Bayes AI

Unit 4: Bayesian Hypothesis Tests

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Lindley's Paradox

Often evidence which, for a Bayesian statistician, strikingly supports the null leads to rejection by standard classical procedures.

- Do Bayes and Classical always agree?

Bayes computes the probability of the null being true given the data $p(H_0|D)$. That's not the p-value. Why?

- Surely they agree asymptotically?
- How do we model the prior and compute likelihood ratios $L(H_0|D)$ in the Bayesianwork?

Bayes t-ratio

Edwards, Lindman and Savage (1963)

Simple approximation for the likelihood ratio.

$$L(p_0) \approx \sqrt{2\pi}\sqrt{n}\exp\left(-\frac{1}{2}t^2\right)$$

- Key: Bayes test will have the factor $\sqrt{n}$

This will asymptotically favour the null.

- There is only a big problem when $2 < t < 4$ – but this is typically the most interesting case!

Coin Tossing

Intuition: Imagine a coin tossing experiment and you want to determine whether the coin is “fair” $H_0 : p = \frac{1}{2}$.

There are four experiments.

	Expt 1	2	3	4
n 50	100	400	10,	000
r 32	60	220	50	98
$L(p_0)$	0.81	1.09	2.17	11.68

Coin Tossing

Implications:

- Classical: In each case the t-ratio is approx 2. They we just H_0 (a fair coin) at the 5% level in each experiment.
- Bayes: $L(p_0)$ grows to infinity and so they is overwhelming evidence for H_0. Connelly shows that the Monday effect disappears when you compute the Bayes version.