Shri Shankaracharya Institute of Professional Management & Technology, Raipur

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Course: B.Tech **Semester:** 3rd

Branch: Computer Science And Engineering

Subject Name: Mathematics-3

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Unit - I

(A)

Sol^n $\rightarrow$ $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{s}}$ $\leftarrow$ To prove

$$\Rightarrow \frac{1}{\sqrt{\pi}} L\left\{\frac{1}{\sqrt{t}}\right\}$$

$$\Rightarrow \frac{1}{\sqrt{\pi}} L\left\{t^{-1/2}\right\}$$

By the elementary transform

$$\left(L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{\pi}} L\left(t^{-1/2}\right) = \frac{1}{\sqrt{\pi}} \frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-1/2+1}}$$

$$= \frac{1}{\sqrt{\pi}} \frac{\Gamma\,1/2}{s^{1/2}}$$

$$= \frac{1}{\sqrt{\pi}} \times \sqrt{\frac{\pi}{s}}$$

$$= \frac{1}{\sqrt{s}}$$

(B)

Sol → (i) To ~~p~~ find Laplace transform of $\frac{1-\cos at}{t^2}$

$$\Rightarrow \quad L\{1-\cos at\} = \frac{1}{s} - \frac{s}{s^2+a^2}$$

Using division property.

$$L\left\{\frac{1-\cos at}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+a^2}\right) ds$$

$$= \left|\log s\right|_s^\infty - \frac{1}{2}\int_s^\infty \frac{2s}{s^2+a^2}\, ds$$

$$= \left|\log s - \frac{1}{2}\log(s^2+a^2)\right|_s^\infty$$

$$= \left|\log \frac{s}{(s^2+a^2)^{1/2}}\right|_s^\infty$$

$$= \lim_{s\to\infty} \log \frac{s}{s\left(1+\frac{a^2}{s^2}\right)} - \log \frac{s}{(s^2+a^2)}$$

$$= \lim_{s\to\infty} \cancel{\log(1)}^{0} - \log \frac{s}{(s^2+a^2)^{1/2}}$$

$$= \log \frac{(s^2+a^2)^{1/2}}{s}$$

Again Using division property

$$L\left\{\frac{1-\cos at}{t\cdot t}\right\} = \int_s^\infty \log\frac{(s^2+a^2)^{1/2}}{s}\,ds$$

→ Using By-parts,

$$= \frac{1}{2}\left[\log(1+s^{-2})\int ds - \int\left\{\frac{d}{ds}\log(1+s^{-2})\int ds\right\}ds\right]_s^\infty$$

$$= \frac{1}{2}\left[s\log(1+s^{-2}) - \int\frac{1}{1+s^{-2}}(-2)\,s^{-2}\cdot s\,ds\right]_s^\infty$$

$$= \frac{1}{2}\left[s\log(1+s^{-2}) + 2\int\frac{s^{-2}}{s^{-2}\left(\frac{1}{s^{-2}}+1\right)}ds\right]_s^\infty$$

$$= \frac{1}{2}\left[s\log(1+s^{-2}) + 2\int\frac{ds}{1+s^2}\right]_s^\infty$$

$$= \frac{1}{2}\left[s\log(1+s^{-2}) + 2\tan^{-1}s\right]$$

$$= \frac{1}{2}\left[s\log(1+s^{-2}) + 2\tan^{-1}s\right]_s^\infty$$

$$= \frac{1}{2}\left[0 + 2\tan^{-1}\infty - s\log(1+s^{-2}) - 2\tan^{-1}s\right]$$

$$= \frac{1}{2}\left[2\times\frac{\pi}{2} - 2\tan^{-1}s - s\log(1+s^{-2})\right]$$

$$= \frac{1}{2}\left[2\left(\frac{\pi}{2} - \tan^{-1}s\right) - s\log(1+s^{-2})\right]$$

$$= \frac{1}{2}\left[2\cot^{-1}s - s\log(1+s^{-2})\right]$$

$$= \cot^{-1}s - \frac{1}{2}s\log\left(1+\frac{1}{s^2}\right) \quad \text{Ans}$$

(ii) Evaluate:

$$f(t) = \int_0^\infty \frac{e^{-t} \sin^2 t}{t} \, dt$$

Sol^n → $L\{\sin^2 t\} = 1 - \cos^2 t$

$$= 1 - \left(\frac{\cos 2t + 1}{2}\right)$$

$$L\left\{1 - \frac{\cos 2t}{2} + \frac{1}{2}\right\}$$

$$= \frac{1}{s} - \frac{s}{s^2+4}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{s}\right)$$

$$= \frac{3}{2}\left(\frac{1}{s}\right) - \frac{s}{(s^2+4)2}$$

Using Division property.

$$L\left\{\frac{\sin^2 t}{t}\right\} = \int_s^\infty \left[\frac{3}{2}\left(\frac{1}{s}\right) - \frac{s}{2(s^2+4)}\right] ds$$

$$= \int_s^\infty \left[\frac{3}{2}\left(\frac{1}{s}\right) - \frac{1}{4}\frac{2s}{(s^2+4)}\right] ds$$

$$= \left[\frac{3}{2} \log s - \frac{1}{4} \log (s^2+4)\right]_s^\infty$$

$$= \left[\log \frac{s^{3/2}}{(s^2+4)^{5/4}}\right]_s^\infty$$

~~Using~~

$$= \lim_{s \to \infty} \log \frac{s^{3/2}}{s^{3/2}\left(1+\frac{4}{s^2}\right)^{1/4}} - \log\left(\frac{s^{3/2}}{(s^2+4)^{3/4}}\right)$$

$$= \log(1) - \log\left(\frac{s^{3/2}}{(s^2+4)^{1/4}}\right)$$

$$= \log \frac{(s^2+4)^{1/4}}{s^{3/2}} = \frac{1}{4} \log\left[\frac{s^2+4}{s^{5/4}}\right]$$

Using shifting property

$$L\left\{e^{-t} \frac{\sin^2 t}{t}\right\} = \frac{1}{4} \log\left[\frac{(s+1)^2+4}{(s+1)^{5/4}}\right]$$

Now by definition of Laplace Transformation.

$$L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

$$L\left\{e^{-t} \frac{\sin^2 t}{t}\right\} = \int_0^\infty e^{-st} \cdot e^{-t} \frac{\sin^2 t}{t}\, dt \quad \text{-(1)}$$

On comparing eq^n (1) with given eq^n.

$$e^{-st} \longrightarrow 1$$

$$\Rightarrow \quad s \longrightarrow 0$$

$$\boxed{\int_0^\infty e^0 \cdot e^{-t} \frac{\sin^2 t}{t}\, dt = \frac{1}{4} \log_e 5}$$

(D)

Sol^n → Taking Laplace transform of both sides of the equation.

$L\{t(f(t))\} = -\frac{d}{ds}[L\{f(t)\}]$, we get.

$$-\frac{d}{ds}\left[s^2\bar{y} - s\,y(0) - y'(0)\right] + 2\left[s\bar{y} - y(0)\right] - \frac{d}{ds}(\bar{y}) = \frac{1}{s^2+1}$$

or $$-\left(s^2\frac{d\bar{y}}{ds} + 2s\bar{y}\right) + y(0) + 0 + 2s\bar{y} - 2y(\cdot) - \frac{d}{ds}(\bar{y}) = \frac{1}{s^2+1}$$

or $$-(s^2+1)\frac{d\bar{y}}{ds} - 1 = \frac{1}{s^2+1}$$

$$-(s^2+1)\frac{d\bar{y}}{ds} = \frac{1}{s^2+1} + 1$$

$$\frac{d\bar{y}}{ds} = \frac{-1}{(s^2+1)^2} - \frac{1}{(s^2+1)}$$

on inversion

$L^{-1}(f^{-1}(s)) = -t f(t)$, we get

$$-ty = -\sin t - \left(\frac{1}{2}\sin t - \frac{t\cos t}{2}\right)$$

$$= \frac{1}{2}(-3\sin t + t\cos t)$$

$$\Rightarrow \boxed{Y = \frac{1}{2}\left(\frac{3\sin t}{t} - \cos t\right)}$$

Ans

Unit-III

(A)

Sol^n → (i) Mean of a binomial distribution $b(n,p)$ is np and the variance is npq where $q = 1-p$

now, $np = 5$ and variance is

$npq = 3^2 = 9 \Rightarrow q = \frac{9}{5} = 1.8$ when isn't possible because q is a ~~prob~~ probability $(0 < q < 1)$.

Hence, a binomial distribution can't have a mean 5 and standard deviation 3.

(A)

Solⁿ → (ii) Given that the function

$$f(x) = ae^{-2x}$$

By the property of distribution function.

$$\int_0^\infty f(x)\,dx = 1$$

$$a\int_0^\infty e^{-2x}\,dx = 1$$

$$\frac{a}{2}\left[e^{-2x}\,dx = 1\right]$$

$$-\frac{a}{2}\left[e^{-2x}\right]_0^\infty = 1$$

$$-\frac{a}{2}\left[e^{-2x}\right]_0^\infty = 1$$

$$-\frac{a}{2}\left[e^{-\infty} - e^{0}\right] = 1$$

$$-\frac{a}{2}\left[0-1\right] = 1$$

$$\frac{a}{2} = 1$$

$$\boxed{a = 2}$$

(B)

Sol^n → Given that:

probability of bomb will strike $= 50\%$

So, $p = \frac{1}{2}$, $q = \frac{1}{2}$

here, x: number of hits.

for two ~~dist~~ direct hits the probability would be

$$P(x \geq 2) \geq 0.98$$

$$[1 - P(x < 2)] \geq 0.98$$

$$[1 - 0.98] \geq P(x < 2)$$

$$0.02 \geq P(x < 2)$$

$$0.02 \geq P(x=0) + P(x=1)$$

$$0.02 \geq \left[{}^nC_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^n\right] + \left[{}^nC_1 \left(\tfrac{1}{2}\right)^1 \left(\tfrac{1}{2}\right)^{n-1}\right]$$

$$0.02 \geq \left[1 \cdot 1 \cdot \frac{1}{2^n}\right] + \left[n \cdot \frac{1}{2} \cdot \frac{1}{2^{n-1}}\right]$$

$$0.02 \geq \frac{1}{2^n} + \frac{n}{2^n}$$

$$\frac{1+n}{2^n} \leq 0.02$$

$$1+n \leq (0.02)\, 2^n$$

for, $n = 9$

$$1+9 \leq (0.02)\, 2^9$$

$$10 \leq (0.02) \cdot 512$$

$$10 \leq 10.24$$

So, for $n=9$ True

Now,

for $n=0$

$$1+0 \leq (0.02)\, 2^0$$

$$1 \leq (0.02) \cdot 1$$

$1 \leq 0.02$ false.

similarly,

for, $n = 9$

$$1+9 \leq (0.02) \times 2^9$$

$$10 \leq 0.02 \cdot 512$$

$10 \leq 10.24$ True

~~So,~~ for $\boxed{n=9}$ True

(c)

Sol^n → Mean $m = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{100}{200} = 0.5$, $N = 200$

By Possion distributions.

Theoretical frequency for r success given by $\frac{N e^{-m} m^r}{r!} = \frac{200 \cdot e^{-0.5} (0.5)^r}{r!}$

$r = 0, 1, 2, 2, 4$

for $r = 0$, $\frac{200 \times e^{-0.5} (0.5)^0}{0!} = 122$

$r = 1$, $\frac{200 \times e^{-0.5} (0.5)^1}{1!} = 61$

$r = 2$, $\frac{200 \times e^{-0.5} (0.5)^2}{2!} = 15.25$

$r = 3$, $\frac{200 \times e^{-0.5} (0.5)^3}{3!} = 2.54$

$r = 4$, $\frac{200 \times e^{-0.5} (0.5)^4}{4!} = 0.32$

∴ Theoretical frequence filled by poission distribution:

x	0	1	2	3	4
f	122	61	15	2	0