

$$\begin{aligned}
\dot{\mathbf{P}}^S \cdot \mathbf{P}^S &= \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cdot \mathbf{E} \cos \vartheta + (\dot{\mathbf{k}}\dot{\mathbf{k}} + \dot{\mathbf{k}}\mathbf{k})(1 - \cos \vartheta) \cdot \mathbf{E} \cos \vartheta + \\
&\quad + (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cdot \dot{\mathbf{k}}\mathbf{k} (1 - \cos \vartheta) + (\dot{\mathbf{k}}\dot{\mathbf{k}} + \dot{\mathbf{k}}\mathbf{k})(1 - \cos \vartheta) \cdot \mathbf{k}\mathbf{k} (1 - \cos \vartheta) = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + (\dot{\mathbf{k}}\dot{\mathbf{k}} + \dot{\mathbf{k}}\mathbf{k}) \cos \vartheta (1 - \cos \vartheta) + (\dot{\mathbf{k}}\dot{\mathbf{k}} \cdot \mathbf{k}\mathbf{k} + \dot{\mathbf{k}}\mathbf{k} \cdot \mathbf{k}\mathbf{k}) (1 - \cos \vartheta)^2 = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}}\dot{\mathbf{k}} \cos \vartheta (1 - \cos \vartheta) + \\
&\quad + \dot{\mathbf{k}}\mathbf{k} \cos \vartheta - \dot{\mathbf{k}}\mathbf{k} \cos^2 \vartheta + \dot{\mathbf{k}}\mathbf{k} - 2 \dot{\mathbf{k}}\mathbf{k} \cos \vartheta + \dot{\mathbf{k}}\mathbf{k} \cos^2 \vartheta = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}}\dot{\mathbf{k}} \cos \vartheta - \dot{\mathbf{k}}\mathbf{k} \cos^2 \vartheta + \dot{\mathbf{k}}\mathbf{k} (1 - \cos \vartheta),
\end{aligned}$$

$$\begin{aligned} \dot{\mathbf{P}}^{\mathbf{A}} \cdot \mathbf{P}^{\mathbf{S}} &= \\ &= (\mathbf{k} \times \mathbf{E}) \cdot \mathbf{E} \dot{\vartheta} \cos^2 \vartheta + (\dot{\mathbf{k}} \times \mathbf{E}) \cdot \mathbf{E} \sin \vartheta \cos \vartheta + \\ &\quad + \overline{(\mathbf{k} \times \mathbf{E}) \cdot \mathbf{k} \mathbf{k}} \dot{\vartheta} \cos \vartheta (1 - \cos \vartheta) + (\dot{\mathbf{k}} \times \mathbf{E}) \cdot \mathbf{k} \mathbf{k} \sin \vartheta (1 - \cos \vartheta) = \\ &= \mathbf{k} \times \mathbf{E} \dot{\vartheta} \cos^2 \vartheta + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}} \times \mathbf{k} \mathbf{k} \sin \vartheta (1 - \cos \vartheta), \end{aligned}$$

$$\begin{aligned}
\dot{\mathbf{P}}^S \cdot \mathbf{P}^A &= \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cdot (\mathbf{k} \times \mathbf{E}) \sin \vartheta + (\mathbf{k}\dot{\mathbf{k}} + \dot{\mathbf{k}}\mathbf{k})(1 - \cos \vartheta) \cdot (\mathbf{k} \times \mathbf{E}) \sin \vartheta = \\
&= \overline{\mathbf{k}\mathbf{k} \cdot (\mathbf{k} \times \mathbf{E})} \dot{\vartheta} \sin^2 \vartheta - \mathbf{E} \cdot (\mathbf{k} \times \mathbf{E}) \dot{\vartheta} \sin^2 \vartheta + \left( \mathbf{k}\dot{\mathbf{k}} \cdot (\mathbf{k} \times \mathbf{E}) + \dot{\mathbf{k}}\mathbf{k} \cdot (\mathbf{k} \times \mathbf{E}) \right) \sin \vartheta (1 - \cos \vartheta) = \\
&= -\mathbf{k} \times \mathbf{E} \dot{\vartheta} \sin^2 \vartheta + \mathbf{k}\dot{\mathbf{k}} \times \mathbf{k} \sin \vartheta (1 - \cos \vartheta),
\end{aligned}$$

$$\begin{aligned}\dot{\mathbf{P}}^A \cdot \mathbf{P}^A &= (\mathbf{k} \times \mathbf{E}) \dot{\vartheta} \cos \vartheta \cdot (\mathbf{k} \times \mathbf{E}) \sin \vartheta + (\dot{\mathbf{k}} \times \mathbf{E}) \cdot (\mathbf{k} \times \mathbf{E}) \sin^2 \vartheta = \\ &= (\mathbf{k} \mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \mathbf{k} \dot{\mathbf{k}} \sin^2 \vartheta;\end{aligned}$$

$$\begin{aligned}
\dot{\mathbf{P}} \cdot \mathbf{P}^\top &= \dot{\mathbf{P}}^S \cdot \mathbf{P}^S + \dot{\mathbf{P}}^A \cdot \mathbf{P}^S - \dot{\mathbf{P}}^S \cdot \mathbf{P}^A - \dot{\mathbf{P}}^A \cdot \mathbf{P}^A = \\
&= (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta + \mathbf{k}\dot{\mathbf{k}} \cos \vartheta - \mathbf{k}\dot{\mathbf{k}} \cos^2 \vartheta + \dot{\mathbf{k}}\mathbf{k} (1 - \cos \vartheta) + \\
&\quad + \mathbf{k} \times \mathbf{E} \dot{\vartheta} \cos^2 \vartheta + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + \dot{\mathbf{k}} \times \mathbf{k} \sin \vartheta (1 - \cos \vartheta) + \\
&\quad + \mathbf{k} \times \mathbf{E} \dot{\vartheta} \sin^2 \vartheta - \mathbf{k}\dot{\mathbf{k}} \times \mathbf{k} \sin \vartheta (1 - \cos \vartheta) - (\mathbf{k}\mathbf{k} - \mathbf{E}) \dot{\vartheta} \sin \vartheta \cos \vartheta - \mathbf{k}\dot{\mathbf{k}} \sin^2 \vartheta = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + (\dot{\mathbf{k}}\mathbf{k} - \mathbf{k}\dot{\mathbf{k}}) (1 - \cos \vartheta) + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + (\dot{\mathbf{k}} \times \mathbf{k} \mathbf{k} - \mathbf{k} \dot{\mathbf{k}} \times \mathbf{k}) \sin \vartheta (1 - \cos \vartheta) = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + \mathbf{k} \times \dot{\mathbf{k}} \times \mathbf{E} (1 - \cos \vartheta) + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + \mathbf{k} \times (\dot{\mathbf{k}} \times \mathbf{k}) \times \mathbf{E} \sin \vartheta (1 - \cos \vartheta) = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta \cos \vartheta + (\dot{\mathbf{k}}\mathbf{k} \cdot \mathbf{k} - \mathbf{k}\dot{\mathbf{k}} \cdot \mathbf{k}) \times \mathbf{E} \sin \vartheta (1 - \cos \vartheta) + \mathbf{k} \times \dot{\mathbf{k}} \times \mathbf{E} (1 - \cos \vartheta) = \\
&= \mathbf{k} \times \mathbf{E} \dot{\vartheta} + \dot{\mathbf{k}} \times \mathbf{E} \sin \vartheta + \mathbf{k} \times \dot{\mathbf{k}} \times \mathbf{E} (1 - \cos \vartheta).
\end{aligned}$$

$$O_{i'k} = \begin{bmatrix} o_{1'1} & o_{1'2} & o_{1'3} \\ o_{2'1} & o_{2'2} & o_{2'3} \\ o_{3'1} & o_{3'2} & o_{3'3} \end{bmatrix} = \begin{bmatrix} \cos \angle(\mathbf{e}'_1, \mathbf{e}_1) & \cos \angle(\mathbf{e}'_1, \mathbf{e}_2) & \cos \angle(\mathbf{e}'_1, \mathbf{e}_3) \\ \cos \angle(\mathbf{e}'_2, \mathbf{e}_1) & \cos \angle(\mathbf{e}'_2, \mathbf{e}_2) & \cos \angle(\mathbf{e}'_2, \mathbf{e}_3) \\ \cos \angle(\mathbf{e}'_3, \mathbf{e}_1) & \cos \angle(\mathbf{e}'_3, \mathbf{e}_2) & \cos \angle(\mathbf{e}'_3, \mathbf{e}_3) \end{bmatrix}$$

$$(O_{i'k})^\top = \begin{bmatrix} o_{1'1} & o_{2'1} & o_{3'1} \\ o_{1'2} & o_{2'2} & o_{3'2} \\ o_{1'3} & o_{2'3} & o_{3'3} \end{bmatrix} = \begin{bmatrix} \cos \angle(\mathbf{e}'_1, \mathbf{e}_1) & \cos \angle(\mathbf{e}'_2, \mathbf{e}_1) & \cos \angle(\mathbf{e}'_3, \mathbf{e}_1) \\ \cos \angle(\mathbf{e}'_1, \mathbf{e}_2) & \cos \angle(\mathbf{e}'_2, \mathbf{e}_2) & \cos \angle(\mathbf{e}'_3, \mathbf{e}_2) \\ \cos \angle(\mathbf{e}'_1, \mathbf{e}_3) & \cos \angle(\mathbf{e}'_2, \mathbf{e}_3) & \cos \angle(\mathbf{e}'_3, \mathbf{e}_3) \end{bmatrix}$$