Q1. Difference between t-test and z-test

Feature	z-test	t-test
Population SD	Known	Unknown
Sample size	Large ($n \geq 30$)	Small ($n < 30$)
Distribution	Normal (Z)	Student’s t

Examples

- **z-test:** Testing average height when population SD is known.
- **t-test:** Testing average marks with small sample and unknown SD.

Q2. One-tailed vs Two-tailed tests

Aspect	One-tailed	Two-tailed
Direction	One direction	Both directions
Hypothesis	$\mu > \mu_0$ or $\mu < \mu_0$	$\mu \neq \mu_0$
Usage	Specific direction	Any difference

Example:

- One-tailed: “New drug increases weight”
- Two-tailed: “New drug changes weight”

Q3. Type I and Type II Errors

- **Type I Error (α):** Rejecting a true null hypothesis

- **Type II Error (β):** Failing to reject a false null hypothesis

Example

Medical test:

- Type I: Diagnosing disease when patient is healthy
- Type II: Missing disease when patient is sick

Q4. Bayes' Theorem

Formula

P(A∣B)=P(B∣A)P(A)P(B)P(A|B) = \frac{P(B|A)P(A)}{P(B)}P(A∣B)=P(B)P(B∣A)P(A)

Example

- Disease prevalence = 1%
- Test accuracy = 95%

Probability person has disease given positive test:

P(D∣+)=0.95×0.010.95×0.01+0.05×0.99=0.16P(D|+) = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.05 \times 0.99} = 0.16P(D∣+)=0.95×0.01+0.05×0.990.95×0.01=0.16

Q5. Confidence Interval

A **confidence interval (CI)** is a range of values likely to contain the population parameter.

Formula (mean)

CI=xˉ±zσnCI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}}CI=xˉ±znσ

Example

Mean = 100, SD = 10, n = 25

$$CI = 100 \pm 1.96(10/5) = (96.08,\ 103.92)$$

Q6. Bayes' Theorem – Sample Problem

Problem

- P(Fraud) = 0.02
- P(Alert | Fraud) = 0.9
- P(Alert | No Fraud) = 0.1

Solution

$$P(Fraud|Alert) = \frac{0.9 \times 0.02}{0.9 \times 0.02 + 0.1 \times 0.98} = 0.155$$

Q7. 95% Confidence Interval

Given:
Mean = 50, SD = 5, n = 25

$$CI = 50 \pm 1.96 \times \frac{5}{5} = 50 \pm 1.96$$

✅ Answer

$$(48.04,\ 51.96)$$

Q8. Margin of Error

Definition

$$\text{Margin of Error} = z \times \frac{\sigma}{\sqrt{n}}$$

Effect of sample size

- Larger n → smaller margin of error

Example

Election polling:
1000 voters → more accurate than 100 voters

Q9. z-score calculation

z=75−705=1z = \frac{75 - 70}{5} = 1z=575−70=1

Interpretation

The value is **1 standard deviation above the mean**.

Q10. Weight loss drug (t-test)

Given:
x̄=6\bar{x} = 6x̄=6, s=2.5s = 2.5s=2.5, n=50n = 50n=50
H_0: $\mu = 0$
H_1: $\mu > 0$

t=6−02.5/50=16.97t = \frac{6 - 0}{2.5/\sqrt{50}} = 16.97t=2.5/506−0=16.97

Critical t ($\alpha = 0.05$) ≈ 1.68
👉 **Reject H_0**

✅ Conclusion

Drug is **significantly effective**.

Q11. CI for population proportion

Given:
$\hat{p} = 0.65$, n = 500

CI=0.65±1.960.65(0.35)500CI = 0.65 \pm 1.96\sqrt{\frac{0.65(0.35)}{500}}CI=0.65±1.965000.65(0.35) CI=0.65±0.042CI = 0.65 \pm 0.042CI=0.65±0.042

✅ Answer

$(0.608,\ 0.692)$

Q12. Teaching methods t-test

$$t = \frac{85 - 82}{\sqrt{\frac{6^2}{n} + \frac{5^2}{n}}} \approx 2.58$$

Critical t ($\alpha = 0.01$) ≈ 2.58
👉 **Reject H_0**

✅ Conclusion

Teaching methods differ significantly.

Q13. 90% Confidence Interval

Given:
Mean = 65, SD = 8, n = 50
$z_{0.90} = 1.645$

$$CI = 65 \pm 1.645\frac{8}{\sqrt{50}} = 65 \pm 1.86$$

✅ Answer

$(63.14,\ 66.86)$

Q14. Caffeine effect (t-test)

Given:
$\bar{x} = 0.25$, $s = 0.05$, $n = 30$
H_0: $\mu = 0.30$

$$t = \frac{0.25 - 0.30}{0.05/\sqrt{30}} = -5.48$$

Critical t ($\alpha = 0.10$) ≈ ±1.70
👉 **Reject H_0**

✅ Conclusion

Caffeine has a **significant effect** on reaction time.
