

A Generalized Algorithm for producing Integer Power Reduction Formulas of Cosine

March 29, 2018

The idea behind this algorithm was inspired by a video entitled *cos(1) + ... + cos(n)* by Peyam R. Tabrizian
<https://www.youtube.com/watch?v=7LBQTpiK-Xg>

Prior to seeing this video, I understood that complex numbers stemmed from the idea that $\sqrt{-1} = i$, and that complex numbers had some properties and could be dealt with in particular ways, but I did not believe they had any utility.

After watching the video however, I became a believer. So I present a way a way I've discovered, for generating the integer power reduction formulas for cosine which hinges on the imaginary representation of cosine.

This analysis begins with (1)Euler's Formula: $e^{i\theta} = \cos(\theta) + i \sin(\theta)$
Plugging in $-\theta$, we see that $e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos(\theta) - i \sin(\theta)$

We can then add these two equations to get the following identity(2):
$$e^{i\theta} + e^{-i\theta} = 2 \cos(\theta) \rightarrow \cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

This will be very important, because it means that whenever we can turn,
for instance, $e^{3i} + e^{-3i} \rightarrow 2 \cos(3x)$