
   Fanchart for investment.goods.imports", xlab = "Horizon", ylab = "
   investment.goods.imports")
fanchart(forecast, names = "agricultural.exports", main = "Fanchart
   for agricultural.exports", xlab = "Horizon", ylab = "agricultural.
   exports")
fanchart(forecast, names = "industrial.exports", main = "Fanchart for
    industrial.exports", xlab = "Horizon", ylab = "industrial.exports
   ")
fanchart(forecast, names = "mineral.exports", main = "Fanchart for
   mineral.exports", xlab = "Horizon", ylab = "mineral.exports")
fanchart(forecast, names = "Rice.Imports.", main = "Fanchart for Rice
   .Imports.", xlab = "Horizon", ylab = "Rice.Imports.")
fanchart(forecast, names = "Sugar.Imports.", main = "Fanchart for
   Sugar.Imports.", xlab = "Horizon", ylab = "Sugar.Imports.")
fanchart(forecast, names = "Crude.Oil.Imports.", main = "Fanchart for
    Crude.Oil.Imports.", xlab = "Horizon", ylab = "Crude.Oil.Imports.
   ")
fanchart(forecast, names = "Tourist.Earnings.", main = "Fanchart for
   Tourist.Earnings.", xlab = "Horizon", ylab = "Tourist.Earnings.")
fanchart(forecast, names = "monthly.average.exchange.rates", main = "
   Fanchart for monthly.average.exchange.rates", xlab = "Horizon",
   ylab = "monthly.average.exchange.rates")

# Reverse the first difference to get the integrated time series
integrated_forecast <- cumsum(c(ts10[length(ts10)], forecast$fcst$
   monthly.average.exchange.rates))
integrated_forecast <- ts(integrated_forecast, start = c(2021,1),
   frequency = frequency(ts10))
# Plotting the fan chart
plot(integrated_forecast, type = "l", col = "blue", lwd = 2,
     main = "Fanchart for Monthly Average Exchange Rates",
     xlab = "Horizon", ylab = "Monthly Average Exchange Rates")
```
Listing 5.1: VAR model for monthly average exchange rate