Task #3

A. The angle θ_1, which corresponds to the shortest time t

1. The dependence of time t on angle θ_1 is given by the formula

$$t(\theta_1) = \frac{1}{v_{sand}}(l_1 + nl_2), \tag{1}$$

where

$$l_1 = \sqrt{x^2 + d_1^2}, \tag{2}$$

$$l_2 = \sqrt{(h-x)^2 + d_2^2}, \tag{3}$$

$$x = d_1\, tg(\theta_1). \tag{4}$$

The lengths d_1, d_2, velocity v_{sand} and coefficient n are input parameters.

2. The dependence from Eq.(1) is depicted on Fig.1 for the set of input parameters from Table I. It is clearly seen from Fig.1 that the graph of this dependence has a minimum value, which equals to

$$t = 23.1\, sec, \tag{5}$$

which corresponds to the angle

$$\theta_1 = 80.6^\circ. \tag{6}$$

B. The analytical solution

1. The angle θ_1, which corresponds to a shortest time t can be also found as a solution of the extremal condition for the dependence $t = t(\theta_1)$, i.e. as a solution of the equation

$$\frac{\partial t(\theta_1)}{\partial \theta_1} = 0. \tag{7}$$

2. First of all we calculate the lhs of Eq.(7). To this end we have from Eq.(1) that

$$\frac{\partial t}{\partial \theta_1} = \frac{1}{v_{sand}} \frac{(\partial l_1 + nl_2)}{\partial x} \frac{\partial x}{\partial \theta_1}, \tag{8}$$

$$\frac{\partial t}{\partial \theta_1} = \frac{1}{v_{sand}} (\frac{\partial l_1}{\partial x} + n\frac{\partial l_2}{\partial x})\frac{\partial x}{\partial \theta_1}. \tag{9}$$

3. Substitution Eqs.(2), (3), (4) into Eq.(9) gives us

$$\frac{\partial t}{\partial \theta_1} = \frac{1}{v_{sand}} (\frac{x}{l_1} + n\frac{(x-h)}{l_2})\frac{d_1}{(\cos\theta_1)^2}, \tag{10}$$

where the dependence of the lengths x, l_1 and l_2 on angle θ_1 is given in Eqs.(2),(3),(4).

4. It looks that to solve analytically the equation in Eqs.(7) and (10) is very demanding task since it includes a complicated algebraic and trigonometric function of angle θ_1. Instead of doing this we are solving the equation in Eqs.(7) and (10) numerically. To this end we plot the dependence of $\frac{\partial t}{\partial \theta_1}$ on angle θ_1 from Eq.(10) on Fig.2.

5. From Fig.2 we found that the condition $\frac{\partial t}{\partial \theta_1} = 0$ in Eq.(7) is executed when the angle θ_1 equals to $80.6°$, i.e. to the same value, which was previously found in Eq.(6). Substitution of $\theta_1 = 80.6°$ into Eq.(1) gives us again the value of time $t = 23.1\,sec$ in Eq.(5). The values of lengths x, l_1 and l_2, corresponding to $\theta_1 = 80.6°$ are presented in Table II.

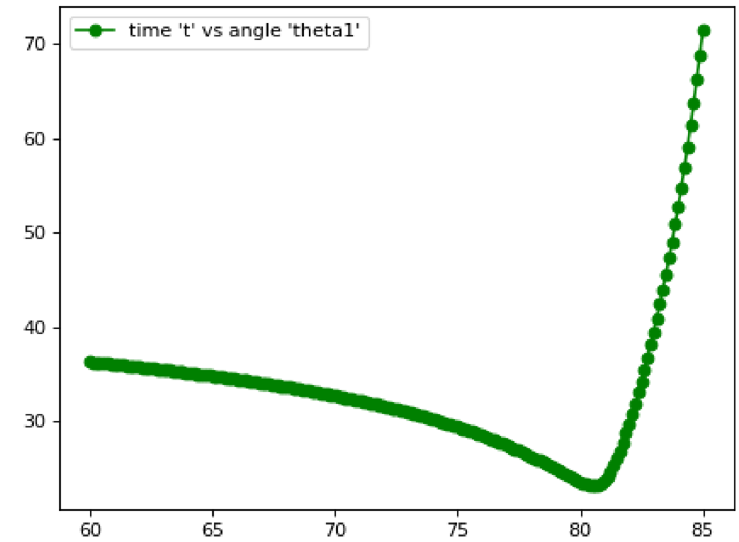

FIG. 1: The dependence of time t(sec) on angle $\theta_1(\angle^\circ)$. This Figure is a result of the execution of the Python code Practice03-1.py

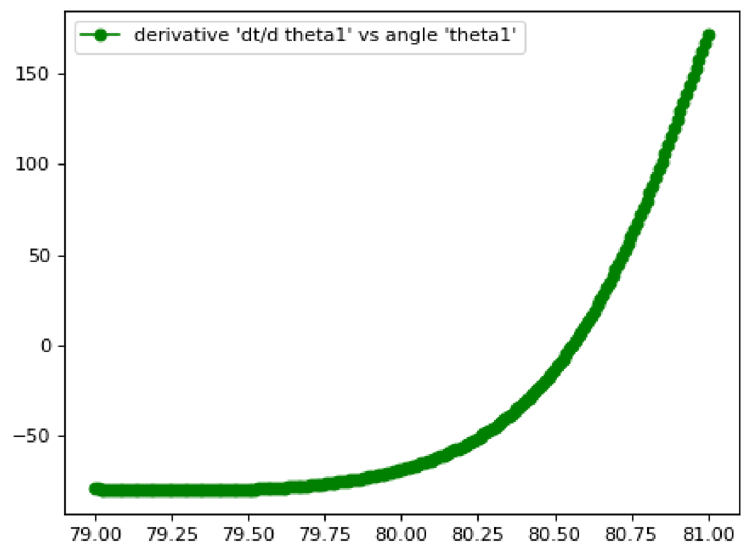

FIG. 2: The dependence of the derivative $\frac{\partial t}{\partial \theta_1}(\frac{sec}{\angle^\circ})$ on angle $\theta_1(\angle^\circ)$. This Figure is a result of the execution of the Python code Practice03-2.py

TABLE I: The set of input parameters from Task #1.

parameter		units	value
d_1	distance	yard	8
d_2	distance	foot	10
h	distance	yard	50
v_{sand}	velocity	$\frac{mile}{hour}$	5
n	coefficient		2

TABLE II: The set of output parameters, corresponding to the input ones from Table I as well as to the angle $\theta_1 = 80.6°$.

parameter		units	value
t	time	sec	23.1
x	distance	yard	144.2
l_1	distance	yard	146.2
l_2	distance	yard	11.6