



$$\begin{aligned} \sum_{i=1}^3 \left(\mathbf{a} + \frac{1}{2} d\mathbf{a} \cdot \mathbf{n}_i \mathbf{n}_i \right) \cdot \mathbf{n}_i d\mathcal{O}_i + \sum_{i=1}^3 \left(\mathbf{a} - \frac{1}{2} d\mathbf{a} \cdot \mathbf{n}_i \mathbf{n}_i \right) \cdot (-\mathbf{n}_i) d\mathcal{O}_i = \\ = d\mathbf{a} \cdot \mathbf{n}_i d\mathcal{O}_i = da_i d\mathcal{O}_i \end{aligned}$$

$$\sum_{i=1}^3 \left(a + \frac{1}{2} da \cdot n^i n_i \right) \cdot n^i d\mathcal{O}_i + \sum_{i=1}^3 \left(a - \frac{1}{2} da \cdot n^i n_i \right) \cdot (-n^i) d\mathcal{O}_i = da \cdot n^i d\mathcal{O}_i = da^i d\mathcal{O}_i$$

$$\begin{aligned}
\mathbf{r} &= \mathbf{r}(\xi^i) \neq \text{linear } \xi_i \mathbf{n}_i, \quad \mathbf{n}_i = \frac{\partial \mathbf{r}}{\partial \xi^i} = \partial_i \mathbf{r}, \quad d\mathbf{r} = d\xi^i \mathbf{n}_i, \quad \nabla = \mathbf{n}^i \partial_i \\
\mathbf{a} &= a^i \mathbf{n}_i, \quad d\mathbf{a} = da^i \mathbf{n}_i \\
d\mathbf{a} &= d\mathbf{r} \cdot \nabla \mathbf{a} = d\xi^i \mathbf{n}_i \cdot \mathbf{n}^j \partial_j \mathbf{a} = d\xi^i \partial_i \mathbf{a} = \frac{\partial \mathbf{a}}{\partial \xi^i} d\xi^i \\
d\mathbf{a} \cdot \mathbf{n}^i &= d\xi^h \partial_h \mathbf{a} \cdot \mathbf{n}^i = d\xi^h \partial_h a^i = \frac{\partial a^i}{\partial \xi^h} d\xi^h = da^i \\
d\mathbf{a} \cdot \mathbf{E} &= d\mathbf{a} \cdot \frac{1}{2} \mathbf{n}^i \in_{ijk} \mathbf{n}^j \times \mathbf{n}^k = d\mathbf{a} \cdot \frac{1}{2} \mathbf{n}^i \in_{ijk} \in^{jkh} \mathbf{n}_h = d\mathbf{a} \cdot \mathbf{n}^i \mathbf{n}_i
\end{aligned}$$

$$\hat{\mathbf{n}} = ?$$

$$\mathbf{n}^i d\mathcal{O}_i = \mathbf{n}^i \mathbf{n}_i \cdot d\xi^j \mathbf{n}_j \times d\xi^k \mathbf{n}_k, \quad d\mathcal{O}_i = \in_{ijk} d\xi^j d\xi^k$$

$$d\mathbf{a} \cdot \mathbf{n}^i d\mathcal{O}_i = d\mathbf{a} \cdot \mathbf{n}^i \mathbf{n}_i \cdot d\xi^j \mathbf{n}_j \times d\xi^k \mathbf{n}_k = da^i \in_{ijk} d\xi^j d\xi^k$$

$$d\mathcal{V} = d\xi^i \mathbf{n}_i \times d\xi^j \mathbf{n}_j \cdot d\xi^k \mathbf{n}_k = \in_{ijk} d\xi^i d\xi^j d\xi^k$$

$$\nabla \cdot \mathbf{a} d\mathcal{V} = \mathbf{n}^i \partial_i \cdot a^j \mathbf{n}_j d\mathcal{V} = \partial_i a^i d\mathcal{V}$$

$$\begin{aligned}
da^i d\mathcal{O}_i &= da^i d\xi^j d\xi^k \in_{ijk} = d\xi^h \partial_h a^i d\xi^j d\xi^k \in_{ijk} \\
\partial_h a^h d\mathcal{V} &= \partial_h a^h d\xi^i d\xi^j d\xi^k \in_{ijk}
\end{aligned}$$

$$\partial_h a^i d\xi^h d\xi^j d\xi^k \in_{ijk} \quad \partial_h a^h d\xi^i d\xi^j d\xi^k \in_{ijk}$$

$$\begin{aligned}
&\partial_h a^i d\xi^h d\xi^j d\xi^k \in_{ijk} = \\
&= \partial_h a^1 d\xi^h d\xi^2 d\xi^3 \in_{123} + \partial_h a^1 d\xi^h d\xi^3 d\xi^2 \in_{132} + \\
&+ \partial_h a^2 d\xi^h d\xi^3 d\xi^1 \in_{231} + \partial_h a^2 d\xi^h d\xi^1 d\xi^3 \in_{213} + \\
&+ \partial_h a^3 d\xi^h d\xi^1 d\xi^2 \in_{312} + \partial_h a^3 d\xi^h d\xi^2 d\xi^1 \in_{321} = \\
&= \partial_h a^1 d\xi^h d\xi^2 d\xi^3 \in_{123} - \partial_h a^1 d\xi^h d\xi^3 d\xi^2 \in_{123} + \\
&+ \partial_h a^2 d\xi^h d\xi^3 d\xi^1 \in_{123} - \partial_h a^2 d\xi^h d\xi^1 d\xi^3 \in_{123} + \\
&+ \partial_h a^3 d\xi^h d\xi^1 d\xi^2 \in_{123} - \partial_h a^3 d\xi^h d\xi^2 d\xi^1 \in_{123}
\end{aligned}$$