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Is the Motherhood Penalty Larger for Low-Wage Women? A Comment on Quantile Regression

Alexandra Killewald[a] and Jonathan Bearak[b]

Abstract

In this comment, we offer a nontechnical discussion of conventional (conditional) multivariate quantile regression, with an emphasis on the appropriate interpretation of results. We discuss its distinction from *unconditional* quantile regression, an analytic method that can be used to estimate varying associations between predictors and outcome at different points of the outcome distribution. We argue that the research question posed by Budig and Hodges (2010)—whether the motherhood penalty is larger for low-wage women—cannot be answered with the authors' conditional quantile regression models. Using more appropriate unconditional quantile regression models, we find, in contrast to Budig and Hodges's claims, that the motherhood penalty is not largest for low-wage women.

Keywords

earnings, family, working parents, quantitative methods, quantile regression

In a 2010 *American Sociological Review* (*ASR*) article, Budig and Hodges (BH) claim that motherhood differentially affects women's wages at different points in the wage distribution. Their work is also, we believe, the first use of quantile regression published in *ASR*.[1] Because of this methodological innovation and the substantive importance of their claim, we take this opportunity to discuss the interpretation of quantile regression results. We show that BH made an interpretation error when discussing their results and that, when a method more appropriate to their research question is used, their results are not robust.

To assess whether the motherhood penalty varies across the wage distribution, BH used fixed-effects models and data on white women from the 1979 to 2004 waves of the National Longitudinal Survey of Youth (NLSY79). While prior research has focused on estimates of the average motherhood penalty, BH's innovation was to use quantile regression to generate multiple estimates of the motherhood penalty at various points in the wage distribution. BH proposed that the size of the motherhood penalty may vary for women at different wage levels. For example, they noted that "relative to low-wage workers, high-earning women are likely to live in households with greater resources (e.g., a marital partner and higher family income), possess greater human

[a]Harvard University
[b]New York University

Corresponding Author:
Alexandra Killewald, 612 William James Hall, Harvard University, 33 Kirkland Street, Cambridge, MA 02138
E-mail: killewald@fas.harvard.edu

capital (e.g., education), and hold jobs with more family-friendly characteristics (e.g., health benefits, greater autonomy, and flexibility)" (p. 706). After presenting their results, they conclude that "mothers who can least afford to lose wages are the ones incurring the largest proportionate penalties" (p. 725). Their hypotheses and conclusions are framed in terms of the unconditional wage distribution: the comparison is among mothers with different wage levels.

BH employ conventional quantile regression to answer their question. Although the literature typically refers to this method simply as quantile regression, we refer to it as conditional quantile regression (hereafter, CQR), to distinguish it from the unconditional quantile regression (hereafter, UQR) approach we also discuss. As we describe in the following sections, CQR estimates the association between motherhood and wages at different points of the conditional wage distribution, thereby estimating variation in the motherhood penalty among women with different values of hourly wages but similar covariate values. Women at high quantiles of the conditional wage distribution have higher wages than we would expect, given their covariates, but are not necessarily high-wage in an absolute sense. CQR is therefore inappropriate for the question posed by BH, because conditional quantiles do not identify women along the unconditional wage distribution. This problem is exacerbated by the fact that BH estimated individual fixed effects, which de-mean (center) data at the individual level. This means each observation records only whether a woman's wage in a particular period was higher or lower than her own average wage, not whether she was a high- or low-wage worker in the population. Thus, BH's analysis cannot answer the question they pose, which relates to variation in the motherhood penalty at different points of the unconditional wage distribution: high- versus low-wage workers.

Instead, their question can be answered with UQR, an analytic approach recently proposed in the econometrics literature (Firpo, Fortin, and Lemieux 2009 [hereafter, FFL]). Using UQR, we show the motherhood penalty is larger for women in the middle of the wage distribution than for women below or above the median.

Our intention is not to single out BH's analysis for criticism, but to use it as a motivating example. Throughout, we adopt a nontechnical tone. Formal discussions of CQR are widely available (Hao and Naiman 2007; Koenker 2005), and technical material concerning UQR, including links to computer code, is available in FFL. We emphasize interpretation of results of CQR and UQR with multiple covariates, making frequent use of examples. We hope to provide practical guidance to sociologists interested in the meaning and interpretation of quantile regression.

QUANTILE REGRESSION WITH A SINGLE INDEPENDENT VARIABLE

Before moving to the more complicated case with multiple covariates, we discuss the intuition behind quantile regression with only a single independent variable. In a quantile regression with the log of wages as the outcome and motherhood as the only covariate, we can imagine dividing the wage distribution into two subgroups: one of mothers and the other of childless women. We could then compare quantiles of the two wage distributions. The difference between the median wage among mothers and the median wage among childless women would indicate the association between motherhood and wages for women at the median of the wage distribution.[2] In similar fashion, we could estimate the motherhood penalty at other points of the unconditional wage distribution. Penner and Paret (2008) used this approach to examine whether the female disadvantage in math test scores varies across the score distribution for elementary school children.

For the question Penner and Paret (2008) posed, quantile regression with a single independent variable is appropriate: the article is purely descriptive and there is no concern of selection into being female by unobserved

Table 1. Stylized Example of the Effect of De-meaning Individual Wage Data

	Jennifer			Allison		
	Wage	Ln(wage)	De-meaned ln(wage)	Wage	Ln(wage)	De-meaned ln(wage)
Year 1	$5.75	1.75	–.25	$42.52	3.75	–.25
Year 2	$7.39	2.00	.00	$54.60	4.00	.00
Year 3	$9.49	2.25	.25	$70.11	4.25	.25
Average	$7.54	2.00		$55.74	4.00	

traits. For the motherhood penalty, the situation is not so simple. To estimate the causal effect of motherhood on wages at different points of the wage distribution, it is necessary to adjust for spurious factors that may be associated with both motherhood and wages, such as age or education. Multiple control variables are thus necessary.

CONDITIONAL QUANTILE REGRESSION WITH MULTIPLE INDEPENDENT VARIABLES

In models with covariates, CQR estimates the magnitude of the motherhood penalty at different points in the conditional wage distribution (BH p. 712). In other words, the conditional distribution measures whether individuals have higher or lower wages than would be expected, given their other characteristics. As an example, imagine a quantile regression with the log of wages as the outcome and just two regressors: motherhood and education. If the motherhood penalty is 5 percent at the 75th percentile and 15 percent at the 25th percentile, it means mothers who are high-wage conditional on their educational attainment have a smaller wage penalty than mothers who are low-wage conditional on their educational attainment. Of course, women's wages depend on their education. Therefore, the conditional quantiles cannot be interpreted as the effect of motherhood on high- and low-wage workers, because low-wage workers are much more likely than high-wage workers to have low education. Instead, results can be interpreted as indicating the motherhood penalty at different points of the wage distribution within each educational group. Although this may be a useful interpretation in some circumstances, BH's stated question is to explore variation in the motherhood penalty "among low-wage, middle-wage, and high-wage workers" (p. 705), rather than within subgroups defined by covariates. Thus, CQR is not an appropriate analytic tool for BH's question of interest.

The distinction between conditional and unconditional quantiles is particularly important because BH de-mean their data at the individual level prior to analysis (p. 713). In the quantile regression context, however, it is not possible to interpret results of models estimated on de-meaned data as indicative of the experiences of low- versus high-wage women in an unconditional sense.

To illustrate this point, consider two women, Jennifer and Allison, each of whom is observed three times in the data, with the hourly wages shown in Table 1. Jennifer is a low-wage worker, with an average hourly wage of $7.54; Allison is a high-wage worker, with an average hourly wage of $55.74. Once the wage data are logged and de-meaned, however, the women have identical wage distributions: –.25 in Year 1, 0 in Year 2, and .25 in Year 3. In this de-meaned dataset, Year 1 wages for both women are low-wage observations and Year 3 observations are high-wage. CQR on de-meaned data cannot be used to draw conclusions about the magnitude of the motherhood penalty at different points of the between-woman unconditional wage distribution, because they compare each woman to herself rather than to other women.

AN ALTERNATIVE APPROACH: UNCONDITIONAL QUANTILE REGRESSION

UQR (FFL 2009), by contrast, defines quantiles *pre-regression*: covariates help net out spurious associations between motherhood and wages, but their inclusion has no effect on which observations are defined to be at the median or other quantiles of the wage distribution. UQR can therefore be used to answer BH's question of the effect of motherhood on wages for women at different positions in the unconditional wage distribution.

UQR models can be estimated using a simple ordinary least squares (OLS) regression on a transformed dependent variable, the recentered influence function (RIF), which is defined as follows:

$$RIF\left(Y; q_{\tau}, F_Y\right) = q_{\tau} + \left(\tau - \mathbf{1}\left\{Y \leq q_{\tau}\right\}\right) \Big/ f_Y\left(q_{\tau}\right)$$

τ is a given quantile. q_{τ} is the value of the outcome variable, Y, at the τth quantile. $f_Y(q_{\tau})$ is the density of Y at q_{τ} and F_Y is the cumulative distribution function of Y. **1** is the indicator function. For example, suppose we are interested in the motherhood penalty for women at the median of the wage distribution ($\tau = .5$). The median wage in the sample is then an estimate of q_{τ}. The estimated density of the wage distribution at the median sample wage is an estimate of $f_Y(q_{\tau})$. The indicator function $\mathbf{1}\{Y \leq q_{\tau}\}$ creates a dummy variable set to 1 if a given woman's wage is below the median wage in the sample. The UQR estimate of the motherhood penalty for women at the median of the wage distribution can then be obtained by OLS regression on the transformed dependent variable.[3] Standard errors are bootstrapped to incorporate the uncertainty involved in the estimation of the RIF. Other quantiles can similarly be examined by choosing a different value of τ.

To illustrate the difference between UQR and CQR, FFL use the same example discussed in Koenker's (2005) *Quantile Regression*: Chamberlain's (1994) analysis of the association between union membership and wages. Chamberlain (1994) used CQR to examine whether union membership compresses the distribution of wages, after controlling for individuals' education, potential experience, occupation, and various other covariates. He found that, among older workers, union membership is more beneficial for individuals at lower conditional quantiles than for those at higher conditional quantiles, indicating that union membership compresses wages among individuals with similar observed wage-relevant characteristics. FFL (2009:963) write: "Finding that the effect of unions (for short) estimated using conditional quantile regressions is smaller at the 90th than at the 10th quantile simply means that unions reduce within-group dispersion, where the 'group' consists of workers who share the same values of the covariates X (other than union status)." By contrast, UQR addresses how the association between union status and wages (net of controls) varies across the wage distribution. FFL found that the association between unionization and wages rises over about the first one-third of the wage distribution, then declines, and is actually negative for workers in the top wage quintile. This is in contrast to the monotonic declines across conditional quantiles found by Chamberlain (1994). The two approaches thus answer different research questions and produce different point estimates.

Both Chamberlain (1994) and FFL (2009) include control variables, such as education, in their quantile regressions. In CQR and UQR, the coefficient on the independent variable of interest can be interpreted as the association between the independent and dependent variables, net of spurious association due to their joint association with the control variables. In both approaches, the inclusion of covariates mitigates selection concerns.

However, in CQR, the control variables have the additional effect of redefining quantiles. Under CQR, the motherhood penalty at the 75th percentile estimates the motherhood penalty for women at the 75th percentile of the wage distribution for women who are otherwise identical on all covariates. In UQR, by contrast, quantiles are defined with reference to the unconditional wage distribution. This is apparent in the fact that the RIF is defined only on the basis of Y—the outcome variable—regardless of what covariates are to be included later in the OLS model. The motherhood penalty at the 75th percentile thus indicates the association between motherhood and wages at the 75th percentile of the unconditional wage distribution, with the association adjusted for confounding effects as measured by the control variables. In a similar way, fixed effects can be thought of as a set of covariates for person-level effects and incorporated into the UQR framework.

DATA, METHOD, RESULTS

We have shown that BH's methods do not produce results that answer their primary research question: Do women at the top of the unconditional wage distribution experience a larger or smaller motherhood penalty compared to women at the bottom? Nonetheless, it is possible their conclusion—that the motherhood penalty is largest for women with the lowest hourly wages—is correct.

To provide evidence on this point, we constructed a panel of white female respondents from the 1979 to 2004 waves of the NLSY79, as in BH. We follow their sample restrictions and variable specifications as closely as possible. Following BH, we do not adjust wages for inflation, and we top- and bottom-code wages at \$200 and \$1, respectively. Also consistent with BH, we drop from the sample all women who do not have at least two years of valid wage observations. Our dataset contains 37,235 person-year observations for 3,238 women, compared with 36,361 observations reported by BH.[4]

We begin by replicating as closely as possible BH's baseline model (Model 1 of Table 2 in BH), using CQR and de-meaned data, as well as controls for age, region, population density, and year. Figure 1 reports point estimates and 95 percent confidence intervals.[5] Our estimates do not perfectly align with those of BH. BH estimated a motherhood penalty of about 13 percent at the 5th percentile, falling to 2.5 percent at the 95th percentile. Our own results estimate a maximum penalty of 20 percent at the 5th percentile, falling to 6 percent at the 90th and 95th percentiles. Nonetheless, we replicate the basic pattern found in BH's results: women with conditionally low wages—that is, relative to their own average wages, and conditional on their age and other covariates—exhibit a larger motherhood penalty. After removing person fixed effects, the penalty still largely decreases across quantiles, but the pattern is much flatter. At the 5th percentile the estimated penalty is 22 percent, but the penalty ranges from 14 to 10 percent from the 10th to 95th percentiles, none of which significantly differ (see Figure S1 in the online supplement [http://asr.sagepub.com/supplemental]).

As noted earlier, this model does not provide evidence as to whether high- or low-wage women experience the largest motherhood penalty. To address BH's question, we re-estimate the same model, but use UQR. In this approach, inclusion of control variables such as age, region, and individual fixed effects addresses concerns that motherhood is selective on observed time-varying traits associated with wages and unobserved individual-specific time-invariant traits, but, unlike CQR, does not redefine what it means to be a high-wage worker.

Figure 2 shows results from the UQR model. The motherhood penalty increases from the 5th percentile to the median of the unconditional wage distribution, then decreases. At the 5th and 10th percentiles, the penalty is 8 percent, rising to 13 percent at the median, and then falling to 0 percent at the 95th percentile, in sharp contrast to results reported in BH.

As shown, the motherhood penalty does not decrease monotonically from the bottom to the top of the unconditional wage distribution. Rather, this pattern of results indicates that women with high wages and low wages tend to pay smaller motherhood penalties than do women close to the median of the wage distribution.

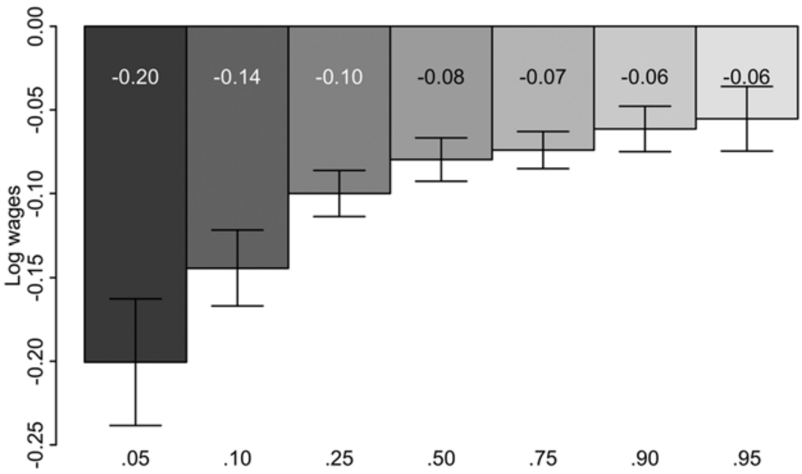

Figure 1. Motherhood Penalty by Conditional Quantile, Adjusting for Year, Age, Region of Country, Population Density, and Person Fixed Effects
Note: All figures report change in log wages, with crosshairs showing 95% bootstrap confidence intervals.

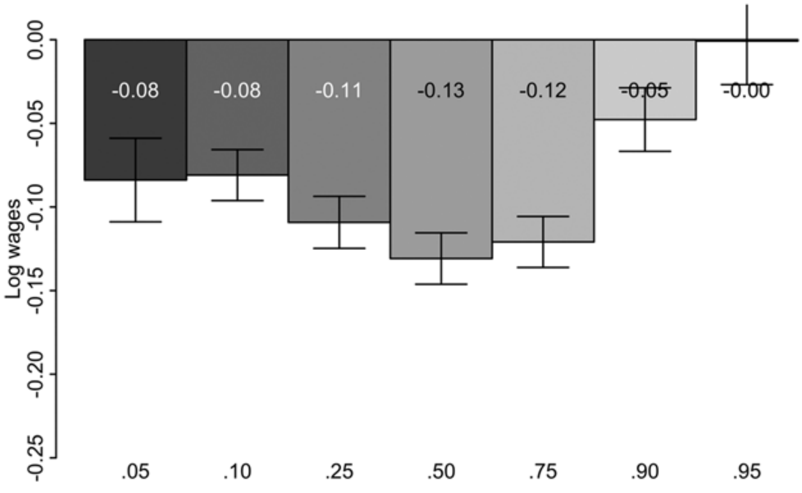

Figure 2. Motherhood Penalty by Unconditional Quantile, Adjusting for Year, Age, Region of Country, Population Density, and Person Fixed Effects
Note: All figures report change in log wages, with crosshairs showing 95% bootstrap confidence intervals.

CONCLUSIONS

Budig and Hodges (2010) engage an important substantive question: Which mothers bear the largest wage penalties for parenthood? However, as we documented, their analysis examines only within-individual variation in wages, rather than variation in wages across the unconditional wage distribution, and therefore cannot answer their question. Furthermore, even if they had not de-meaned their data at the individual level, inclusion of covariates necessarily changes the interpretation of quantiles in the CQR framework. Therefore, CQR with multiple covariates, even in cross-sectional data, cannot answer questions about whether associations are larger at the top or bottom of the unconditional outcome distribution.

We described UQR, a powerful new approach useful for answering the type of question raised by BH. This approach allows researchers to adjust for selection without redefining the quantiles of the outcome variable. Empirically, we showed that, using the same data and control variables, UQR produces a pattern of variation in the motherhood penalty inconsistent with BH's results using CQR. The motherhood penalty does not uniformly diminish at higher wage levels, but instead is larger for women at the middle of the wage distribution than for women above or below the median.

Quantile regression is a promising analytic tool for sociologists, particularly given sociologists' long-standing interest in inequality. Whether CQR or UQR is more appropriate will depend on the research question. As discussed earlier, FFL show that both CQR and UQR can be meaningfully used to assess the association between unions and wages: CQR examines the effect of unions on within-group wage dispersion, and UQR investigates whether union membership has greater benefits for low- or high-wage workers. Of course, both CQR and UQR could be applied to other substantive topics. For example, CQR could be used to test whether individuals with a high tolerance for risk have a wider dispersion of wages, net of other traits, like education and age. The shared feature of questions suited to CQR is that they are concerned with dispersion of the outcome variable within groups, as defined by other covariates in the model. Questions best answered by UQR, on the other hand, are chiefly concerned with the possibility that covariates will matter differently for individuals at different positions of the outcome distribution. For example, UQR might be used to test whether race or gender gaps in wages are larger for low- or high-wage workers.

BH made a valuable contribution by introducing quantile regression to *ASR,* and we hope this comment will provide a useful guide for sociologists interested in employing this technique in future work.

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Notes

1. We queried *ASR* articles for the terms "quantile regression" and "quantile" using JSTOR Arts and Sciences 1 and SAGE Complete, which together have full coverage of *ASR* issues since 1936. Other than BH, the term "quantile regression" was found in a single article, and only in the references. We read six additional articles that included the term "quantile," none of which included quantile regression.
2. This interpretation relies on the assumption of rank preservation (see Djebbari and Smith 2008). We assume rank preservation throughout our discussion.
3. FFL also discuss alternative estimation procedures.
4. We reviewed BH's description of their analysis and tried a number of alternative specifications, but we were unable to replicate perfectly their results. For all the models shown here, we performed analyses that varied whether inflation was adjusted for, whether listwise deletion was used only for covariates replicated in this comment or for all the variables used at least once in BH, whether wages were top- and bottom-coded, whether observations from women under the age of 18 or enrolled in school were excluded, whether individuals observed only once after listwise deletion were included or excluded, and all combinations of these variations. We emphasize the robustness of our pattern of empirical results across these 32 specifications, although the estimate at the 5th percentile is somewhat sensitive to sample specification in the UQR models. We further note that our fundamental point regards the correct interpretation of quantile regression coefficients. This theoretical concern remains regardless of the empirical results.
5. Standard errors were calculated by performing 200 (following, e.g., FFL) bootstraps of the design matrix, clustered around each respondent.

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Alexandra Killewald is Assistant Professor of Sociology at Harvard University. Her research takes a quantitative, demographic approach to the study of social stratification, with a focus on the relationships among gender, family, and economic inequality. She is also interested in quantitative methodology and research design. Current and previous projects explore how employment and family influence one another, in particular the influence of women's financial resources on their family and household behavior.

Jonathan Bearak is a PhD candidate in Sociology at New York University. His research analyzes inequality in earnings and education, with a focus on gender, family, and relationship formation. He is also interested in quantitative methods and statistical software. Current and past projects explore the relationship between men's economic standing and marriage prospects, and the effects of education on non-academic outcomes including sexual risk behaviors and adolescent births.