Shri Shankaracharya Institute of Professional Management & Technology, Raipur

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April-May-2021- Class Test-1 (July-2021)

Date: 05 / 07 /2021

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Roll No.:

3	0	3	3	0	2	2	2	0	0	2	0

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B	J	4	5	9	9

Course: B.Tech **Semester:** 2nd

Branch: COMPUTER SCIENCE AND ENGINEERING

Subject Name: MATHEMATICS - 2

Subject Code:

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Q1>

(i)

Solⁿ → ~~x^2~~ Given differential Eqⁿ is,

$$(x^2+y^2)\,dx + 2xy\,dy = 0 \quad -①$$

Now, $\cancel{\frac{\partial M}{\partial y}}$ $M\,dx + N\,dy = 0$

Here on comparing both eqⁿ, we get

$$M = x^2+y^2 \quad \& \quad N = 2xy$$

Now, partially differentiating M & N ~~to~~ w. r. to. y and x respectively.

$$\frac{\partial M}{\partial y} = 0 + 2y = 2y$$

$$\frac{\partial N}{\partial x} = 2y$$

$$\Rightarrow \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

∴ The ~~comp~~ given Differential Eqⁿ is Exact differential Eqⁿ.

⇒ The complete solution will be

$$\underset{(y \text{ is constant})}{\int M\,dx} + \underset{(\text{term free from } x)}{\int N\,dy} = c$$

$\Rightarrow \int M\,dx + \int N\,dy = c$

$\Rightarrow \int (x^2+y^2)\,dx + \int 0\,dy = c$ ($\because$ In M → y as constant. In N → term free from x)

$\Rightarrow \boxed{\frac{x^3}{3} = c}$

Hence, $\frac{x^3}{3} = c$ is a Ans

(ii)

Solⁿ → From given, we have an equation,

$p = \log(px - y)$

$\Rightarrow \log_e(px-y) = p$

$\Rightarrow px - y = e^p$

$\therefore y = px - e^p$ — ①

differentiating both sides w.r.t. x, we get,

$\frac{dy}{dx} = \frac{d}{dx}(px - e^p)$

$\frac{dy}{dx} = p + x\frac{dp}{dx} - e^p\frac{dp}{dx}$

$\therefore p = p + (x - e^p)\frac{dp}{dx}$ ($\because \frac{dy}{dx} = p$)

$(x - e^p)\frac{dp}{dx} = 0$ — ②

~~$(x - e^p)\frac{dp}{dx} = 0$~~

$(x - e^p) \neq 0$, when $\frac{dp}{dx} = 0$

$\therefore \frac{dp}{dx} = 0$

$\boxed{dp = 0\, dx}$

integrating on both sides, we get,

$\int dp = \int 0\, dx$

$\boxed{p = c}$ $(c = \text{constant})$ — ③

from ① and ③, we get,

$\boxed{y = cx - e^c}$

from ②,

if $x - e^p = 0$

$\Rightarrow x = e^p$

$\log x = p$

$\therefore p = \log x$ — ④

from ① and ④, we get,

$y = \log x \cdot x - e^{\log x}$

$y = x \log x - x$

$\boxed{y = x(\log x - 1)}$

Q 2>

Sol^n → $(xy^2 - e^{1/x^3})\,dx - x^2 y\,dy = 0$ —— ①

Here, $M = xy^2 - e^{1/x^3}$, $N = -x^2 y$

$\frac{\partial M}{\partial y} = 2xy$, $\frac{\partial N}{\partial x} = -2xy$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, Therefore the given differential Equation ~~to~~ make it an exact differential equation, we will be introducing an integrating factor such ~~too~~ that,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2xy - (-2xy)}{-x^2 y} = \frac{-4}{x}$$

= which is only of x.

$$\therefore \text{I.F.} = e^{\int -\frac{4}{x}dx}$$

$$= e^{-4\log x}$$

$$= x^{-4}$$

Multiplying IF through given differential Equation ~~by x~~, i.e., x^{-4}, we get

$$\left[\frac{y^2}{x^3} - \frac{1}{x^4}e^{1/x^3}\right]dx - \frac{y}{x^2}\,dy = 0 \quad \text{—— ②}$$

For the differential Eqⁿ ②, we have

$$M = \frac{y^2}{x^3} - \frac{1}{x^4} e^{1/x^3} \quad \text{and } N = \frac{-y}{x^2}$$

Now, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} = -\dfrac{1}{x^3}$

Hence the differential equation is an exact differential Eqⁿ,

Then it's solution is

$$\underset{y\text{-constant}}{\int M\,dx} + \underset{x \text{ not containing term}}{\int N\,dy} = c$$

$$\underset{y\text{ constant}}{\int} \left(\frac{y^2}{x^3} - \frac{1}{x^4} e^{x^{-3}}\right) dx = c$$

$$\Rightarrow \frac{-y^2}{2x^2} - \int \frac{e^{x^{-3}}}{x^4}\,dx = c$$

let $x^{-3} = t$

$-3x^{-4}\,dx = dt$

$-x^{-4}\,dx = \dfrac{dt}{3}$

$$\Rightarrow \frac{-y^2}{2x^2} + \int \frac{e^t}{3}\,dt = c$$

$$\Rightarrow \frac{-y^2}{2x^2} + \frac{e^t}{3} = c$$

$$\Rightarrow \boxed{\frac{-y^2}{2x^2} + \frac{e^{x^{-3}}}{3} = c} \quad \text{Ans}$$

Q4) $p^2 + 2py\cot x = y^2$

Soln → The given eqn we have,

$$p^2 + 2py\cot x + y^2\cot^2 x = y^2 + y^2\cot^2 x$$

$$(p + y\cot x)^2 = y^2[1 + \cot^2 x]$$

$$p + y\cot x = \pm y\sqrt{(1+\cot^2 x)}$$

$$p + y\cot x = \pm y\,(\text{cosec}\,x)$$

$$p + y\cot x = y\,\text{cosec}\,x,$$

and, $p + y\cot x = -y\,\text{cosec}\,x$

taking, $p + y\cot x = y\,\text{cosec}\,x$

$$\frac{dy}{dx} + y\cot x = y\,\text{cosec}\,x$$

$$dy + y\cot x\,dx = y\,\text{cosec}\,x\,dx$$

$$dy = y(\text{cosec}\,x - \cot x)\,dx$$

$$\frac{dy}{y} = (\text{cosec}\,x - \cot x)\,dx$$

By integration,

$$\int \frac{dy}{y} = \int (\text{cosec}\,x - \cot x)\,dx$$

$$\log y = \log\tan\frac{x}{2} - \log\sin x + \log c$$

$$\log y = \log\left[\frac{c\cdot\tan\frac{x}{2}}{\sin x}\right]$$

$$y = \frac{c \tan \frac{x}{2}}{\sin x}$$

$$= \frac{c \tan x/2}{2 \sin x/2 \cdot \cos x/2} = \frac{c \cdot \frac{\sin x/2}{\cos x/2}}{2 \sin x/2 \cdot \cos x/2}$$

$$= \frac{c}{2 \cos^2 x/2} = \frac{c}{1 + \cos x}$$

or $y (1 + \cos x) = c$

Similarly, $y = \frac{c}{1 - \cos x}$

or $y(1 - \cos x) = c$

Hence the solution is

$$\boxed{y(1 \pm \cos x) = c}$$ Ans

P.T.O

Unit - 2

Q1>

(i) $\frac{d^3y}{dx^3} + y = 0$

Sol^n → Given Eq^n is, $\frac{d^3y}{dx^3} + y = 0$

we can write ⇒ $(D^3+1)\, y = 0$

Auxilliary Eq^n ⇒ $D^3+1 = 0$

⇒ $(D+1)(D^2-D+1) = 0$

⇒ $D = -1$, $D = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

⇒ hence, the general equation of CF for one real and imaginary root is,

⇒ $\boxed{CF = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)}$

where α = real root
β = Imaginary root.

⇒ hence the solution of given eq^n will be here, $\alpha = -1$, $\beta = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

⇒ $\boxed{C.F. = C_1 e^{-x} + e^{x/2}\left(C_2 \cos\frac{\sqrt{3}}{2}x + C_3 \sin\frac{\sqrt{3}}{2}x\right)}$

Ay

(ii)

Sol^n → $\frac{d^2y}{dx^2} + 4y = 0$

The given Diff eq is

$$\frac{d^2y}{dx^2} + 4y = 0 \quad — ①$$

which is linear Diff. eq^n

Now from eq ①

$$D^2y + 4y = 0$$

$$y(D^2 + 4) = 0$$

$$D^2 + 4 = 0 \quad — ②$$

which is ~~thy~~ the required symbolic form,

Now for Auxilliary equation $f(D) = 0$

$$D^2 + 4 = 0$$

$$D^2 \cancel{+4} = -4$$

$$D = \sqrt{-4}$$

$$D = \pm 2i$$

Since the value of D is Imaginary $+2i, -2i$,

So, $CF = C_1 \cos 2x + C_2 \sin 2x$

finding of P.I.

$$P.I. = \frac{1}{f(D)} \text{ (RHS)}$$

$$P.I. = \frac{1}{D^2+4} (0)$$

$$P.I. = 0$$

Now compare solution of linear Diff. eq^n is written as,

$$y = C.F. + P.I.$$

$$y = C_1 \cos 2x + C_2 \sin 2x + 0$$

$$\boxed{y = C_1 \cos 2x + C_2 \sin 2x} \text{ Ans.}$$

Q2>

Sol^n→ Given Eq^n, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$

we can write,

$$(D^2 - 6D + 9)\, y = \frac{e^{3x}}{x^2}$$

Auxilliary Eq^n $(D-3)^2 = 0$

$$\boxed{D = 3, 3}$$

Both real and Equal roots hence CF is.

$\Rightarrow$ C.F. $= (C_1 + C_2 x)\, e^{mx}$ here, $m = 3$

$\Rightarrow \boxed{\text{C.F.} = (C_1 + C_2 x)\, e^{3x}}$

Now,

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \Rightarrow \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & e^{3x} + 3x e^{3x} \end{vmatrix}$$

$$\boxed{W = e^{6x}}$$

Now, P.I.

$$\text{P.I.} = -y_1 \int \frac{y_2 x}{W} \cdot dx + y_2 \int \frac{y_1 x}{W}\, dx$$

$$= -e^{3x} \int \frac{x \cdot e^{3x}}{e^{6x}} \cdot \frac{e^{3x}}{x^2}\, dx +$$

$$x e^{3x} \int \frac{e^{3x}}{e^{6x}} \cdot \frac{e^{3x}}{x^2}\, dx$$

$$= -e^{3x} \int \frac{dx}{x} + x e^{3x} \int x^{-2}\, dx$$

$$\Rightarrow \boxed{\text{P.I.} = -e^{3x} (\log x + 1)}$$

$\Rightarrow \therefore$ CS $=$ CI $+$ P.I.

$$\Rightarrow \boxed{\text{C.S.} = (C_1 + C_2 x) e^{3x} - e^{3x} (\log x + 1)}$$

Q3>

Sol^n → $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$

putting $x = e^t$

i.e. $t = \log x$, the eq^n becomes

$(D(D-1) + D+1)\, y = t \sin t$ — ①

$\{(D^2+1)y = t\sin t\}$

Its Auxilliary Eq^n is $D^2+1 = 0$ i.e.

$D = \pm i$

$$\boxed{C.F. = c_1 \cos t + c_2 \sin t}$$

$$P.I. = \frac{1}{D^2+1}\, t \sin t = \frac{1}{D^2+1}\,(I.P.\ of\ e^{it})$$

$$= I.P.\ of\ e^{it}\left(\frac{1}{(D+i)^2+1}\, t\right)$$

$$= I.P.\ of\ e^{it}\left(\frac{1}{D^2+2iD}\, t\right)$$

$$= I.P.\ of\ e^{it}\left(\frac{1}{D^2+2iD}\, t\right)$$

$$= I.P.\ of\ e^{it}\ \frac{1}{2iD(D/2i)}\ t$$

$$= I.P\ of\ \frac{1}{2i} \times e^{it} \times \frac{1}{D}\left(1 - \frac{iD}{2}\right)^{-1} t$$

$$= I.P.\ of\ \frac{1}{2i}\, e^{it}\, \frac{1}{D}\left(1 + \frac{iD}{2} + \dots\right) t$$

{By Binomial expansion}

$$= \text{I.P. of } \frac{e^{it}}{2i} \int \left(t + \frac{i}{2}\right) dt$$

$$= \text{I.P. of } \frac{e^{it}}{2p} \left(\frac{t^2}{2} + \frac{it}{2}\right)$$

$$= \text{I.P. of } e^{it} \left(-\frac{p}{4} t^2 + \frac{t}{4}\right)$$

$$= \text{I.P. of } (\cos t + i \sin t) \left(-\frac{it^2}{4} + \frac{t}{4}\right)$$

$$= -\frac{t^2}{4} \cos t + \frac{t}{4} \sin t$$

$$\boxed{P.I. = -\frac{t^2}{4} \cancel{\cot} \cos t + \frac{t}{4} \sin t}$$

Hence,

complete solution,

$$\boxed{y = C_1 \cos t + C_2 \sin t - \frac{t^2}{4} \cos t + \frac{t}{4} \sin t}$$

Ans.