Shri Shankaracharya Institute of Professional Management & Technology, Raipur

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Course: B.Tech Semester: 4th

Branch: Computer Science And Engineering

Subject Name: Discrete Mathematics

Subject Code: | B | 0 | 2 | 2 | 4 | 1 | 1 | (| 0 | 1 | 4 |) |

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Q1)

(i) Sol^n => The word ALLAHABAD contains 4 A's and 2 L's, therefore the total number of arrangement of nine letters in the ~~words~~ word.

'ALLAHABAD'

$$= \frac{9!}{4!\,2!} = 7560 \text{ ways.}$$

(ii) Sol^n → We apply the principle of mathematical ~~into~~ induction in two steps:

Let $P(n) = n! \geq 2^{n-1}$ for $n \geq 1$

Step 1: Basis of induction:

For $n = 1$, $P(1)$ is $1! \geq 2^{1-1}$

$\Rightarrow 1 = 1$

Step 2: Induction step:

For $n = r$, $r! \geq 2^{r-1}$

for $n = r+1$, $(r+1)! \geq 2^{(r+1)-1} = 2^r$

$(r+1 \geq 2)$

Now, $(r+1)! = (r+1)\,r! \geq (r+1)\,2^{r-1}$

$\geq 2 \cdot 2^{r-1}$ since, $r+1 \geq 2$

$\geq 2^r$

Hence $P(r+1)$ is true

Thus by mathematical Induction $P(n)$ is true for all $n \geq 1$.

Q2>

Sol(i)> Here there are 1000 people who were born in 12 months. Hence, by the generalized pigeonhole principle there are at least

$$\left\lceil \frac{1000-1}{12} \right\rceil + 1 = 83 + 1 = 84 \text{ people}$$

who were born in the same month.

(ii) Formula => $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Calculation:

Given $1 \leq n \leq 250$

Let

A : integers divisible by 2

B : integers divisible by 3

C : integers divisible by 7

Therefore,

$n(A)$ = number divisible by 2 $= \frac{250}{2} = \cancel{150}\ 125$

$n(B)$ = number divisible by 3

$= \frac{250}{3} = 83.33 \approx 83$

$n(C)$ = number divisible by 7 $= \frac{250}{7}$

$= 35.71$

≈ 35

$n(A \cap B)$ = no. divisible ~~by 7 = 250/7 = 35.71~~ by 2 & 3 (i.e. 6) $= \frac{250}{6} = 41.66$

≈ 41

$n(B \cap C)$ = no. divisible by 3 & 7

$= \frac{250}{21} = 11.09 \approx 11$

$n(C \cap A)$ = no. divisible by 7 & 2

$= \frac{250}{14} = 17.85 \approx 17.$

$n(A \cap B \cap C)$ = number divisible by 2, 3 & 7 (i.e. 42) $= \frac{250}{42} = 5.95 \approx 5$

By using the above ~~fol~~ formula.

$n(A \cup B \cup C) = 125 + 83 + 35 - 41 - 11 - 17 + 5$

$= \underline{179}.$

Q2)
(ii) Solⁿ → Let A_1, A_2, A_3, A_4 be the sets of integers

P.T.O

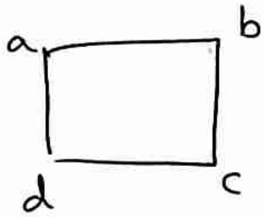
Q4>

(i) Sol^n → ① Euler Graph:

(i) Euler path

Path contain each edge exactly one and vertex ~~con~~ cover ~~all all~~ atleast one time (means vertex may be repeat but edge can't be repeat).

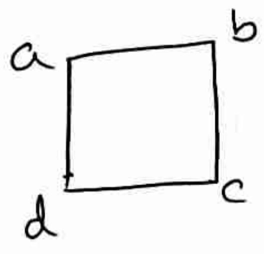
eg.



a-b-c-d-a
eular path

(ii) Eular Circuit:

Eular path which have starting vertex same as ending vertex is called euler circuit.



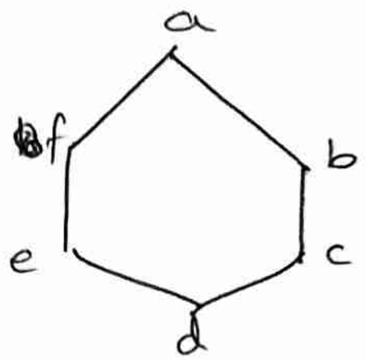
$\left.\begin{array}{l} a-b-c-d-a \\ a-d-c-b-a \end{array}\right]$ Euler circuit.

A connected graph G is called a Euler graph, if there is a closed trail which includes every edge of the graph a.

OR

Euler graph is a graph which contains Euler circuit.

eg:

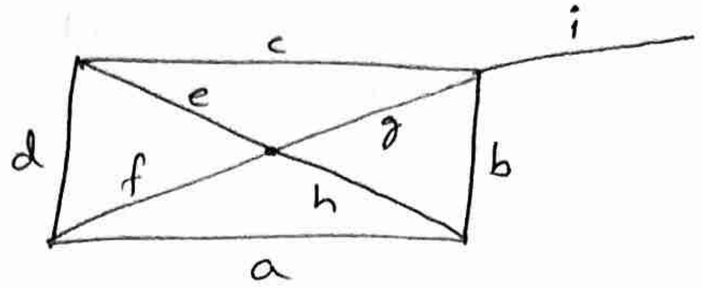


a-b-c-d-e-f-a

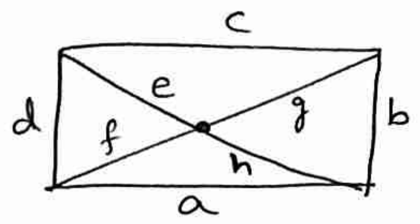
(ii) Cut set

Let G (V, E) be a connected graph. A subset E' of E is called a cut set of G if deletion of all the edges of E' from G make Graph dis connected.

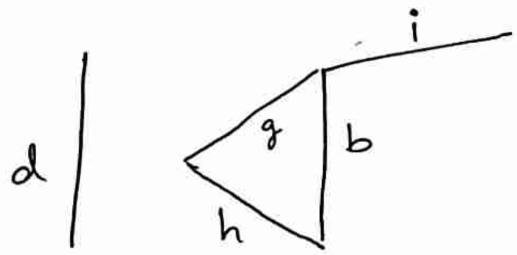
eg: considering a graph



~~in~~ its cut set is [i]



and [c, e, f, a]



(ii) Solⁿ → The initial labeling is given by

vertex v	a	b	c	d	e	f
L(v)	0	α	α	α	α	α
T	{a,	b,	c,	d,	e,	f}

Iteration 1: $u = a$ has $L(u) = 0$

T becomes $T - \{a\}$

There are two edges incident with a i.e. ab and ac where b and $c \in T$

~~Hence~~ $L(b) = \min\{\text{old } L(b), L(a) + \omega(ab)\} =$
$\min\{\alpha, 0 + 1.0\} = 1.0$
$L(c) = \min\{\text{old } L(c), L(a) + \omega(a,c)\}$
$= \min\{\alpha, 0 + 4.0\} = 4.0$

Hence minimum label is $L(b) = 1.0$

Vertex v	a	b	c	d	e	f
L(v)	0	1.0	4.0	α	α	α
T	{	b,	c,	d,	e,	f}

<u>Iteration 2:</u> $u = b$ has $L(u) = 1.0$

T becomes $T - \{b\}$

There are three edges incident with b i.e. bc, and be where $c, d, e \in T$

$L(c) = \min\{\text{old } L(c), L(b) + \omega(bc)\} =$
$\min\{4.0, 1.0 + 2.0\} = 3.0$

$L(d) = \min\{\text{old } L(d), L(b) + \omega(bd)\} =$
$\min\{\alpha, 1.0 + 6.0\} = 7.0$

$L(e) = \min\{\text{old } L(e), L(b) + \omega(be)\}$
$= \min\{\alpha, 1.0 + 5.0\} = 6.0$

Vertex v	a	b	c	d	e	f
L(v)	0	1.0	3.0	7.0	6.0	α
T	{		c,	d,	e,	f}

Thus min. label is $L(c) = 3.0$

Iteration 3: $u = c$ has $L(u) = 3.0$

T becomes $T - \{c\}$

There is only edge incident with c i.e. ce where $e \in T$

$L\{e\} = \min\{\text{old } L(e), L(c) + w(ce)\}$

$= \min\{6.0, 3.0 + 1.0\} = 4.0$

The minimum label is $L(e) = 4.0$

vertex v	a	b	c	d	e	f
L(v)	0	1.0	3.0	7.0	4.0	∞
T	{			d,	e,	f}

Iteration 4: $u = e$ has $L(e) = 4.0$

vertex v	a	b	c	d	e	f
L(v)	0	1.0	3.0	7.0	4.0	∞
T	{			d,	e,	f}

T becomes $T - \{e\}$.

There are two edges incident with e i.e. ef and ed where $d, f \in T$

$L(d) = \min\{\text{old } L(d), L(e) + w(ed)\}$

$= \min\{7.0, 4.0 + 3.0\} = 7.0$

$L(f) = \min\{\text{old } L(f), L(e) + w(ef)\}$

$= \min\{\infty, 4.0 + 7.0\} = 11.0$

vertex v	a	b	c	d	e	f
L(v)	0	1.0	3.0	7.0	4.0	11.0
T	{			d,		f}

Thus minimum label is $L(d) = 7.0$

<u>Iteration 5:</u> $u = e$ has $L(u) = 7.0$

T becomes $T - \{d\}$

There is one edge incident with d i.e. df where $f \in T$

$L(f) = \min \{\text{old } L(f), L(d) + w(df)\}$

$= \min \{11.0, 7.0 + 2.0\} = 9.0$

vertex	a	b	c	d	e	f
L(v)	0	1.0	3.0	7.0	4.0	9.0
T	{					f }

shortest path is <a-b-c-e-d-f>

shortest distance b/w a & f is 9.

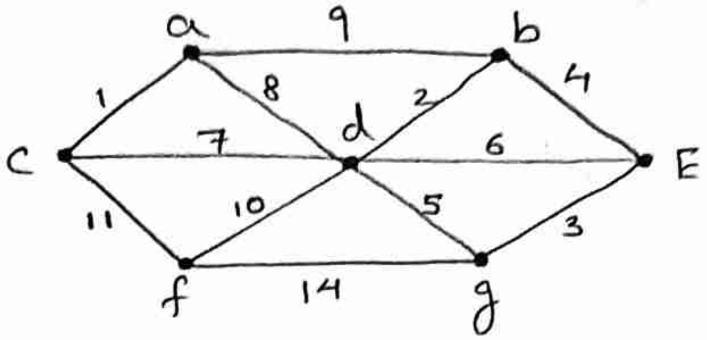
Q5>

Sol => In the mathematical field of graph theory, a spanning ~~th~~ tree T of an undirected graph G is a subgraph that is a tree which includes all of the vertices of G. In General, a graph may have several spanning trees, but a graph that is not connected will not contain a spanning tree.

~~Q2)~~

(ii) ~~Sol^n → Let A~~

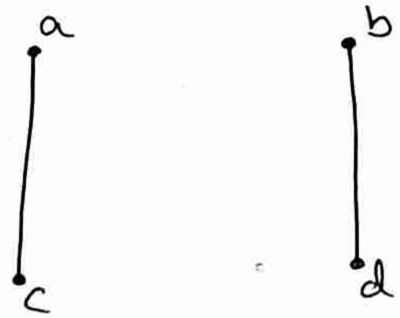
~~List~~



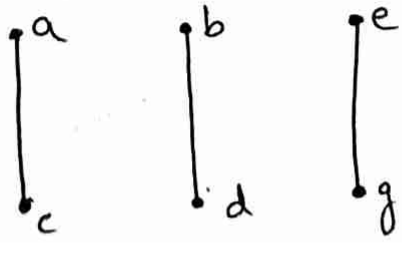
Edge	weight
(a, c)	1
(b, d)	2
(e, g)	3
(b, e)	4
(d, g)	5
(d, e)	6
(d, c)	7
(a, d)	8
(a, b)	9
(d, f)	10
(c, f)	11
(f, g)	14

Select the edge (a, c) since it has the smallest weight, include it in T.

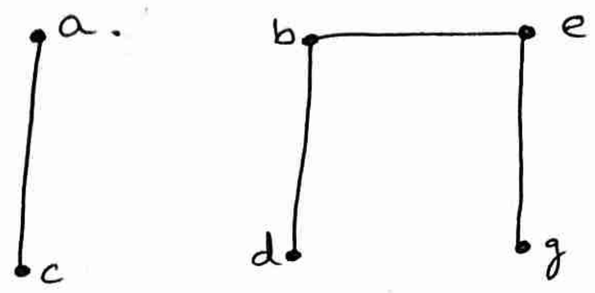
Select an edge with the next smallest weight (b, d) since it does not form cycle with the existing edges in T, so including it in T.



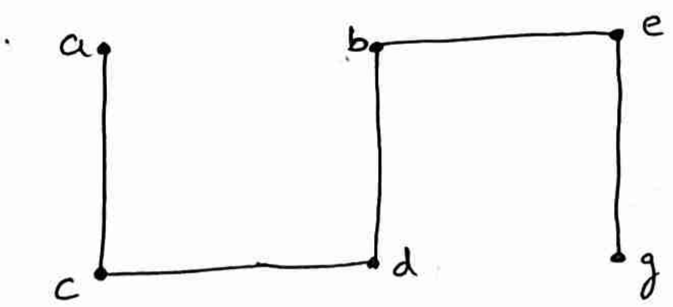
Select an edge with the next smallest weight (e, g) since it does not form cycle with the existing edges in T, so include it in T.



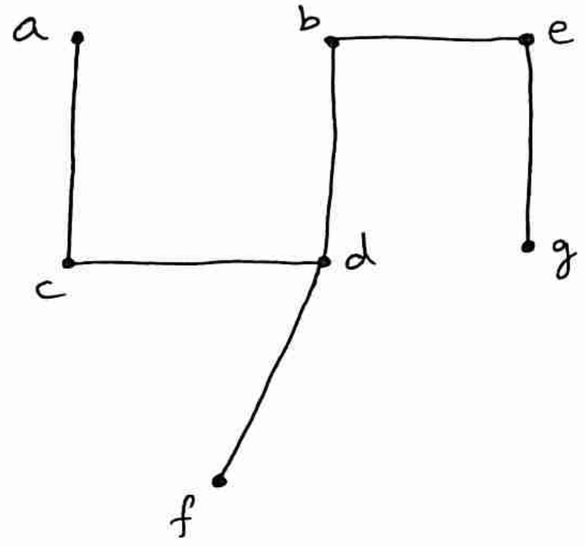
Select an edge with the next smallest weight (b, e) since it does not form cycle with the existing edges in T, so include it in T.



Select an edge with the next smallest weight (d, c) since it does not form cycle with the existing edges in T, so including it in T.



Select an edge with the next smallest weight (d, f) since it does not form cycle with the existing edges in T, so includ it in T.



Since G contains 7 vertices and have chosen 6 edges, the process terminants and the minimal spanning tree is produced.

Q 37

Sol^n →

$$a_{r+2} - 4a_{r+1} + 4a_r = r^2$$

Characteristics eq^n

$$\alpha^2 - 4\alpha + 4 = 0$$

$$\boxed{\alpha = 2, 2}$$

$$a_r^{(h)} = (c_1 + c_2 r) 2^r \quad — ②$$

Trial sol

$$a_r^{(p)} = (P_1 r^2 + P_2 r + P_2) \quad — ③$$

substitute in ①

$$[P_1 (r+2)^2 + P_2 (r+2) + P_3] - 4[P_1 (r+1)^2 + P_2(r+1) + P_3] + 4[P_1 r^2 + P_2 r + P_3] = r^2$$

$$P_1 [(r+2)^2 - 4(r+1)^2 + 4r^2] + P_2[(r+2) - 4(r+1) + 4r] + P_3 [1 - 4 + 4] = r^2$$

$$P_1 [r^2 - 4r] + P_2 [r - 2] + P_3 = r^2$$

equating coeff.

r^2, $\boxed{P_1 = 1}$

r, $-4P_1 + P_2 = 0$

$-4 + P_2 = 0$

$\boxed{P_2 = 4}$

const,

$$-2P_2 + P_3 = 0$$

$$-2(4) + P_3 = 0$$

$$\boxed{P_3 = 8}$$

from 3,

$$a_r^{(P)} = (r^2 + 4r + 8) \quad - 4$$

Total sol^n = $a_r^{(h)} + a_r^{(P)}$

$$= (C_1 + C_2 r)2^r + (r^2 + 4r + 8)$$