№3258

$\frac{\partial^6 u}{\partial x^3 \partial y^3}$; $u = x^3 \sin y + y^3 \sin x$

$$\frac{\partial^3 u}{\partial x^3} = 6\sin y + y^3 \sin\left(x + \frac{3\pi}{2}\right) =$$

$$= 6\sin y - y^3 \cos x$$

$$\frac{\partial^6 u}{\partial x^3 \partial y^3} = 6\sin\left(y + \frac{3\pi}{2}\right) - 6\cos x = -6(\cos y + \cos x)$$

№3256

$u = x - y + x^2 + 2xy + y^2 + x^3 - 3x^2 y - y^3 + x^4 - 4x^2 y^2 + y^4$

$\frac{\partial^4 u}{\partial x^4}$ - ? $\quad \frac{\partial u}{\partial x} = 1 + 2x + 2y + 3x^2 - 6xy + 4x^3 - 8xy^2$

$$\frac{\partial^2 u}{\partial x^2} = 2 + 6x - 6y + 12x^2 - 8y^2$$

$$\frac{\partial^3 u}{\partial x^3} = 6 + 24x$$

$$\frac{\partial^4 u}{\partial x^4} = 24$$

№3260

$u = e^{xyz}$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = \frac{\partial^2}{\partial y \partial z}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2}{\partial y \partial z}\left(yz e^{xyz}\right) =$$

$$= \frac{\partial}{\partial z}\left(z e^{xyz} + xyz^2 e^{xyz}\right) = e^{xyz} + xyz e^{xyz} +$$

$$+ 2xyz e^{xyz} + x^2 y^2 z^2 e^{xyz} = e^{xyz}(1 + 3xyz - y^2 x^2 z^2)$$

№3269

$u = x^3 + y^3 - 3xy(x-y)$ $\quad d^3u - ?$

$$d^3u = \left(\frac{\partial}{\partial x}dx + \frac{\partial}{\partial y}dy\right)^3 u = \frac{\partial^3 f}{\partial x^3}dx^3 + 3\frac{\partial^3 f}{\partial x^2 \partial y}dx^2 dy +$$

$$+ 3\frac{\partial^3 f}{\partial x \partial y^2}dx\,dy^2 + \frac{\partial^3 f}{\partial y^3} = 6dx^3 - 18dx^2 dy + 18dx\,dy^2 + 6dy^3 =$$

$$= 6(dx^3 - 3dx^2 dy + 3dx\,dy^2 + dy^3)$$

№3273

$u = xyz$, $d^3u - ?$

$$d^3u = \left(dx\frac{\partial}{\partial x} + dy\frac{\partial}{\partial y} + dz\frac{\partial}{\partial z}\right)u$$

$$\frac{\partial u}{\partial x} = yz \qquad \frac{\partial u}{\partial y} = xz \qquad \frac{\partial u}{\partial z} = yx$$

$$d^2x = d^2y = d^2z = 0$$

$$d^3u = d^3(xyz) = 3dx\,d^2(yz) = 6dx\,dy\,dz$$