Discussion of "Multivariate generalized hyperbolic laws for modeling financial log-returns – empirical and theoretical considerations"

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The central idea of the paper is to use the multivariate generalized hyperbolic (MGH) distribution for modeling log-returns of assets Eberlein [2001]. The MGH distribution is presented as an alternative to widely used modeling approaches, based on the Lévy processes, that have been studied in the literature. The authors here are to be congratulated on a stimulating paper, which showed an alternative way to tackling a problem which is more than a century-old. The main applications of such a statistical model include the problems of asset pricing, or debt-and-credit risk assessment Merton [1974], Korteweg and Polson [2008]. In this note, I will discuss the history of the asset pricing models, starting with the work of Bachelier in 1900 and ending with machine learning models of today. I will add some little known historical facts about the researchers that have made major contributions in this field.

Historically, the most widely used models relied on the assumption that asset returns follow a normal or a lognormal distribution. The lognormal model for the asset returns was challenged after the October 1987 crash of the American stock market. On October 19 (black Monday) the Dow Jones index had fallen 508 points, or 23 percent. It was the worst single day in history for the US markets. The reason for the crash was rather simple, it was caused by the portfolio insurance product created by one of the financial firms. The idea of this insurance was to switch from equities to the US Treasury bills, as markets go down. Although the lognormal model does a good job at describing the historical data, the jump observed on that day had a probability close to zero, according to the lognormal model. The lognormal model underestimates the probability of a large change (thin tail). The widely used then Black-Sholes model for asset pricing was relying on the lognormal model, it was incapable of correctly pricing in the possibility of such a large drop. The MGH provides a much better fit to the historical data and might address this issue.

The normal assumption of the asset returns was first proposed in 1900 in the PhD thesis of Louis Bachelier, who was a student of Henri Poincaré. Bachelier was interested in developing statistical tools for pricing options (predicting asset returns) on the Paris stock exchange. Although Bachelier's work laid the foundation for the modern theory of stochastic processes, he was never given credit by his contemporaries, including Einstein,

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Levy and Borel. In 1905 Einstein published a paper which used the same statistical model as Bachelier to describe the 1827 discovery by a botanist Robert Brown, who observed that pollen particles suspended in water followed irregular random trajectories. Thus, we call the stochastic process that describes these phenomena a Brownian motion. Einstein's advisor at the University of Zurich was Hermann Minkowski who was a friend and a collaborator of Poincaré. Thus, it is likely Einstein knew about the work of Bachelier, but he never mentioned it in his paper. This was not the first instance of when Einstein did not give proper credit. Poincaré published a paper Poincaré [1898] on the relativity theory in 1898, seven years before Einstein. This paper was published in a philosophy journal and thus Poincaré had avoided using any mathematical formulas except for the famous $E = mc^2$. Poincaré did discussed his results on the relativity theory with Minkowski. Minkowski asked Einstein to read Poincaré's work Arnol'd [2006]. However, Einstein never referenced the work of Poincaré until 1945. One of the reviewers for the 1905 paper on relativity by Einstein was Poincaré and he wrote a very positive review mentioning it as a breakthrough. When Minkowski asked Poincaré why he did not claim his priority on the theory, Poincaré replied that our mission is to support young scientists. More about why credit is mistakenly given to Einstein for the relativity theory is discussed by Logunov Logunov [2004].

Einstein was not the only one who ignored the work of Bachelier, Paul Lévy did so as well. Paul Lévy was considered a pioneer and authority on stochastic processes during Bachelier's time, although Bruno de Finetti introduced a dual concept of infinite divisibility in 1929, before the works of Lévy in early 1930s on this topic. Lévy never mentioned the work of the obscure and little known mathematician Bachelier. The first to give credit to Bachelier was Kolmogorov in his 1931 paper Kolmogoroff [1931] (Russian translation Kolmogorov [1938] and English translation Shiryayev [1992]). Later Leonard Jimmie Savage translated Bachelier's work to English and showed it to Paul Samuelson. Samuelson extended the work of Bachelier by considering the log-returns rather than absolute numbers, popularized the work of Bachelier among economists and the translation of Bachelier's thesis was finally published in English in 1964 Cootner [1967]. Many economists who extended the work of Bachelier won Nobel prizes, including Eugine Fama known for work on the efficient markets hypothesis, Paul Samuelson, and Myron Scholes for the Black-Sholes model, as well as Robert Merton. Robert Merton, who was a student of Samuelson, who proposed a major extension to the work of Bachelier, by introducing jumps to the model. The additive jump term addresses the same issues as authors mentioned in the paper, specifically, leptokurticity, asymmetry, and heavy tails. Merton's Jump Stochastic volatility model has a discrete-time version for log-returns, y_t, with jump times, J_t, jump sizes, Z_t, and spot stochastic volatility, V_t, given by the dynamics

$$\begin{aligned} y_t &\equiv \log\left(S_t/S_{t-1}\right) = \mu + V_t\varepsilon_t + J_t Z_t \\ V_{t+1} &= \alpha_v + \beta_v V_t + \sigma_v \sqrt{V_t}\varepsilon_t^v \end{aligned}$$

where $\mathbb{P}\left(J_t = 1\right) = \lambda$, S_t denote a stock or asset price and log-returns y^t is the log-return. The errors $(\varepsilon_t, \varepsilon_t^v)$ are possibly correlated bivariate normals. The investor must obtain optimal filters for (V_t, J_t, Z_t), and learn the posterior densities of the parameters $(\mu, \alpha_v, \beta_v, \sigma_v^2, \lambda)$. These estimates will be conditional on the information available at each time.

From the computational perspective, performance of MCMC algorithms proposed by the authors and in other works Johannes and Polson [2010], Jacquier et al. [2004], Eraker et al. [2003], Li et al. [2008], Fulop et al. [2012] have one major caveat: MCMC algorithms are computationally demanding, and are not feasible for sequential analysis. Essentially, the MCMC algorithm needs to be re-applied to the entire data set, when a new observation becomes available.

An alternative approach to an MCMC algorithm for analyzing sequential data is to use particle filtering. Particle filters were shown to be effective for the analysis of financial time series data. For example, Warty et al. [2018] applied particle filter to the problem of inference of stochastic volatility with variance-gamma (SVVG) jumps in returns and Jacquier et al. [2016] developed particle filters to analyze spot volatility and jump times, together with sequentially updating (learning) of jump and volatility parameters.

Finally, I would like to mention an alternative approach to asset pricing that relies on machine learning algorithms Heaton et al. [2017], Sokolov [2017], Dixon [2018], Dixon et al. [2019]. One of the major advantages of machine learning algorithms is the ability to analyze high-dimensional data sets and to extract highly non-linear patterns. The high dimensional analysis allows to analyze many assets at different time scales using a single model that can be estimated using a data set and does not require constructing economic models of price movements.

References

Vladimir I Arnol'd. Forgotten and neglected theories of Poincaré. *Russian Mathematical Surveys*, 61(1):1, 2006.

Paul H Cootner. *The random character of stock market prices*. MIT press, 1967.

Matthew Dixon. Sequence classification of the limit order book using recurrent neural networks. *Journal of computational science*, 24:277–286, 2018.

Matthew Dixon, Nicholas Polson, and Vadim Sokolov. Deep learning for spatio-temporal modeling: Dynamic traffic flows and high frequency trading. *Applied Stochastic Models in Business and Industry*, 35(3):788–807, 2019.

Ernst Eberlein. Application of generalized hyperbolic Lévy motions to finance. In *Lévy processes*, pages 319–336. Springer, 2001.

Bjørn Eraker, Michael Johannes, and Nicholas Polson. The impact of jumps in volatility and returns. *The Journal of Finance*, 58(3):1269–1300, 2003.

Andras Fulop, Junye Li, and Jun Yu. Bayesian learning of impacts of self-exciting jumps in returns and volatility. *Working Paper*, 2012.

James B Heaton, Nick G Polson, and Jan Hendrik Witte. Deep learning for finance: deep portfolios. *Applied Stochastic Models in Business and Industry*, 33(1):3–12, 2017.

Eric Jacquier, Nicholas Polson, and Peter Rossi. Bayesian Inference for SV models with Correlated Errors. *Journal of Econometrics*, 2004.

Eric Jacquier, Nicholas Polson, and Vadim Sokolov. Sequential Bayesian learning for Merton's jump model with stochastic volatility. *arXiv preprint arXiv:1610.09750*, 2016.

Michael Johannes and Nicholas Polson. MCMC Methods for Continuous-Time Financial Econometrics. In Lars Peter Hansen and Yacine Aït-Sahalia, editors, *Handbook of Financial Econometrics: Applications*, volume 2 of *Handbooks in Finance*, pages 1–72. Elsevier, San Diego, 2010.

Andrei Kolmogoroff. Über die analytischen methoden in der wahrscheinlichkeitsrechnung. *Mathematische Annalen*, 104(1):415–458, 1931.

A. N. Kolmogorov. On the analytic methods of probability theory. *Uspekhi Mat. Nauk*, (5): 5–41, 1938.

Arthur Korteweg and Nicholas Polson. Volatility, Liquidity, Credit Spreads and Bankruptcy Prediction. Technical report, Stanford University, Working Paper, 2008.

Haitao Li, Martin T Wells, and L Yu Cindy. A Bayesian analysis of return dynamics with Lévy jumps. *Review of Financial Studies*, 21(5):2345–2378, 2008.

A. A. Logunov. Henri poincare and relativity theory, 2004.

Robert C Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of finance*, 29(2):449–470, 1974.

Henri Poincaré. La mesure du temps. *Revue de métaphysique et de morale*, 6(1):1–13, 1898.

A. N. Shiryayev. *On Analytical Methods In Probability Theory*, pages 62–108. Springer Netherlands, Dordrecht, 1992.

Vadim Sokolov. Discussion of Deep learning for finance: deep portfolios. *Applied Stochastic Models in Business and Industry*, 33(1):16–18, 2017.

Samir P Warty, Hedibert F Lopes, and Nicholas G Polson. Sequential Bayesian learning for stochastic volatility with variance-gamma jumps in returns. *Applied Stochastic Models in Business and Industry*, 34(4):460–479, 2018.