

$$\begin{aligned}
\dot{\mathbf{P}}^S \cdot \mathbf{P}^S &= \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cdot \mathbf{E} \cos \vartheta + (\dot{\mathbf{k}}\mathbf{k} + \dot{\mathbf{k}}\mathbf{k})(1 - \cos \vartheta) \cdot \mathbf{E} \cos \vartheta + \\
&\quad + (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cdot \dot{\mathbf{k}}\mathbf{k}(1 - \cos \vartheta) + (\dot{\mathbf{k}}\mathbf{k} + \dot{\mathbf{k}}\mathbf{k})(1 - \cos \vartheta) \cdot \dot{\mathbf{k}}\mathbf{k}(1 - \cos \vartheta) = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + (\dot{\mathbf{k}}\mathbf{k} + \dot{\mathbf{k}}\mathbf{k}) \cos \vartheta (1 - \cos \vartheta) + (\dot{\mathbf{k}}\mathbf{k} \cdot \dot{\mathbf{k}}\mathbf{k} + \dot{\mathbf{k}}\mathbf{k} \cdot \mathbf{k}\mathbf{k})(1 - \cos \vartheta)^2 = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}}\mathbf{k} \cos \vartheta (1 - \cos \vartheta) + \\
&\quad + \dot{\mathbf{k}}\mathbf{k} \cos \vartheta - \dot{\mathbf{k}}\mathbf{k} \cos^2 \vartheta + \dot{\mathbf{k}}\mathbf{k} - 2 \dot{\mathbf{k}}\mathbf{k} \cos \vartheta + \dot{\mathbf{k}}\mathbf{k} \cos^2 \vartheta = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}}\mathbf{k} \cos \vartheta - \dot{\mathbf{k}}\mathbf{k} \cos^2 \vartheta + \dot{\mathbf{k}}\mathbf{k} (1 - \cos \vartheta),
\end{aligned}$$

$$\begin{aligned} \dot{\mathbf{P}}^A \cdot \mathbf{P}^S &= \\ &= (\mathbf{k} \times \mathbf{E}) \cdot \mathbf{E} \dot{\vartheta} \cos^2 \vartheta + (\dot{\mathbf{k}} \times \mathbf{E}) \cdot \mathbf{E} \sin \vartheta \cos \vartheta + \\ &\quad + \overline{(\mathbf{k} \times \mathbf{E}) \cdot \mathbf{k} \mathbf{k}} \dot{\vartheta} \cos \vartheta (1 - \cos \vartheta) + (\dot{\mathbf{k}} \times \mathbf{E}) \cdot \mathbf{k} \mathbf{k} \sin \vartheta (1 - \cos \vartheta) = \\ &= \mathbf{k} \times \mathbf{E} \dot{\vartheta} \cos^2 \vartheta + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}} \times \mathbf{k} \mathbf{k} \sin \vartheta (1 - \cos \vartheta), \end{aligned}$$

$$\begin{aligned} \dot{\mathbf{P}}^S \cdot \mathbf{P}^A &= \\ &= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cdot (\mathbf{k} \times \mathbf{E}) \sin \vartheta + (\mathbf{k}\dot{\mathbf{k}} + \dot{\mathbf{k}}\mathbf{k})(1 - \cos \vartheta) \cdot (\mathbf{k} \times \mathbf{E}) \sin \vartheta = \\ &= \cancel{\mathbf{k}\mathbf{k} \cdot (\mathbf{k} \times \mathbf{E})} \dot{\vartheta} \sin^2 \vartheta - \mathbf{E} \cdot (\mathbf{k} \times \mathbf{E}) \dot{\vartheta} \sin^2 \vartheta + \left( \mathbf{k}\dot{\mathbf{k}} \cdot (\mathbf{k} \times \mathbf{E}) + \dot{\mathbf{k}}\mathbf{k} \cdot (\mathbf{k} \times \mathbf{E}) \right) \sin \vartheta (1 - \cos \vartheta) = \\ &= -\mathbf{k} \times \mathbf{E} \dot{\vartheta} \sin^2 \vartheta + \mathbf{k}\dot{\mathbf{k}} \times \mathbf{k} \sin \vartheta (1 - \cos \vartheta), \end{aligned}$$

$$\begin{aligned}\dot{\mathbf{P}}^A \cdot \mathbf{P}^A &= (\mathbf{k} \times \mathbf{E}) \dot{\vartheta} \cos \vartheta \cdot (\mathbf{k} \times \mathbf{E}) \sin \vartheta + (\dot{\mathbf{k}} \times \mathbf{E}) \cdot (\mathbf{k} \times \mathbf{E}) \sin^2 \vartheta = \\ &= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \mathbf{k}\dot{\mathbf{k}} \sin^2 \vartheta;\end{aligned}$$

$$\begin{aligned}
\dot{\mathbf{P}} \cdot \mathbf{P}^\top &= \dot{\mathbf{P}}^S \cdot \mathbf{P}^S + \dot{\mathbf{P}}^A \cdot \mathbf{P}^S - \dot{\mathbf{P}}^S \cdot \mathbf{P}^A - \dot{\mathbf{P}}^A \cdot \mathbf{P}^A = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \mathbf{k}\dot{\mathbf{k}} \cos \vartheta - \mathbf{k}\dot{\mathbf{k}} \cos^2 \vartheta + \dot{\mathbf{k}}\mathbf{k} (1 - \cos \vartheta) + \\
&\quad + \mathbf{k} \times \mathbf{E} \dot{\vartheta} \cos^2 \vartheta + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}} \times \mathbf{k} \mathbf{k} \sin \vartheta (1 - \cos \vartheta) + \\
&\quad + \mathbf{k} \times \mathbf{E} \dot{\vartheta} \sin^2 \vartheta - \mathbf{k}\dot{\mathbf{k}} \times \mathbf{k} \sin \vartheta (1 - \cos \vartheta) - (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta - \mathbf{k}\dot{\mathbf{k}} \sin^2 \vartheta = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + (\dot{\mathbf{k}}\mathbf{k} - \mathbf{k}\dot{\mathbf{k}})(1 - \cos \vartheta) + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + (\dot{\mathbf{k}} \times \mathbf{k} \mathbf{k} - \mathbf{k}\dot{\mathbf{k}} \times \mathbf{k}) \sin \vartheta (1 - \cos \vartheta) = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + \mathbf{k} \times \dot{\mathbf{k}} \times \mathbf{E} (1 - \cos \vartheta) + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + \mathbf{k} \times (\dot{\mathbf{k}} \times \mathbf{k}) \times \mathbf{E} \sin \vartheta (1 - \cos \vartheta) = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + (\dot{\mathbf{k}}\mathbf{k} \cdot \mathbf{k} - \mathbf{k}\dot{\mathbf{k}} \cdot \mathbf{k}) \times \mathbf{E} \sin \vartheta (1 - \cos \vartheta) + \mathbf{k} \times \dot{\mathbf{k}} \times \mathbf{E} (1 - \cos \vartheta) = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta + \mathbf{k} \times \dot{\mathbf{k}} \times \mathbf{E} (1 - \cos \vartheta).
\end{aligned}$$