# Limiting behavior of spatial population process

## Spatially uniform limit

Observation of numerous simulation runs gave rise to a hypothesis that under certain circumstances spatial population process degenerates into a spatially uniform process birth-death process with $C(r,t),\ T(r_1,r_2,t),\ Q(r_1,r_2,r_3,t),\ ...$ being constant at any r and t. At equilibrium, it works as a sum of uniform Poisson birth process and uniform thinning process, which could be approximated by a discrete birth-death process.

### Discrete birth death process

Discrete birth-death process is characterised by Poisson birth rate, Poisson death rate, competitive death rate and starting population.

$B(t) = b \bullet N(t)$ , $D(t) = d \bullet N(t) + d \bullet N(t) \ \bullet (N(t) - 1)$

$\Lambda(t) = B(t) + D(t)$

$\tau(t) \sim Poisson(\Lambda(t))$

$P(Event(t) = Birth) = \frac{B}{B+D}$ , $P(Event(t_0) = Death) = \frac{D}{B+D}$

$b\ - poisson\ birth\ rate,\ d - poisson\ death\ rate,\ b, d > 0$

$d'\ - competitive\ death\ rate,\ d' > 0$

$N(t)\ - population\ at\ time\ t,\ N(t) \in \mathbb{N} \cup \{0\}$

$B(t)\ - population\ birth\ rate\ at\ time\ t$

$D(t)\ - population\ death\ rate\ at\ time\ t$

$\Lambda(t)\ - population\ event\ rate\ at\ time\ t$

$\tau(t)\ - interoccurence\ time\ at\ time\ t$

### Logistic equation

logistic equation is a common model of population growth, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal.

$$P(t) = \frac{KP_0 e^{rt}}{K + P_0(e^{rt} - 1)}$$

$P(t)\ -\ represent\ population\ size$

$r = b - d\ -\ defines\ the\ growth\ rate$

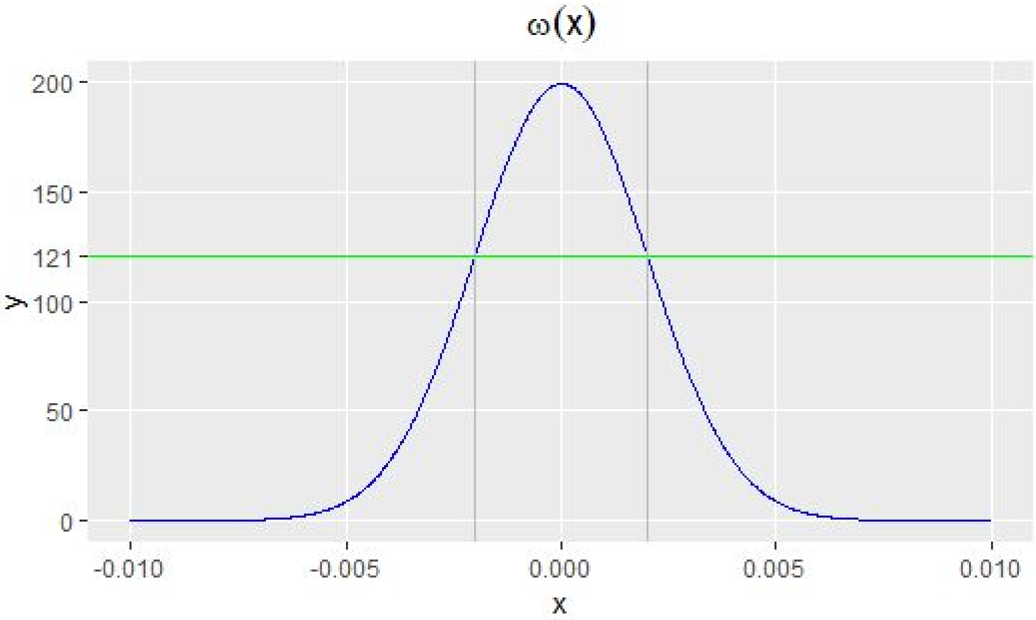
$K = (b - d) / d'\ -\ is\ the\ carrying\ capacity$

## Non-stationary second moment case in two species model

Given local nature of birth in the model, it could be that given two populations with exact same inter- and intrapopulation interaction terms to create clusters of increasing size purely by random walk model. For any finite area of carrying capacity K, sum of specimens of two populations remains near K, while any single population experience random walk with single consuming state N=0, therefore leading to eventual extinction of one species with 50% chance.

## Flattening of death response

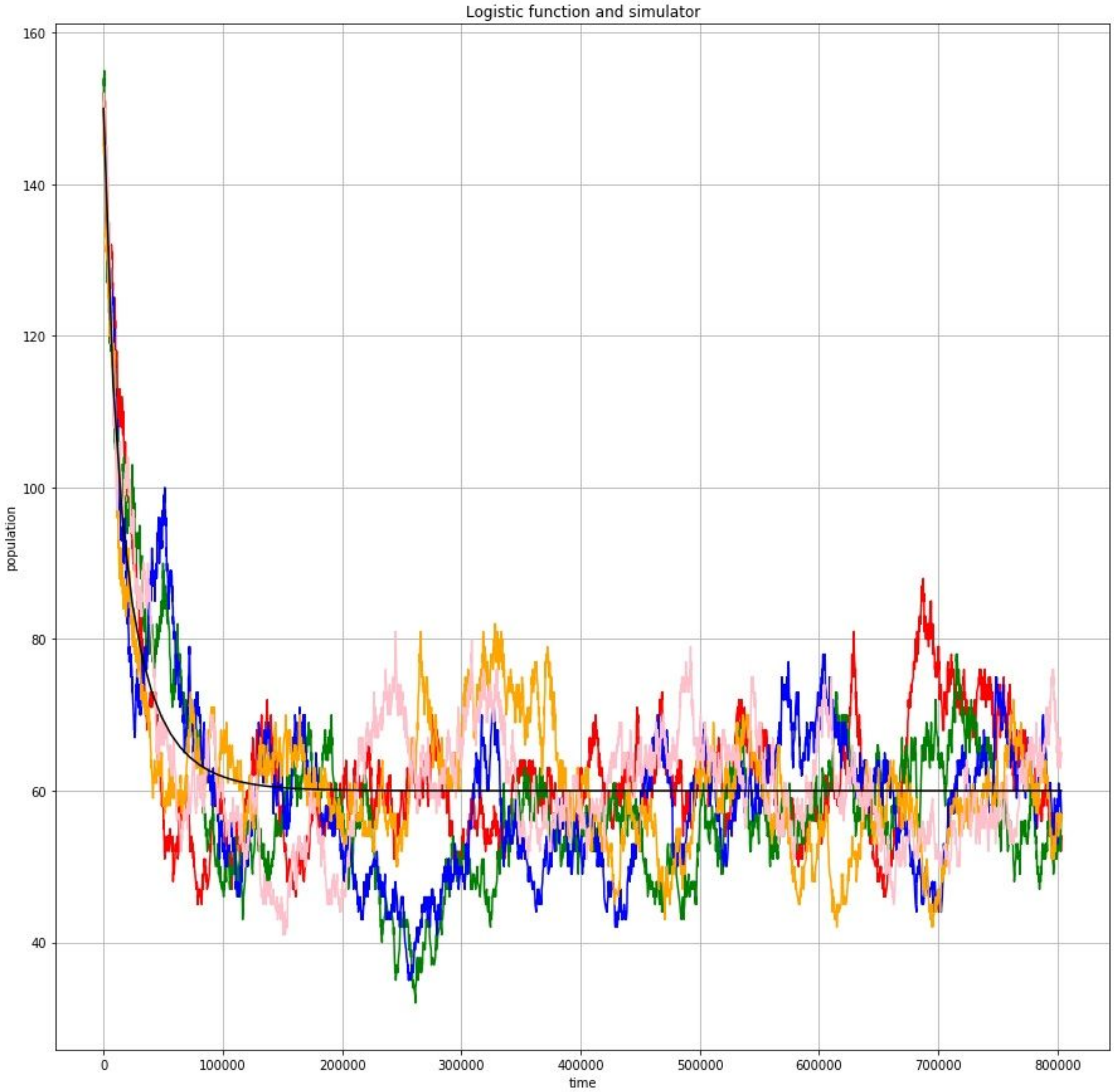
Numerous simulation runs showed that with current parameter picking scheme, where death kernel is normalized to 1, could lead in conjunction with small death dispersion to extreme nearby death interaction values.
Example:
$b = 0.6, d = 0.2, d' = 0.1$
$\omega_b \sim Normal(0, 0.001),\ \omega_d \sim Normal(0, 0.002)$




Effective competitive death rate at distance 0.002 equals 121, killing any neighbours almost instantly. Thus after some $d'$ which guarantees death to neighbours further increase would not change the behavior of simulation. Under some assumptions, this could be approximated by a random walk model with jumps and neighbour consumption.

# Simulator

(1)
probability of birth: 0.00004
probability of death: 0.00001
probability of murder: 0.0000005
population: 150
end time: 803212.8514056226



(2)
probability of birth: 0.00004
probability of death: 0.00002
probability of murder: 0.0000009
population: 200
end time: 418235.04809703055