Let dx be a small change in the variable x

$$dx = x' - x\,, \tag{1}$$

where $x + dx = x'$ is near x, but nonetheless $x' \neq x \Leftrightarrow dx \neq 0$.

The change dx is *infinitesimal* if it approaches zero being very ("vanishingly") small, but not exactly zero, while the higher powers of dx, such as $(dx)^2$, $(dx)^3$, $(dx)^4$ and so on, are infinitesimally smaller than dx — and therefore are equal to zero. Here

$$dx \neq 0, \quad \text{but} \ \ (dx)^2 = 0,\ (dx)^3 = 0,\ \ldots \tag{2}$$

is what defines the infinitesimality of dx. Elements defined in this way are known as *nilsquare* or *nilpotent* of the second degree. An infinitesimal change (infinitesimal difference) is also called a *differential*, and the operation of obtaining an infinitesimal difference — *differentiation*.

The properties of differentiation :

✓ linearity $d\big(\lambda p + \mu q\big) = \lambda\, dp + \mu\, dq$
✓ $d(\mathsf{constant}) = 0$

"The product rule" :

✓ $d(uv) = (u+du)(v+dv) - uv = (du)v + u(dv) + (du)(dv)$,
$(du)(dv) = (du)\frac{du}{du}(dv) = (du)^2\big(\frac{dv}{du}\big) = 0$ or $= (du)\frac{dv}{dv}(dv) = \big(\frac{du}{dv}\big)(dv)^2 = 0$

For example, differential of the square $d\big(w^2\big)$ is

✓ $d\big(w^2\big) = (w + dw)^2 - w^2 = 2w dw + (dw)^2 = 2w dw$
or, applying "the product rule",
✓ $d\big(w^2\big) = d(ww) = (dw)w + w(dw) = 2w dw$

....

$\Rightarrow \forall \lambda, \mu \in \mathbb{R}\,(\neq\infty)\ \sin(\lambda dx + \mu dx) = (\lambda + \mu)\sin(dx)$

For infinitesimal dx, the sine function $\sin(dx)$ behaves linearly and is therefore equal to its argument :

$$\sin(dx) = dx\,. \tag{3}$$