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«AMALIY MATEMATIKA VA MATEMATIK FIZIKA» KAFEDRASI

ALGEBRA VA SONLAR NAZARIYASI

FANIDAN MUSTAQIL ISH UCHUN MASALALAR

(«Matematika», «Amaliy matematika va informatika» va «Mexanika» bakalavriat yo'nalishi talabalari uchun)

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1§. Chiziqli tenglamalar sistemasi. Gauss usuli.

1-21 chiziqli tenglamalar sistemasini Gauss usuli bilan yeching:

1. $\begin{cases} 2x+3y+4z=9, \\ 4y+11z=1, \\ 7x-5y=-1. \end{cases}$ **2.** $\begin{cases} 2x-y+5z=27, \\ 5x+2y+13z=70, \\ 3x-z=-2. \end{cases}$ **3.** $\begin{cases} 3x-5y+3z=46, \\ x+2y+z=8, \\ x-7y-2z=5. \end{cases}$

4. $\begin{cases} 4x+y-3z=-1, \\ 8x+3y-z=-1, \\ x+y-z=-1. \end{cases}$ **5.** $\begin{cases} x-4y-2z=0, \\ 3x-5y-6z=21, \\ 3x+y+z=-4. \end{cases}$ **6.** $\begin{cases} 4x-3y+z=43, \\ x+y-z=3, \\ 2x+y=13. \end{cases}$

7. $\begin{cases} 3x+y-2z=6, \\ 5x-3y+2z=-4, \\ 4x-2y-3z=-2. \end{cases}$ **8.** $\begin{cases} 3x+y+2z=11, \\ 2x+2y-3z=9, \\ x-5y-8z=23. \end{cases}$ **9.** $\begin{cases} 5x+6y-2z=12, \\ 2x+5y-3z=9, \\ 4x-3y+2z=-15. \end{cases}$

10. $\begin{cases} 2x+3y+4z=15, \\ x+y+5z=16, \\ 3x-2y+z=1. \end{cases}$ **11.** $\begin{cases} 2x-3y+z=11, \\ x+2y-z=-6, \\ x-4y-2z=3. \end{cases}$ **12.** $\begin{cases} 2x-5y=19, \\ 3x+5y-z=-10, \\ x-4y+2z=16. \end{cases}$

13. $\begin{cases} 3x+y-2z=6, \\ 2y-z=0, \\ 4x-3y+5z=19. \end{cases}$ **14.** $\begin{cases} 2x-3y-2z=16, \\ 3x+4y-5z=-10, \\ 2x-3z=4. \end{cases}$ **15.** $\begin{cases} 2x+4y-z=1, \\ 3x+5y-2z=3, \\ x-2y=-7. \end{cases}$

16. $\begin{cases} x-4y+3z=11, \\ x+y-z=-3, \\ 3x+4y-2z=-10. \end{cases}$ **17.** $\begin{cases} x+3y-z=-2, \\ 2x-y+4z=7, \\ x-3z=11. \end{cases}$ **18.** $\begin{cases} 5x-2y+3z=-7, \\ 2x+y-z=-5, \\ 3x+5y-z=38. \end{cases}$

19. $\begin{cases} 2x-y+5z=-4, \\ x+y+4z=4, \\ 2x+3y-z=14. \end{cases}$ **20.** $\begin{cases} x-3y=-10, \\ 4x+3z=-7, \\ 5x+2y-4z=38. \end{cases}$ **21.** $\begin{cases} x+4y-3z=-13, \\ 2x+y-z=0, \\ 4x+2y+5z=3. \end{cases}$

22-42 berilgan tenglamalar sistemasining birgalikda ekanligini tekshiring, agar birgalikda bo'lsa, ularni:
A) Kramer qoidasidan foydalanib,
B) Gauss usuli bilan yeching:

22. $\begin{cases} 3x+2y+z=5, \\ 2x+3y+z=1, \\ 2x+y+3z=11. \end{cases}$ **23.** $\begin{cases} 4x-3y+2z=9, \\ 2x+5y-3z=4, \\ 5x+6y+2z=18. \end{cases}$ **24.** $\begin{cases} 2x-y-z=4, \\ 3x+4y-2z=11, \\ 3x-2y+4z=11. \end{cases}$

25. $\begin{cases} x+y-z=1, \\ 8x+3y-6z=2, \\ -4x-y+3z=-3. \end{cases}$ **26.** $\begin{cases} 7x-5y=31, \\ 4x+11z=-43, \\ 2x+3y+4z=-20. \end{cases}$ **27.** $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=20, \\ 3x-2y-5z=6. \end{cases}$

28. $\begin{cases} x+y+3z=-1, \\ 2x-y+2z=-4, \\ 4x+y+4z=-2. \end{cases}$ **29.** $\begin{cases} 3x+4y+2z=8, \\ 2x-y-3z=-1, \\ x+5y+z=-7. \end{cases}$ **30.** $\begin{cases} x-4y-2z=-7, \\ 3x+y-z=5, \\ -3x+5y+6z=7. \end{cases}$

31. $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=20, \\ 3x-y+z=0. \end{cases}$ **32.** $\begin{cases} x+5y+z=-2, \\ 2x-4y-3z=0, \\ 3x+4y+2z=3. \end{cases}$ **33.** $\begin{cases} 2x-3y+2z=-6, \\ 5x+8y-z=0, \\ x+2y+3z=6. \end{cases}$

34. $\begin{cases} x-4y-2z=0, \\ 3x-5y-6z=7, \\ 3x+y+z=6. \end{cases}$ **35.** $\begin{cases} 2x-y+5z=10, \\ 5x+2y-13z=21, \\ 3x-y+5z=12. \end{cases}$ **36.** $\begin{cases} 2x+y-5z=-1, \\ x+y-z=-2, \\ 4x-3y+z=13. \end{cases}$

37. $\begin{cases} 2x+3y+4z=-10, \\ 4x+11z=-29, \\ 7x-5y=7. \end{cases}$ **38.** $\begin{cases} 2x+7y-z=10, \\ 3x-5y+3z=-14, \\ x+2y+z=-1. \end{cases}$ **39.** $\begin{cases} 4x+y-3z=-6, \\ 8x+3y-6z=-15, \\ x+y-z=-4. \end{cases}$

40. $\begin{cases} 3x-2y-5z=-14, \\ x-2y+3z=0, \\ 2x+3y-4z=-10. \end{cases}$ **41.** $\begin{cases} 5x+6y-2z=-9, \\ 2x+5y-3z=-1, \\ 4x-3y+2z=-15. \end{cases}$ **42.** $\begin{cases} 2x+y-z=9, \\ 2x-3y=0, \\ 5x-4y-2z=9. \end{cases}$

43-63 bir jinsli tenglamalar sistemasini yeching.

43. $\begin{cases} 3x+4y+2z=0, \\ x-y+4z=0, \\ 5x+2y+10z=0. \end{cases}$ **44.** $\begin{cases} 2x-3y+z=0, \\ x+y+z=0, \\ 3x-2y+2z=0. \end{cases}$ **45.** $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+3y-4z=0. \end{cases}$

46. $\begin{cases} 2x-y+3z=0, \\ x+2y-5z=0, \\ 3x+y-2z=0. \end{cases}$ **47.** $\begin{cases} 3x-2y+z=0, \\ 5x-14y+15z=0, \\ x+2y-3z=0. \end{cases}$ **48.** $\begin{cases} 3x+4y+2z=0, \\ x-y+4z=0, \\ 5x+2y+10z=0. \end{cases}$

49. $\begin{cases} 2x-3y+z=0, \\ x+y+z=0, \\ 3x-2y+2z=0. \end{cases}$ **50.** $\begin{cases} 3x+2y-z=0, \\ 2x-y+3z=0, \\ x+3y-4z=0. \end{cases}$ **51.** $\begin{cases} 2x-y+3z=0, \\ x+2y-5z=0, \\ 3x+y-2z=0. \end{cases}$

52. $\begin{cases} 3x-2y+z=0, \\ 4x+3y-5z=0, \\ x+5y-6z=0. \end{cases}$ **53.** $\begin{cases} 2x+2y-3z=0, \\ 4x-2y-9z=0, \\ x-2y+7z=0. \end{cases}$ **54.** $\begin{cases} 3x-3y+2z=0, \\ 4x+3y-5z=0, \\ x+5y-6z=0. \end{cases}$

55. $\begin{cases} 5x-2y+3z=0, \\ 2x+3y-4z=0, \\ 3x-5y+7z=0. \end{cases}$ **56.** $\begin{cases} 3x+y-z=0, \\ 5x+3y-z=0, \\ 4x+2y-z=0. \end{cases}$ **57.** $\begin{cases} x-2y-2z=0, \\ x+3y-5z=0, \\ 2x-y-7z=0. \end{cases}$

58. $\begin{cases} 2x-y-5z=0, \\ 5x+3y-z=0, \\ 4x+2y-z=0. \end{cases}$ **59.** $\begin{cases} 2x-y+4z=0, \\ 2x+3y-5z=0, \\ 4x+2y-z=0. \end{cases}$ **60.** $\begin{cases} 4x-3y-2z=0, \\ 2x-y+2z=0, \\ x-y-2z=0. \end{cases}$

61. $\begin{cases} 2x-3y-3z=0, \\ 2x+5y-z=0, \\ 2x+y-2z=0. \end{cases}$ **62.** $\begin{cases} 7x-y+2z=0, \\ x+3y-4z=0, \\ 3x-2y+3z=0. \end{cases}$ **63.** $\begin{cases} x-3y-2z=0, \\ 3x+y+4z=0, \\ 5x-y+z=0. \end{cases}$

2§. Matritsani pog'onali ko'rinishga keltirish. Matritsaning rangi va asosiy ustunlari.

64-68 misollarda matritsani pog'onali ko'rinishga keltiring.

64. $\begin{pmatrix} 1 & 0 & 2 & -1 \\ -2 & -1 & 0 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}$ **65.** $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 1 & 2 & 2 \end{pmatrix}$

66. $\begin{pmatrix} 1 & 2 & 0 & 1 \\ -2 & -1 & 0 & 0 \\ 0 & -1 & 1 & 2 \end{pmatrix}$ **67.** $\begin{pmatrix} 2 & -3 \\ 1 & 4 \\ 5 & 2 \end{pmatrix}$ **68.** $\begin{pmatrix} 5 & 4 \\ 1 & -3 \\ -2 & 6 \end{pmatrix}$

69-94. Quyida berilgan A matritsa rangini toping:

69. $\begin{pmatrix} 6 & -2 & 0 \\ 1 & 3 & 4 \\ 7 & -5 & 1 \end{pmatrix}$ **70.** $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 0 & 0 \\ 3 & -1 & -1 \\ 2 & 0 & 1 \end{pmatrix}$

71. $\begin{pmatrix} 1 & 0 & -1 & -1 \\ 2 & -1 & 1 & 2 \\ 1 & 0 & 4 & -1 \end{pmatrix}$ **72.** $\begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 1 \\ 5 & 2 & 1 \end{pmatrix}$

73. $\begin{pmatrix} 1 & 4 & -3 & 61 \\ 2 & 5 & 1 & -23 \\ 17 & -10 & 20 & 0 \end{pmatrix}$ **74.** $\begin{pmatrix} 2 & 1 & -2 & 3 \\ -3 & 0 & 1 & 1 \\ 5 & 1 & -3 & 2 \end{pmatrix}$ **75.** $\begin{pmatrix} 1 & -3 & -4 & 1 & 1 \\ 5 & -8 & -2 & 8 & 3 \\ -2 & -1 & -10 & -5 & 0 \end{pmatrix}$

76. $\begin{pmatrix} 4 & 2 & 1 & -2 \\ 2 & 0 & 3 & 6 \\ 3 & 1 & 2 & 2 \end{pmatrix}$ **77.** $\begin{pmatrix} 5 & -5 & 10 & 12 \\ 3 & 1 & 7 & 11 \\ 1 & 7 & 4 & 30 \end{pmatrix}$ **78.** $\begin{pmatrix} 1 & 2 & -1 & 2 \\ 3 & -4 & 5 & 0 \\ -1 & 3 & -3 & 1 \end{pmatrix}$

79. $\begin{pmatrix} 7 & 5 & 3 & 6 & 3 \\ 2 & -1 & -1 & 4 & 1 \\ 1 & 8 & 6 & -6 & 0 \end{pmatrix}$ **80.** $\begin{pmatrix} 5 & 0 & -4 & 2 \\ 3 & 2 & 2 & -6 \\ 4 & 1 & -1 & -2 \end{pmatrix}$ **81.** $\begin{pmatrix} 1 & 3 & -1 & 2 & 2 \\ 2 & 5 & -8 & -5 & 6 \\ 1 & 4 & 5 & -1 & 0 \end{pmatrix}$

82. $\begin{pmatrix} -1 & 2 & 3 & 4 \\ 2 & 0 & 1 & -1 \\ 0 & 4 & 7 & 7 \end{pmatrix}$ **83.** $\begin{pmatrix} 3 & 5 & -1 & 2 & 4 \\ 2 & 4 & -1 & 3 & 2 \\ 1 & 3 & -1 & 4 & 0 \end{pmatrix}$ **84.** $\begin{pmatrix} 3 & 4 & -4 & 0 \\ 1 & -1 & 2 & -1 \\ 5 & 2 & 0 & -2 \end{pmatrix}$

85. $\begin{pmatrix} 5 & -3 & 4 & 2 & 0 \\ 3 & 2 & -1 & 3 & 1 \\ 1 & 7 & -6 & 4 & 2 \end{pmatrix}$ **86.** $\begin{pmatrix} 4 & 1 & 2 & 0 \\ -1 & 2 & 1 & -1 \\ 5 & -1 & 1 & 1 \end{pmatrix}$ **87.** $\begin{pmatrix} 7 & 5 & -3 & 1 & 8 \\ 3 & 2 & -3 & 2 & 2 \\ 1 & 1 & 3 & -3 & 4 \end{pmatrix}$

88. $\begin{pmatrix} 3 & -2 & 2 & -1 \\ 5 & 2 & 4 & -1 \\ 4 & 0 & 3 & -1 \end{pmatrix}$ **89.** $\begin{pmatrix} 1 & 1 & -3 & 2 & 1 \\ 2 & -3 & 1 & -1 & 2 \\ 4 & -1 & -5 & 3 & 4 \end{pmatrix}$ **90.** $\begin{pmatrix} 2 & -3 & 1 & 1 \\ 3 & -1 & 2 & 4 \\ 1 & 2 & 1 & 3 \end{pmatrix}$

91. $\begin{pmatrix} 3 & -2 & 1 & -4 & 1 \\ 2 & -3 & -2 & 1 & 2 \\ 4 & -1 & 4 & -9 & 0 \end{pmatrix}$ **92.** $\begin{pmatrix} -2 & 1 & 3 & 0 \\ 4 & 3 & -5 & 2 \\ 1 & 2 & -1 & 1 \end{pmatrix}$ **93.** $\begin{pmatrix} -6 & 1 & 11 & 9 \\ 2 & 5 & 0 & 7 \\ 1 & -3 & 2 & 3 \end{pmatrix}$

94. $\begin{pmatrix} 4 & 3 & -2 & -1 \\ 2 & -1 & 4 & -3 \\ 3 & 1 & 1 & -2 \end{pmatrix}$

94-99 misollarda matritsani normal pog'onali ko'rinishga keltiring.

95. $\begin{pmatrix} 3 & 2 & 0 & 2 \\ -1 & 0 & -1 & -1 \\ 1 & -2 & 1 & 3 \end{pmatrix}$ **96.** $\begin{pmatrix} 3 & 0 & 2 & -1 \\ -2 & -1 & 0 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}$

97. $\begin{pmatrix} -1 & 0 & -2 \\ 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ **98.** $\begin{pmatrix} 3 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 2 & 0 \\ 2 & 1 & 2 \end{pmatrix}$

99. $\begin{pmatrix} 2 & -1 & 3 & 0 \\ 0 & 1 & 2 & 1 \\ 3 & 2 & -1 & 1 \end{pmatrix}$

100-102 misollarda matritsaning asosiy ustunlarini toping.

100. $\begin{pmatrix} 1 & -2 & 3 & 1 \\ -1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{pmatrix}$ **101.** $\begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -1 & 1 & 2 \\ 2 & 1 & 0 & 2 \\ -1 & 0 & 1 & 0 \end{pmatrix}$

102. $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 0 \\ 2 & -1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$

3§. Chiziqli tenglamalar sistemasi. Kroneker-Kapelli teoremasi.

Teorema (Kroneker-Kapelli). n ta noma'lumli m ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

birgalikda bo'lishi uchun

$$rang\, A = rang\, \overline{A}$$

bo'lishi zarur va yetarlidir.

Bu yerda

$$A=\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

sistemaning asosiy matritsasi,

$$\overline{A}=\begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} & b_1 \\ a_{21} & a_{22} & . & . & . & a_{2n} & b_2 \\ . & . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} & b_m \end{pmatrix}$$

sistemaning kengaytirilgan matritsasi.

103-107 misollarda berilgan chiziqli tenglamalar sistemasining kengaytirilgan matritsasini normal pog'onali ko'rinishga keltirish usuli bilan yechimini toping.

103. a) $\begin{cases} x_1+2x_2+x_3+x_4=4 \\ 3x_1+6x_2+x_3+x_4=2 \end{cases}$ b) $\begin{cases} x_1+2x_2-x_3+x_4=1 \\ 2x_1+5x_2+x_3+2x_4=2 \end{cases}$

c) $\begin{cases} x_1+x_2+x_3=0 \\ 2x_1+x_2-x_3=-4 \end{cases}$ d) $\begin{cases} x_1+2x_2+x_3+5x_4=2 \\ 2x_1+4x_2-x_3+x_4=1 \end{cases}$

104. $\begin{cases} x_1+x_2-2x_3+2x_4=0 \\ 2x_1+3x_2-x_3+x_4=0 \end{cases}$

105. a) $\begin{cases} x_1+2x_2-4x_3=1, \\ 2x_1+x_2-5x_3=-1, \\ x_1-x_2-x_3=-2. \end{cases}$ b) $\begin{cases} 3x_1-5x_2+2x_3+4x_4=2, \\ 7x_1-4x_2+x_3+3x_4=5, \\ 5x_1+7x_2-4x_3-6x_4=3. \end{cases}$

106. a) $\begin{cases} 2x_1+x_2-x_3-3x_4=2, \\ 4x_1+x_3-7x_4=3, \\ 2x_2-3x_3+x_4=1, \\ 2x_1+3x_2-4x_3-2x_4=3 \end{cases}$ b) $\begin{cases} 3x_1-2x_2-5x_3+x_4=3, \\ 2x_1-3x_2+x_3+5x_4=-3, \\ x_1+2x_2-4x_4=-3, \\ x_1-x_2-4x_3+9x_4=22 \end{cases}$

107. a) $\begin{cases} 9x_1 - 3x_2 + 5x_3 + 6x_4 = 4, \\ 6x_1 - 2x_2 + 3x_3 + 4x_4 = 5, \\ 3x_1 - x_2 + 3x_3 + 14x_4 = -8. \end{cases}$ b) $\begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1, \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2, \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4. \end{cases}$

108-110 misollarda berilgan bir jinsli tenglamalar sistemasining yechimini va fundamental yechimlar sistemasini toping.

108. a) $\begin{cases} x_1 + 2x_2 - x_3 = 0, \\ 2x_1 + 9x_2 - 3x_3 = 0. \end{cases}$ b) $\begin{cases} x_1 - 2x_2 - 3x_3 = 0, \\ -2x_1 + 4x_2 + 6x_3 = 0. \end{cases}$

109. a) $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 0 \\ -2x_1 + 3x_2 - 2x_3 - x_4 = 0 \\ 2x_2 - x_3 + 7x_4 = 0 \end{cases}$ b) $\begin{cases} x_1 + x_2 - x_3 + x_4 = 0 \\ -2x_1 - 4x_2 + 2x_3 - 2x_4 = 0 \\ -x_2 - x_3 + 2x_4 = 0 \end{cases}$

c) $\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 4x_1 + 5x_2 + 6x_3 = 0 \\ 7x_1 + 8x_2 + 9x_3 = 0 \end{cases}$ d) $\begin{cases} 3x_1 + 2x_2 + x_3 = 0, \\ 2x_1 + 5x_2 + 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 0. \end{cases}$

e) $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 0, \\ x_1 + x_2 + x_3 = 0, \\ 3x_1 - 2x_2 + 2x_3 = 0. \end{cases}$ f) $\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0, \\ 3x_1 - 6x_2 + 4x_3 + 2x_4 = 0, \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 = 0. \end{cases}$

j) $\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0, \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0, \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0, \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0 \end{cases}$

110. Quyida berilgan bir jinslimas sistemaning yechimini unga mos bir jinslimas sistemaning fundamental yechimlari sistemasidan foydalanib toping:

a) $\begin{cases} 2x_1 + x_2 - x_3 - x_4 + x_5 = 1, \\ x_1 - x_2 + x_3 + x_4 - 2x_5 = 0, \\ 3x_1 + 3x_2 - 3x_3 - 3x_4 + 4x_5 = 2, \\ 4x_1 + 5x_2 - 5x_3 - 5x_4 + 7x_5 = 3. \end{cases}$ **b)** $\begin{cases} x_1 - x_2 + x_3 - x_4 + x_5 - x_6 = 1, \\ 2x_1 - 2x_2 + 2x_3 + x_4 - x_5 + x_6 = 1. \end{cases}$

c) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 0, \\ x_1 - 2x_2 - 3x_3 - 4x_4 - 5x_5 = 2, \\ 2x_2 + 3x_3 + 4x_4 + 5x_5 = -1. \end{cases}$

111-114 misollarda shunday bir jinsli chiziqli tenglamalar sistemasini tuzingki, berilgan vektorlar uning yechimlari bo'lsin va to'zilgan sistema iloji boricha sodda ko'rinishga ega bo'lsin.

111. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ **112.** $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

113. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ **114.** $\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

115-117 misollarda shunday bir jinsli bo'lmagan chiziqli tenglamalar sistemasini tuzingki, matritsasining rangi 2 ga teng bo'lib, berilgan vektorlar uning echimlari bo'lsin.

115. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$ **116.** $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

117. $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$

4§. Determinantni hisoblash.

118-132. Quyidagi ikkinchi tartibli determinantni hisoblang:

118. $\begin{vmatrix} -5 & -12 \\ 8 & 46 \end{vmatrix}$ **119.** $\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix}$ **120.** $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ **121.** $\begin{vmatrix} 6 & 9 \\ 8 & 12 \end{vmatrix}$

122. $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$ **123.** $\begin{vmatrix} a^2+b^2 & a^2 \\ a^2 & a^2-b^2 \end{vmatrix}$ **124.** $\begin{vmatrix} x+2b & x-2b \\ x-2b & x+2b \end{vmatrix}$

125. $\begin{vmatrix} y^2+3y+9 & y^2-3y+9 \\ y+3 & y-3 \end{vmatrix}$ **126.** $\begin{vmatrix} \cos x & -3\sin x \\ \sin x & -3\cos x \end{vmatrix}$

127. $\begin{vmatrix} \sin\beta & \cos\beta \\ \cos\alpha & \sin\alpha \end{vmatrix}$

128. $\begin{vmatrix} \sin\alpha+\sin\beta & \cos\beta+\cos\alpha \\ \cos\beta-\cos\alpha & \sin\alpha-\sin\beta \end{vmatrix}$

129. $\begin{vmatrix} 2\sin\varphi\cos\varphi & 2\sin^2\varphi-1 \\ 2\cos^2\varphi-1 & 2\sin\varphi\cos\varphi \end{vmatrix}$

130. $\begin{vmatrix} \dfrac{1-t^2}{1+t^2} & \dfrac{2t}{1+t^2} \\ \dfrac{-2t}{1+t^2} & \dfrac{1-t^2}{1+t^2} \end{vmatrix}$

131. $\begin{vmatrix} \dfrac{(1-t)^2}{1+t^2} & \dfrac{2t}{1+t^2} \\ \dfrac{2t}{1+t^2} & -\dfrac{(1+t)^2}{1+t^2} \end{vmatrix}$

132. $\begin{vmatrix} \dfrac{1+t^2}{1-t^2} & \dfrac{2t}{1-t^2} \\ \dfrac{2t}{1-t^2} & \dfrac{1+t^2}{1-t^2} \end{vmatrix}$

133-154. Quyidagi uchinchi tartibli determinantni hisoblang:

133. $\begin{vmatrix} 3 & 6 & 3 \\ 21 & 32 & 8 \\ 5 & 2 & -4 \end{vmatrix}$

134. $\begin{vmatrix} -9 & 13 & 7 \\ -1 & 5 & -5 \\ 18 & -7 & -10 \end{vmatrix}$

135. $\begin{vmatrix} -5 & -3 & 1 \\ -9 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix}$

136. $\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -4 \\ 5 & -4 & 6 \end{vmatrix}$

137. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$

138. $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$

139. $\begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}$

140. $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$

141. $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}$

142. $\begin{vmatrix} 4 & 2 & -1 \\ 5 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}$

143. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$

144. $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

145. $\begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{vmatrix}$

146. $\begin{vmatrix} 2 & 0 & 3 \\ 0 & 1 & 6 \\ 3 & 0 & 5 \end{vmatrix}$

147. $\begin{vmatrix} 1 & 5 & 25 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \end{vmatrix}$

148. $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 9 \\ 16 & 25 & 81 \end{vmatrix}$

149. $\begin{vmatrix} 2 & 0 & 4 \\ -1 & 5 & -3 \\ 3 & 4 & 2 \end{vmatrix}$

150. $\begin{vmatrix} 9 & 0 & 8 \\ 2 & -5 & 5 \\ 5 & -4 & -3 \end{vmatrix}$

151. $\begin{vmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon^2 & 1 & \varepsilon \\ \varepsilon & \varepsilon^2 & 1 \end{vmatrix}$, *bu yerda* $\varepsilon = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$.

152. $\begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix}$, *bu yerda* $\varepsilon = \cos\dfrac{2}{3}\pi + i\sin\dfrac{2}{3}\pi$.

153. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \varepsilon & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon \end{vmatrix}$, *bu yerda* $\varepsilon = \cos\dfrac{4}{3}\pi + i\sin\dfrac{4}{3}\pi$.

154. a) $\begin{vmatrix} 1 & 0 & 1+i \\ 0 & 1 & i \\ 1-i & -i & 1 \end{vmatrix}$ **b)** $\begin{vmatrix} \cos\alpha & \sin\alpha\cos\beta & \sin\alpha\sin\beta \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{vmatrix}$

155-165 oldin biror qator elementlarining bittasidan boshqasini nollarga aylantirib, determinantni tartibini pasaytirish usuli bilan hisoblang.

155. $\begin{vmatrix} -4 & 2 & 3 & 0 \\ -1 & -3 & 4 & -2 \\ 2 & 4 & -1 & 3 \\ 0 & -5 & 2 & 1 \end{vmatrix}$ **156.** $\begin{vmatrix} 2 & -2 & 0 & 5 \\ 4 & 1 & 1 & -1 \\ 2 & -3 & 4 & -3 \\ 1 & 2 & 3 & -5 \end{vmatrix}$

157. $\begin{vmatrix} 0 & 3 & 2 & -4 \\ -2 & 4 & -3 & -1 \\ 1 & 3 & -2 & 0 \\ 3 & -1 & 4 & 2 \end{vmatrix}$ **158.** $\begin{vmatrix} 2 & -1 & 0 & 5 \\ -1 & -3 & 2 & -4 \\ 4 & 2 & -1 & 3 \\ 3 & 0 & -4 & -2 \end{vmatrix}$

159. $\begin{vmatrix} 1 & -2 & 4 & -3 \\ -4 & 1 & -1 & 2 \\ 0 & 5 & -3 & -4 \\ -3 & 2 & 2 & -1 \end{vmatrix}$ **160.** $\begin{vmatrix} 0 & 4 & 2 & 1 \\ 1 & 0 & 3 & 2 \\ 3 & -3 & -1 & 4 \\ -2 & 2 & 4 & 3 \end{vmatrix}$

161. $\begin{vmatrix} 4 & 2 & 0 & -3 \\ -1 & 3 & 2 & 4 \\ -2 & 4 & 3 & 1 \\ 0 & 5 & -1 & 2 \end{vmatrix}$ **162.** $\begin{vmatrix} 1 & 7 & -1 & 0 \\ 2 & 6 & 2 & -1 \\ 1 & -3 & 4 & 0 \\ 4 & 5 & 1 & 3 \end{vmatrix}$

163. $\begin{vmatrix} 1 & 3 & -2 & 0 \\ -3 & 1 & -4 & -2 \\ 0 & 6 & 4 & -8 \\ -2 & 4 & -3 & -1 \end{vmatrix}$ **164.** $\begin{vmatrix} 0 & -1 & 1 & -4 \\ 2 & 5 & 0 & 6 \\ 1 & -1 & 2 & -1 \\ 4 & 2 & 1 & 0 \end{vmatrix}$

165-175 quyida berilgan determinantlarni uchburchak usulidan foydalanib hisoblang.

165. $\begin{vmatrix} 8 & 5 & -1 & 1 \\ 5 & 3 & 1 & 1 \\ 0 & 4 & -7 & -6 \\ 3 & 2 & -1 & 0 \end{vmatrix}$ **166.** $\begin{vmatrix} 1 & -4 & 0 & 3 \\ -4 & 3 & 2 & -3 \\ -2 & 3 & -1 & 4 \\ 3 & 2 & 5 & 0 \end{vmatrix}$

167. $\begin{vmatrix} 1 & 2 & -1 & 4 \\ -2 & 1 & 3 & -5 \\ 3 & 7 & 5 & 1 \\ 5 & 3 & -1 & 2 \end{vmatrix}$ **168.** $\begin{vmatrix} 1 & 4 & -4 & 2 \\ 3 & 1 & 2 & 1 \\ -4 & -3 & 4 & 2 \\ 1 & -5 & 3 & 0 \end{vmatrix}$

169. $\begin{vmatrix} -4 & 3 & 2 & 3 \\ 1 & -2 & 3 & -4 \\ 3 & 4 & 0 & -3 \\ 0 & -1 & -2 & 5 \end{vmatrix}$ **170.** $\begin{vmatrix} 3 & 0 & 1 & -3 \\ 1 & 7 & 1 & 3 \\ 2 & -1 & 0 & 2 \\ -2 & 3 & 7 & 1 \end{vmatrix}$

171. $\begin{vmatrix} 2 & -1 & -4 & 4 \\ -9 & 3 & 2 & -7 \\ -1 & 0 & 4 & 5 \\ 6 & 4 & 7 & -4 \end{vmatrix}$ **172.** $\begin{vmatrix} 1 & -4 & 0 & 3 \\ -4 & 3 & 2 & -3 \\ -2 & 3 & -1 & 4 \\ 3 & 2 & 5 & 0 \end{vmatrix}$

173. $\begin{vmatrix} 5 & -8 & -4 & 7 \\ 0 & -5 & 4 & 1 \\ 2 & -1 & -3 & -2 \\ 1 & 5 & -5 & -1 \end{vmatrix}$ **174.** $\begin{vmatrix} 3 & -1 & 0 & 3 \\ 5 & 1 & 4 & -7 \\ 5 & -1 & 0 & 2 \\ 1 & -8 & 5 & 3 \end{vmatrix}$

175. $\begin{vmatrix} 2 & 3 & -1 & 0 \\ -4 & -1 & -3 & 3 \\ 3 & 4 & 2 & -2 \\ 1 & 2 & 0 & 4 \end{vmatrix}$

176-186 berilgan determinantni uch usul bilan hisoblang:
a) uni i-satr elementlari bo'yicha yoying.
b) uni j-ustun elementlari bo'yicha yoying.
v) oldin j-ustundagi bittadan boshqa elementlarni nolga aylantirib, so'ngra shu ustun elementlari bo'yicha yoying.

176. $\begin{vmatrix} 2 & -1 & 3 & 0 \\ -5 & 3 & 1 & -2 \\ -3 & -2 & 0 & 4 \\ 1 & 5 & -4 & 2 \end{vmatrix}$ $i=2,\ j=4$

177. $\begin{vmatrix} 6 & -2 & 1 & 8 \\ -6 & 1 & 4 & 5 \\ 2 & 0 & 3 & -3 \\ 4 & 5 & -3 & 1 \end{vmatrix}$ $i=2,\ j=2$

178. $\begin{vmatrix} 2 & -1 & 3 & 2 \\ 9 & 2 & 5 & 0 \\ 2 & 4 & -6 & 3 \\ 0 & 3 & 2 & -1 \end{vmatrix}$ $i=3,\ j=4$

179. $\begin{vmatrix} 3 & -2 & 1 & 4 \\ 0 & 5 & -3 & 2 \\ -4 & 0 & 3 & -1 \\ 6 & 3 & -2 & 0 \end{vmatrix}$ $i=4,\ j=3$

180. $\begin{vmatrix} 3 & -3 & 1 & 2 \\ -4 & 0 & 5 & 7 \\ -2 & -1 & 3 & 4 \\ 0 & 6 & -2 & 3 \end{vmatrix}$ $i=1,\ j=3$

181. $\begin{vmatrix} 2 & 0 & -2 & 1 \\ 4 & 5 & -1 & 0 \\ 3 & -4 & 2 & 5 \\ -3 & 1 & 3 & 6 \end{vmatrix}$ $i=2,\ j=1$

182. $\begin{vmatrix} 6 & -3 & 1 & 2 \\ -1 & 0 & 4 & 5 \\ 2 & 7 & 3 & 4 \\ 0 & -5 & -1 & 3 \end{vmatrix}$

$i=3,\ j=3$

183. $\begin{vmatrix} -3 & 2 & 5 & 7 \\ 1 & -3 & 2 & 0 \\ 4 & 0 & -2 & 1 \\ -4 & 1 & 2 & 3 \end{vmatrix}$

$i=4,\ j=4$

184. $\begin{vmatrix} 1 & -1 & 0 & 2 \\ -2 & 3 & 1 & 5 \\ 4 & -1 & 2 & 4 \\ 6 & 0 & 3 & -2 \end{vmatrix}$

$i=3,\ j=1$

185. $\begin{vmatrix} -3 & 5 & 6 & 7 \\ 0 & -2 & 1 & 3 \\ 4 & -3 & 1 & 0 \\ 2 & 1 & 0 & -3 \end{vmatrix}$

$i=1,\ j=2$

186-196 quyida berilgan determinantlarni Laplas teoremasidan foydalanib hisoblang:

186. $\begin{vmatrix} 1 & -4 & 1 & 7 \\ 0 & 2 & -2 & 3 \\ 1 & -1 & 2 & 0 \\ 4 & 1 & 3 & -6 \end{vmatrix}$

187. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ -5 & 3 & 0 & 2 \\ -2 & 1 & -3 & 4 \\ 5 & 2 & -1 & 0 \end{vmatrix}$

188. $\begin{vmatrix} -3 & 1 & 4 & 5 \\ 0 & 2 & -1 & 3 \\ 4 & -2 & 7 & 0 \\ -5 & 0 & 3 & 2 \end{vmatrix}$

189. $\begin{vmatrix} 3 & -2 & 9 & 0 \\ 4 & -3 & 1 & -2 \\ 2 & 3 & -2 & 6 \\ 5 & 7 & -3 & 1 \end{vmatrix}$

190. $\begin{vmatrix} 4 & -5 & 1 & 0 \\ 3 & 2 & 6 & 4 \\ -5 & 3 & -4 & 8 \\ 0 & -2 & -1 & 3 \end{vmatrix}$

191. $\begin{vmatrix} 4 & -1 & 3 & 2 \\ -5 & 5 & -3 & 0 \\ 2 & 1 & 4 & 7 \\ 0 & -2 & 6 & 3 \end{vmatrix}$

192. $\begin{vmatrix} 4 & 3 & -1 & -2 \\ 6 & 0 & 3 & 4 \\ 1 & -2 & 2 & 3 \\ 5 & -2 & 1 & 4 \end{vmatrix}$

$i=1,\ j=1$

193. $\begin{vmatrix} 6 & -1 & 3 & 0 \\ -2 & 3 & 4 & -5 \\ 2 & 0 & -3 & 1 \\ -5 & 5 & 8 & 2 \end{vmatrix}$

$i=4,\ j=4$

194. $\begin{vmatrix} 1 & -1 & 2 & 3 \\ 4 & 0 & 3 & -2 \\ 3 & 1 & -2 & 5 \\ -3 & 2 & 7 & 0 \end{vmatrix}$

$i=4,\ j=1$

195. $\begin{vmatrix} 3 & -5 & 5 & 1 \\ 7 & 1 & 2 & 3 \\ 2 & 1 & 8 & 0 \\ 1 & -7 & 3 & 2 \end{vmatrix}$

196. $\begin{vmatrix} 2 & 6 & -10 & 3 \\ -1 & 3 & 2 & 7 \\ 5 & 1 & 0 & 3 \\ -2 & 0 & 3 & 6 \end{vmatrix}$

5§. Matritsalarni transponerlash, qo'shish, ko'paytirish, darajaga ko'tarish va teskarisini topish. Determinantlarni ko'paytirish.

197-207 misollarda berilgan matritsalarni ko'paytiring.

197. $A=\begin{pmatrix} 2 & 1 \\ -6 & 3 \\ 1 & 4 \end{pmatrix}, B=\begin{pmatrix} 2 & 6 & -3 \\ 1 & 5 & 4 \end{pmatrix}$

198. $A=\begin{pmatrix} 1 & 3 & -2 \\ -4 & 1 & 7 \end{pmatrix}, B=\begin{pmatrix} 2 & -4 \\ 1 & 0 \\ 3 & -1 \end{pmatrix}$

199. $A=\begin{pmatrix} 1 & 1 & -2 \\ 3 & 0 & -1 \end{pmatrix}, B=\begin{pmatrix} 1 & -2 & 4 \\ 3 & -1 & 0 \\ 2 & -1 & -4 \end{pmatrix}$

200. $A=\begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & -2 \\ 3 & 1 & -1 \end{pmatrix}, B=\begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 4 \\ -1 & 3 & -2 \end{pmatrix}$

201. $A=\begin{pmatrix} 2 & 1 & 0 \end{pmatrix}, B=\begin{pmatrix} 1 & 0 \\ -2 & -1 \\ 3 & 2 \end{pmatrix}$

202. $A=\begin{pmatrix} 1 & -1 & 3 \end{pmatrix}, B=\begin{pmatrix} 1 & 0 & 4 \\ -1 & 3 & 1 \\ 2 & -2 & 0 \end{pmatrix}$

203. $A=\begin{pmatrix} 1 & 5 & 0 \\ -1 & 2 & -3 \\ 4 & 1 & 2 \end{pmatrix}, B=\begin{pmatrix} 1 \\ -1 \\ 6 \end{pmatrix}$

204. $A=\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -2 \end{pmatrix}, B=\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$

205. $A=\begin{pmatrix} 1 & -5 & 2 \end{pmatrix}, B=\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

206. $A=\begin{pmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \end{pmatrix}, B=\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$

207. $A=\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}, B=\begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$

208. $A=\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, B=\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ bo'lsa, $S = AB\text{-}BA$ ni toping.

209. $A=\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}, B=\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$ bo'lsa, $A=C^2-B^2$ ni toping.

210. $A=\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, B=\begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix}$ bo'lsa, $S= AB\text{-}BA$ ni toping.

211. $A=\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}, B=\begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & 3 \end{pmatrix}$ bo'lsa, $S= B\text{-}AB$ ni toping.

212. $A=\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, B=\begin{pmatrix} 1 & 2 \\ -3 & 1 \\ 0 & -1 \end{pmatrix}$ bo'lsa, $C=2AB+(B^T A^T)^T$ ni toping.

213. $A=\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}, B=\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$ bo'lsa, $C=A^T B-B^T A$ ni toping.

214. $A=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 2 & 1 \end{pmatrix}, B=\begin{pmatrix} 2 & 4 \\ 1 & 0 \\ -1 & 3 \end{pmatrix}$ bo'lsa, $C=(AB)^T-B^T A^T$ ni toping.

215. $A=\begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \end{pmatrix}, B=\begin{pmatrix} 2 & 3 \\ -1 & 1 \\ -3 & 0 \end{pmatrix}$ bo'lsa, $C=B^T A^T$ ni toping.

216. $A=\begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & 1 & 1 \end{pmatrix}, B=\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{pmatrix}$ bo'lsa, $C=B^T A-(AB)^T$ ni toping.

217. $A=\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}, B=\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$ bo'lsa, $C=AB+2B$ ni toping.

218. $A=\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}, B=\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$ bo'lsa, $C=A^2+B^2$ ni toping.

219-240. Quyida A va B matritsalar berilgan. AB va BA matritsalar ko’paytmalarini toping.

219. $A=\begin{pmatrix} 2 & 19 & 30 \\ 0 & -5 & -12 \\ 0 & 2 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 3 & 5 \\ 6 & 7 & 1 \\ 9 & 1 & 8 \end{pmatrix}$

220. $A=\begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 3 & -2 \\ -4 & 1 & 2 \\ 3 & -4 & 4 \end{pmatrix}$

221. $A=\begin{pmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -1 & -1 \\ -1 & 4 & 7 \\ 3 & 1 & -1 \end{pmatrix}$

222. $A=\begin{pmatrix} 4 & -5 & 7 \\ 1 & -4 & 9 \\ -4 & 0 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 9 & 3 & 5 \\ 2 & 0 & 3 \\ 0 & 1 & -1 \end{pmatrix}$

223. $A=\begin{pmatrix} 5 & 6 & -3 \\ -1 & 0 & 1 \\ 1 & 2 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 2 & 0 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix}$

224. $A=\begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$, $B=\begin{pmatrix} -3 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix}$

225. $A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$, $B=\begin{pmatrix} 7 & 0 & 4 \\ 0 & 4 & -9 \\ 3 & 1 & 0 \end{pmatrix}$

226. $A=\begin{pmatrix} 7 & -12 & 6 \\ 10 & -19 & 10 \\ 12 & -24 & 13 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 1 & -6 \\ 3 & 0 & 7 \\ 1 & 1 & -1 \end{pmatrix}$

227. $A=\begin{pmatrix} 3 & -1 & 0 \\ -4 & -1 & 0 \\ 4 & -8 & -2 \end{pmatrix}$, $B=\begin{pmatrix} 3 & -1 & 5 \\ 1 & 2 & 4 \\ 3 & 2 & -1 \end{pmatrix}$

228. $A=\begin{pmatrix} 1 & -3 & 4 \\ 4 & -7 & 8 \\ 6 & -7 & 7 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & 1 \end{pmatrix}$

229. $A=\begin{pmatrix} 0 & 7 & 4 \\ 0 & 1 & 0 \\ 1 & 13 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 3 & 2 \\ -2 & 1 & -1 \\ 3 & 1 & 1 \end{pmatrix}$

230. $A=\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 2 & 1 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$

231. $A=\begin{pmatrix} -2 & 1 & 0 \\ 0 & 3 & 4 \\ 1 & -1 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 3 & 8 \\ 6 & 9 & 1 \\ 2 & 1 & 8 \end{pmatrix}$

232. $A=\begin{pmatrix} 5 & 2 & 1 \\ 1 & -1 & 0 \\ 0 & 3 & -1 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 8 & -2 \\ -4 & 3 & 2 \\ 3 & -8 & 5 \end{pmatrix}$

233. $A=\begin{pmatrix} 0 & -2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 1 & -3 & 4 \\ 2 & 1 & -5 \\ -3 & 5 & 1 \end{pmatrix}$

234. $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 2 & 1 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 5 & -3 \\ 1 & -1 & -1 \\ 7 & 0 & 4 \end{pmatrix}$

235. $A=\begin{pmatrix} -1 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 3 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 3 & -6 \end{pmatrix}$

236. $A=\begin{pmatrix} 1 & -1 & 1 \\ -2 & 1 & 0 \\ 0 & 3 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -1 & -5 \\ 7 & 1 & 4 \\ 6 & 4 & -7 \end{pmatrix}$

237. $A=\begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & -1 \\ 5 & 2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 1 & 0 \\ 1 & -2 & -1 \\ 0 & 3 & 2 \end{pmatrix}$

238. $A=\begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 5 \\ 2 & 1 & -3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 2 & 2 \\ 0 & -3 & 1 \\ 2 & 0 & 3 \end{pmatrix}$

239. $A=\begin{pmatrix} 1 & 3 & -1 \\ 2 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 5 & -1 & 3 \\ 1 & -2 & 0 \\ 0 & 7 & -1 \end{pmatrix}$

240. $A=\begin{pmatrix}1 & -3 & 1\\ 4 & 2 & -1\\ 1 & 0 & 2\end{pmatrix},\ B=\begin{pmatrix}2 & 0 & 1\\ 0 & 1 & -5\\ 2 & 0 & 0\end{pmatrix}$

241-245 misollarda berilgan matritsaga teskari matritsani toping.

241. $A=\begin{pmatrix}3 & 5\\ 2 & 1\end{pmatrix}$ **242.** $A=\begin{pmatrix}0 & 0 & 3\\ 0 & 2 & 0\\ 1 & 0 & 0\end{pmatrix}$

243. $A=\begin{pmatrix}6 & -5\\ 7 & -6\end{pmatrix}$ **244.** $A=\begin{pmatrix}2 & 3\\ -3 & 2\end{pmatrix}$ **245.** $A=\begin{pmatrix}2 & 3\\ 3 & -2\end{pmatrix}$

246-267 A matritsa berilgan. A^{-1} teskari matritsani toping va $AA^{-1}=A^{-1}A=E$ ekanini tekshiring.

246. $\begin{pmatrix}1 & 2 & -1\\ 1 & -2 & 3\\ 4 & 1 & -4\end{pmatrix}$ **247.** $\begin{pmatrix}3 & 2 & -1\\ 7 & 3 & 0\\ 1 & 2 & 2\end{pmatrix}$ **248.** $\begin{pmatrix}1 & -3 & 5\\ 2 & 4 & 0\\ 3 & -3 & -1\end{pmatrix}$

249. $\begin{pmatrix}-2 & 3 & 3\\ 4 & 5 & 1\\ -3 & 4 & 0\end{pmatrix}$ **250.** $\begin{pmatrix}0 & 1 & -3\\ 1 & -5 & 4\\ 2 & 3 & 2\end{pmatrix}$ **251.** $\begin{pmatrix}1 & 1 & 3\\ 2 & -2 & -5\\ 1 & 4 & 3\end{pmatrix}$

252. $\begin{pmatrix}-5 & 7 & -4\\ 8 & 0 & -1\\ 4 & -5 & 0\end{pmatrix}$ **253.** $\begin{pmatrix}-1 & 8 & 1\\ -1 & 5 & 5\\ 0 & -1 & 3\end{pmatrix}$ **254.** $\begin{pmatrix}4 & -2 & 1\\ -3 & 4 & 1\\ 1 & -1 & 1\end{pmatrix}$

255. $\begin{pmatrix}3 & -3 & 4\\ -1 & -5 & -7\\ 0 & -1 & 5\end{pmatrix}$ **256.** $\begin{pmatrix}3 & -1 & 4\\ 7 & 8 & -2\\ 2 & -3 & 3\end{pmatrix}$ **257.** $\begin{pmatrix}1 & -1 & 8\\ 1 & -5 & 5\\ -2 & 3 & 10\end{pmatrix}$

258. $\begin{pmatrix}2 & 3 & 4\\ 2 & 1 & 3\\ -7 & 0 & 2\end{pmatrix}$ **259.** $\begin{pmatrix}5 & -1 & 3\\ 4 & -2 & 0\\ 2 & -4 & 5\end{pmatrix}$ **260.** $\begin{pmatrix}1 & -3 & -2\\ -2 & 1 & 3\\ -2 & 4 & 4\end{pmatrix}$

261. $\begin{pmatrix}5 & 6 & 4\\ 2 & 0 & -3\\ 1 & 3 & 4\end{pmatrix}$ **262.** $\begin{pmatrix}3 & 1 & 0\\ 2 & 2 & 1\\ 6 & 3 & 7\end{pmatrix}$ **263.** $\begin{pmatrix}4 & 1 & 2\\ 3 & 1 & 2\\ 4 & 2 & 5\end{pmatrix}$

264. $\begin{pmatrix}4 & 2 & 1\\ 1 & 3 & 3\\ 3 & 2 & -1\end{pmatrix}$ **265.** $\begin{pmatrix}2 & 1 & 5\\ 1 & 3 & 1\\ 1 & 4 & 8\end{pmatrix}$ **266.** $\begin{pmatrix}1 & -1 & 3\\ 2 & 0 & 1\\ 1 & 2 & 2\end{pmatrix}$

267. $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 9 & 7 \\ 4 & -3 & 1 \end{pmatrix}$

268-279. Quyidagi matritsaviy tenglamani yeching:

268. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$

269. $\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} X = \begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$

270. $\begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix} X = \begin{pmatrix} 0 & 0 \\ 1 & 5 \end{pmatrix}$

271. $\begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix} X = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

272. $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix} X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

273. $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} X = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

274. $X \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

275. $X \begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

276. $X \begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$

277. $X \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$

278. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix} X = \begin{pmatrix} 5 & -2 & 1 \\ 1 & 3 & 0 \end{pmatrix}$

279. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix} X = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 3 & 2 \\ 0 & -1 & 1 \end{pmatrix}$

280. Quyidagi chiziqli tenglamalar sistemalarini teskari matritsa topish usuli bilan yeching:

a) $\begin{cases} 3x_1+4x_2=2 \\ 5x_1+7x_2=-3 \end{cases}$ b) $\begin{cases} 5x_1+6x_2=-1 \\ 4x_1+5x_2=2 \end{cases}$

281. Quyida berilgan tenglamalar sistemasining birgalikda ekanligini tekshiring, agar birgalikda bo'lsa, ularni matritsa usuli bilan yeching.

a) $\begin{cases} 3x+2y+z=5, \\ 2x+3y+z=1, \\ 2x+y+3z=11. \end{cases}$ **b)** $\begin{cases} 4x-3y+2z=9, \\ 2x+5y-3z=4, \\ 5x+6y+2z=18. \end{cases}$

c) $\begin{cases} 2x-y-z=4, \\ 3x+4y-2z=11, \\ 3x-2y+4z=11. \end{cases}$ **d)** $\begin{cases} x+y-z=1, \\ 8x+3y-6z=2, \\ -4x-y+3z=-3. \end{cases}$

e) $\begin{cases} 7x-5y=31, \\ 4x+11z=-43, \\ 2x+3y+4z=-20. \end{cases}$ **f)** $\begin{cases} x-2y+3z=6, \\ 2x+3y-4z=20, \\ 3x-2y-5z=6. \end{cases}$

g) $\begin{cases} x+y+3z=-1, \\ 2x-y+2z=-4, \\ 4x+y+4z=-2. \end{cases}$ **h)** $\begin{cases} 3x+4y+2z=8, \\ 2x-y-3z=-1, \\ x+5y+z=-7. \end{cases}$

i) $\begin{cases} x-4y-2z=-7, \\ 3x+y-z=5, \\ -3x+5y+6z=7. \end{cases}$ **j)** $\begin{cases} x+2y+4z=31, \\ 5x+y+2z=20, \\ 3x-y+z=0. \end{cases}$

Determinantlarni ko'paytirish.

Teorema. n-tartibli

$$D_1=\begin{vmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{vmatrix} \quad \text{va} \quad D_2=\begin{vmatrix} b_{11} & b_{12} & . & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & . & b_{2n} \\ . & . & . & . & . & . \\ b_{n1} & b_{n2} & . & . & . & b_{nn} \end{vmatrix}$$

ikki determinantning ko`paytmasini yana n-tartibli

$$D=\begin{vmatrix} c_{11} & c_{12} & . & . & . & c_{1n} \\ c_{21} & c_{22} & . & . & . & c_{2n} \\ . & . & . & . & . & . \\ c_{n1} & c_{n2} & . & . & . & c_{nn} \end{vmatrix}$$

determinant ko`rinishida ifodalash mumkin bo`lib, D ko`paytmaning $c_{ij}$ elementi $D_1$ ning $i$-satridagi hamma elementlarini $D_2$ ning $j$-ustunidagi mos elementlariga ko`paytirib, natijalarni qo`shish bilan vujudga keladi, ya'ni:

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ... + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}. \qquad (7.1)$$

$$(i, j = 1,2,3,...,n)$$

Isbot. Ushbu $2n$-tartibli determinantni olamiz:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & . & . & . & a_{1n} & 0 & 0 & . & . & . & 0 \\ a_{21} & a_{22} & . & . & . & a_{2n} & 0 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} & 0 & 0 & . & . & . & 0 \\ -1 & 0 & . & . & . & 0 & b_{11} & b_{12} & . & . & . & b_{1n} \\ 0 & -1 & . & . & . & 0 & b_{21} & b_{22} & . & . & . & b_{2n} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & -1 & b_{n1} & b_{n2} & . & . & . & b_{nn} \end{vmatrix}.$$

Birinchidan: Δ determinantda birinchi n ta satrni ajratib, ulardan n-tartibli minorlar tuzsak, birinchi minor D_1 va uning algebraik to`ldiruvchisi $D_2$ bo`ladi. Qolgan hamma minorlar esa nolga teng, chunki ularning , eng kamida, bita ustuni nollardan iborat. Demak, Δ ning shu minorlar bo'yicha yoyilmasi

$$\Delta = D_1 \cdot D_2.$$

Ikkinchidan: Δ determinantni shaklan shunday o'zgartiraylikki, natijada hamma b_{ij} Lar o'rinlaridagi elementlar nolga aylansin. Buning uchun, 1-ustunni b_{11} ga, 2-ustunni b_{21} ga,…, $n-$ustunni b_{n1} ga ko'paytirib, natijalarni $(n+1)$-ustunga qo'shamiz. So'ngra 1-ustunni b_{12} ga, 2-ustunni b_{22} ga,…, $n-$ustunni b_{n2} ga ko'paytirib, natijalarni $(n+2)$-ustunga qo'shamiz va hokazo. Eng oxirgi, 1-ustunni b_{1n} ga, 2-ustunni b_{2n} ga,…, $n-$ustunni b_{nn} ga ko'paytirib, natijalarni $2n-$ustunga qo'shamiz.

Bularni bajargandan keyin, Δ determinantni quyidagi shaklni oladi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & . & . & . & a_{1n} & c_{11} & c_{12} & . & . & . & c_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} & c_{21} & c_{22} & . & . & . & c_{2n} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} & c_{n1} & c_{n2} & . & . & . & c_{nn} \\ -1 & 0 & . & . & . & 0 & 0 & 0 & . & . & . & 0 \\ 0 & -1 & . & . & . & 0 & 0 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & -1 & 0 & 0 & . & . & . & 0 \end{vmatrix},$$

Bunda hamma c_{ij} elementlar xuddi (7.1) yig'indilarga teng.

Endi, Δ ni so'nggi n ta satrdan tuzilgan n-tartibli minorlar bo'yicha yoyamiz. Bu minorlar orasida faqat:

$$M = \begin{vmatrix} -1 & 0 & . & . & . & 0 \\ 0 & -1 & . & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & -1 \end{vmatrix} = (-1)^n$$

minor noldan farqli bo'lib, uning algebraik to'ldiruvchisi Ushbu determinantdan iborat:

$$A = (-1)^{[(n+1)+(n+2)+...+2n]+[1+2+...+n]} \begin{vmatrix} c_{11} & c_{12} & . & . & . & c_{1n} \\ c_{21} & c_{22} & . & . & . & c_{2n} \\ . & . & . & . & . & . \\ c_{n1} & c_{n2} & . & . & . & c_{nn} \end{vmatrix} = (-1)^{n(2n+1)} \cdot D.$$

Qolgan hamma minorlar nolga teng. Demak, bizning yoyilmamiz:

$$\Delta = MA = (-1)^n \cdot (-1)^{n(2n+1)} \cdot D = (-1)^{2(n^2+n)} \cdot D = D$$

bo'ladi. Shunday qilib:

$$D = D_1 \cdot D_2.$$

Bu isbot qilingan ko'paytirish qoidasi - «*satrlarni ustunlarga*» qoidasi deyiladi. Determinantda satrlarni ustunlar bilan almashtirish mumkin bo'lgani uchun, Yana uchta ko'paytirish qoidasini hosil qilamiz:

«*satrlarni satrlarga*», «*ustunlarni satrlarga*» *va* «*ustunlarni ustunlarga*» qoidasi.

Misol. Ushbu

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -1 & 4 & 5 \end{vmatrix}, \qquad \Delta_2 = \begin{vmatrix} 3 & 4 & -1 \\ 2 & 0 & 5 \\ -1 & 1 & -3 \end{vmatrix}$$

determinantlarni «*satrlarni satrlarga*» va «*ustunlarni ustunlarga*» qoidasi bo’yicha ko’paytmasini hisoblang.

Yechilishi: «Satrlarni satrlarga» qoidasi bo’yicha quyidagini topamiz:

$$\Delta_1 \cdot \Delta_2 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -1 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 3 & 4 & -1 \\ 2 & 0 & 5 \\ -1 & 1 & -3 \end{vmatrix} =$$

$$= \begin{vmatrix} 1\cdot 3 + 2\cdot 4 + 3\cdot(-1) & 1\cdot 2 + 2\cdot 0 + 3\cdot 5 & 1\cdot(-1) + 2\cdot 1 + 3\cdot(-3) \\ 2\cdot 3 + 3\cdot 4 + 1\cdot(-1) & 2\cdot 2 + 3\cdot 0 + 1\cdot 5 & 2\cdot(-1) + 3\cdot 1 + 1\cdot(-3) \\ (-1)\cdot 3 + 4\cdot 4 + 5\cdot(-1) & (-1)\cdot 2 + 4\cdot 0 + 5\cdot 5 & (-1)\cdot(-1) + 4\cdot 1 + 5\cdot(-3) \end{vmatrix} =$$

$$= \begin{vmatrix} 8 & 17 & -8 \\ 17 & 9 & -2 \\ 8 & 23 & -10 \end{vmatrix} = -286.$$

Bu determinantlarni «ustunlarni ustunlarga» qoidasi bo’yicha ko’paytirsak ham yana o’sha natijaga kelamiz:

$$\Delta_1 \cdot \Delta_2 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -1 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 3 & 4 & -1 \\ 2 & 0 & 5 \\ -1 & 1 & -3 \end{vmatrix} =$$

$$= \begin{vmatrix} 1\cdot 3 + 2\cdot 2 + (-1)\cdot(-1) & 1\cdot 4 + 2\cdot 0 + (-1)\cdot 1 & 1\cdot(-1) + 2\cdot 5 + (-1)\cdot(-3) \\ 2\cdot 3 + 3\cdot 2 + 4\cdot(-1) & 2\cdot 4 + 3\cdot 0 + 4\cdot 1 & 2\cdot(-1) + 3\cdot 5 + 4\cdot(-3) \\ 3\cdot 3 + 1\cdot 2 + 5\cdot(-1) & 3\cdot 4 + 1\cdot 0 + 5\cdot 1 & 3\cdot(-1) + 1\cdot 5 + 5\cdot(-3) \end{vmatrix} =$$

$$= \begin{vmatrix} 8 & 3 & 12 \\ 8 & 12 & 1 \\ 6 & 17 & -13 \end{vmatrix} = -286.$$

282. Quyida berilgan determinantlarni «satrlarni satrlarga» qoidasi bo’yicha ko’paytiring:

1) $\Delta_1 = \begin{vmatrix} 5 & 6 & 4 \\ 2 & 0 & -3 \\ 1 & 3 & 4 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -3 & 7 & 14 \\ 2 & -10 & 32 \\ 22 & -3 & 11 \end{vmatrix}$ **2)** $\Delta_1 = \begin{vmatrix} -3 & 1 & 37 \\ 2 & -12 & 1 \\ -36 & 9 & 5 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -3 & 56 & 20 \\ -6 & 4 & 8 \\ 5 & 29 & -7 \end{vmatrix}$

3) $\Delta_1 = \begin{vmatrix} -3 & 7 & 4 \\ 0 & 1 & 12 \\ 1 & 13 & -5 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 4 & 3 & 2 \\ -2 & 1 & -1 \\ 3 & 1 & 1 \end{vmatrix}$ **4)** $\Delta_1 = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 2 & 1 & 4 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 1 & -2 & -1 \\ 3 & 1 & 2 \\ 1 & 2 & 2 \end{vmatrix}$

5) $\Delta_1 = \begin{vmatrix} -2 & 1 & 0 \\ 0 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 4 & 3 & 8 \\ 6 & -9 & 1 \\ -2 & 1 & -8 \end{vmatrix}$ **6)** $\Delta_1 = \begin{vmatrix} 5 & 2 & 1 \\ 1 & -1 & 0 \\ 0 & 3 & -1 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -1 & 8 & -2 \\ -4 & 3 & 2 \\ 3 & -8 & 5 \end{vmatrix}$

7) $\Delta_1 = \begin{vmatrix} 2 & 19 & 30 \\ 0 & -5 & -12 \\ 0 & 2 & 5 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 4 & 3 & 5 \\ 6 & 7 & 1 \\ 9 & 1 & 8 \end{vmatrix}$ **8)** $\Delta_1 = \begin{vmatrix} 0 & 1 & 0 \\ -4 & 4 & 2 \\ -3 & 5 & 6 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -1 & 3 & -2 \\ -4 & 3 & -7 \\ -4 & 3 & -1 \end{vmatrix}$

9) $\Delta_1 = \begin{vmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 4 & 7 \\ 3 & 1 & -1 \end{vmatrix}$ **10)** $\Delta_1 = \begin{vmatrix} -4 & 25 & -7 \\ 3 & 45 & 2 \\ 9 & 0 & -4 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -14 & 1 & -2 \\ 3 & -51 & 5 \\ -4 & 3 & 67 \end{vmatrix}$

283. Quyida berilgan determinantlarni «ustunlarni ustunlarga» qoidasi bo'yicha ko'paytiring:

1. $\Delta_1 = \begin{vmatrix} 2 & 19 & 30 \\ 0 & -5 & -12 \\ 0 & 2 & 5 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 4 & 3 & 5 \\ 6 & 7 & 1 \\ 9 & 1 & 8 \end{vmatrix}$ **2.** $\Delta_1 = \begin{vmatrix} 0 & 1 & 0 \\ -4 & 4 & 2 \\ -3 & 5 & 6 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -1 & 3 & -2 \\ -4 & 3 & -7 \\ -4 & 3 & -1 \end{vmatrix}$

3. $\Delta_1 = \begin{vmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 4 & 7 \\ 3 & 1 & -1 \end{vmatrix}$ **4.** $\Delta_1 = \begin{vmatrix} -4 & 25 & -7 \\ 3 & 45 & 2 \\ 9 & 0 & -4 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} -14 & 1 & -2 \\ 3 & -51 & 5 \\ -4 & 3 & 67 \end{vmatrix}$

5. $\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 7 & 0 & 4 \\ 0 & 4 & -9 \\ 3 & 1 & 0 \end{vmatrix}$ **6.** $\Delta_1 = \begin{vmatrix} 7 & -12 & 6 \\ 10 & -19 & 10 \\ 12 & -24 & 13 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 0 & 1 & -6 \\ 3 & 0 & 7 \\ 1 & 1 & -1 \end{vmatrix}$

7. $\Delta_1 = \begin{vmatrix} 65 & -1 & 0 \\ 0 & 33 & -1 \\ 5 & 2 & 41 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 3 & 11 & 8 \\ 61 & -2 & -19 \\ 20 & 3 & 27 \end{vmatrix}$ **8.** $\Delta_1 = \begin{vmatrix} 1 & -1 & 2 \\ 0 & -4 & 8 \\ -3 & 1 & -3 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 1 & 2 & 2 \\ 0 & -3 & 1 \\ 2 & 0 & 3 \end{vmatrix}$

9. $\Delta_1 = \begin{vmatrix} 3 & -1 & 0 \\ -4 & -1 & 0 \\ 4 & -8 & -2 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 3 & -1 & 5 \\ 1 & 2 & 4 \\ 3 & 2 & -1 \end{vmatrix}$ **10.** $\Delta_1 = \begin{vmatrix} 4 & -5 & 7 \\ 1 & -4 & 9 \\ -4 & 0 & 5 \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} 9 & 3 & 5 \\ 2 & 0 & 3 \\ 0 & 1 & -1 \end{vmatrix}$

6§. O’rin almashtirishlar, o’rniga qo’yishlar va ular ustida amallar.

284. Quyidagi o’rin almashtirishda inversiyalar sonini aniqlang:

a) 1, 2, 5, 4, 6, 3.

b) 5, 4, 7, 8, 1, 9, 6, 3, 2.

c)8, 7, 4, 5, 2, 3,6, 9, 1.

285. Quyidagi o’rin almashtirishda inversiyalar sonini aniqlang:

7, 6, 3, 4, 1, 2, 9, 8, 5.

286. Quyidagi o’rin almashtirishda inversiyalar sonini aniqlang:

3, 6, 5, 2, 5, 7, 4, 1, 8, 9.

287-291 misollarda berilgan o’riniga qo’yishdagi inversiyalar sonini aniqlang:

287. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$

288. $\begin{pmatrix} 5 & 3 & 4 & 1 & 2 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$

289. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix}$

290. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 1 & 7 & 8 & 2 & 6 & 9 \end{pmatrix}$

291. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 5 & 6 & 4 & 7 & 8 & 2 \end{pmatrix}$

292. Quyidagi o’rniga qo’yishlarni ko’paytiring:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

293. Quyidagi o’rniga qo’yishlarni ko’paytiring:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 2 & 1 \end{pmatrix}.$$

294. Quyidagi o’rniga qo’yishni kvadratini toping:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 1 & 3 \end{pmatrix}.$$

295. Quyidagi o’rniga qo’yish qanday darajada E ga teng:

$$A=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 2 & 1 & 5 & 4\end{pmatrix}.$$

296. Quyidagi o’rniga qo’yish qanday darajada E ga teng:

$$A=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 4 & 3 & 2 & 1 & 6 & 5\end{pmatrix}.$$

297. Quyidagi o’rniga qo’yish qanday darajada E ga teng:

$$A=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7\\ 7 & 6 & 5 & 4 & 3 & 2 & 1\end{pmatrix}.$$

298. Quyidagi o’rniga qo’yishga teskari o’rniga qo’yishni toping:

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 1 & 2\end{pmatrix}.$$

299. Quyidagi tenglamadan X o’rniga qo’yishni toping:

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 3 & 2\end{pmatrix}X=\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}.$$

300. Quyidagi tenglamadan X o’rniga qo’yishni toping:

$$X\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 5 & 4 & 3 & 2 & 1\end{pmatrix}=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 4 & 5 & 2 & 1\end{pmatrix}.$$

301. Quyidagi o’rniga qo’yishlar $AB=BA$ shartni kanoatlantiradimi?

$$A=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 4 & 5 & 2 & 1\end{pmatrix}, B=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 2 & 5 & 4 & 1\end{pmatrix}.$$

302. Quyidagi o’rniga qo’yishni tsikllar ko’paytmasiga yoying:

$$\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\ 5 & 8 & 9 & 2 & 1 & 4 & 3 & 6 & 7\end{pmatrix}.$$

303. Sikllar yordamida yozilgan o’rniga qo’yishni ikki satrli shaklga keltiring:

(7 5 3 1) (2 4 6) (8) (9).

304. Sikllar yordamida yozilgan o’rniga qo’yishni ikki satrli shaklga keltiring:

(14) (25) (36).

7§. Matritsalarning determinanti. Minor va uning algebraik to'ldiruvchisi.

Ta'rif. $n-$tartibli Δ detminantning istalgan r ta satri va r ta ustunlarini ajrataylik $(1 \le r < n)$. Bu satrlar va ustunlarning kesishgan joylaridagi elementlarni Δ detminantdagidek tartibda olib, ulardan r- tartibli M determinantni tuzsak, u Δ *ning r- tartibli minori* deb ataladi.

Ta'rif. Δ detminantda ajratilgan r ta satr va r ta ustunni o'chiraylik. Δ ning qolgan elementlarini shu Δ dagidek tartibda olib, ulardan $(n-r)$ - tartibli $\overline{M}$ determinantni tuzsak, u M ga *qo'shma minor* deyiladi.

Misol. 5 – tartibli

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix}$$

determinantda 1 – va 5 – satrlarni, 3 – va 4 – ustunlarni ajratsak, ularning kesishgan joylaridagi elementlardan 2 – tartibli

$$M = \begin{vmatrix} a_{13} & a_{14} \\ a_{53} & a_{54} \end{vmatrix}$$

minor tuziladi. Bu ajratilgan satr va ustunlarni o'chirsak, qolgan elementlardan ushbu:

$$\overline{M} = \begin{vmatrix} a_{21} & a_{22} & a_{25} \\ a_{31} & a_{32} & a_{35} \\ a_{41} & a_{42} & a_{45} \end{vmatrix}$$

qo'shma minor hosil bo'ladi.

Agar $k-$satr va $l-$ustun ajratilsa, ularning kesishgan joyida a_{kl} element turgani uchun:

$$M = a_{kl} \quad .$$

Bu holda, $\overline{M}$ qo'shimcha minor, odatda, M_{kl} bilan belgilanib, a_{kl} *elementning minori* deyiladi.

Masalan, yuqoridagi 5 – tartibli determinantda 3 – satr va 4 – ustunga qarashli a_{34} elementning minori

$$M_{34}=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{25} \\ a_{41} & a_{42} & a_{43} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{55} \end{vmatrix}$$

bo’lib, bu minor, determinantdan 3 – satr va 4 – ustunni o’chirish bilan hosil qilingan.

Determinant a_{ik} elementining algebraik to’ldiruvchisi

$$A_{ik}=(-1)^{i+k}M_{ik}$$

munosabat bilan aniqlanadi.

Misol.

$$M=\begin{vmatrix} a_{13} & a_{14} \\ a_{53} & a_{54} \end{vmatrix}$$

minorning algebraik to’ldiruvchisi:

$$A=(-1)^{1+5+3+4}\begin{vmatrix} a_{21} & a_{22} & a_{25} \\ a_{31} & a_{32} & a_{35} \\ a_{41} & a_{42} & a_{45} \end{vmatrix}=-\begin{vmatrix} a_{21} & a_{22} & a_{25} \\ a_{31} & a_{32} & a_{35} \\ a_{41} & a_{42} & a_{45} \end{vmatrix}.$$

a_{34} elementning algebraik to’ldiruvchisi esa ushbudan iborat:

$$A_{34}=(-1)^{3+4}\cdot M_{34}=-M_{34}=-\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{25} \\ a_{41} & a_{42} & a_{43} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{55} \end{vmatrix}.$$

305. Quyidagi ko’paytma 6-tartibli determinantning hadi bo’la oladimi? Agar bo’la olsa qanday ishora bilan qatnashadi?

$$a_{61}\,a_{23}\,a_{43}\,a_{36}\,a_{12}\,a_{54}\,.$$

306. Quyidagi ko’paytma 6-tartibli determinantning hadi bo’la oladimi? Agar bo’la olsa qanday ishora bilan qatnashadi?

$$a_{54}\,a_{12}\,a_{34}\,a_{45}\,a_{23}\,a_{61}\,.$$

307. Quyidagi ko’paytma 6-tartibli determinantning hadi bo’la oladimi? Agar bo’la olsa qanday ishora bilan qatnashadi?

$$a_{12}\,a_{34}\,a_{26}\,a_{45}\,a_{53}\,a_{61}\,.$$

308-314. Quyidagi determinantlarni hisoblang:

308. $\begin{vmatrix} 1975 & 0 & 1974 \\ 999 & 2 & 551 \\ 1976 & 0 & 1975 \end{vmatrix}$ **309.** $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 2 & 3 \\ 1 & 3 & 1 \end{vmatrix}$

310. $\begin{vmatrix} 1993 & 1992 \\ 1994 & 1993 \end{vmatrix}$ **311.** $\begin{vmatrix} 1970 & 1969 \\ 1971 & 1970 \end{vmatrix}$

312. $\begin{vmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 3 & 1 & 0 & 1 \\ 0 & 2 & 2 & 0 \end{vmatrix}$ **313.** $\begin{vmatrix} 3 & 0 & 9 & 0 \\ 1 & 2 & 8 & 2 \\ 0 & 0 & 4 & 0 \\ 0 & 1 & 5 & 1 \end{vmatrix}$ **314.** $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{vmatrix}$

315. $A=(a_{ij})$ matritsa berilgan. (A_{ji}) matritsani iloji boricha oson usul bilan toping:

$$A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}.$$

316. $A=(a_{ij})$ matritsa berilgan. (A_{ji}) matritsani iloji boricha oson usul bilan toping:

$$A=\begin{pmatrix} 1 & 2 & -3 \\ 0 & 2 & 1 \\ -1 & 3 & 2 \end{pmatrix}.$$

317. $A=(a_{ij})$ matritsa berilgan. (M_{ji}) matritsani iloji boricha oson usul bilan toping:

$$A=\begin{pmatrix} 1 & 2 & -1 \\ -1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

318. $A=(a_{ij})$ matritsa berilgan. (M_{ji}) matritsani iloji boricha oson usul bilan toping:

$$A=\begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{pmatrix}.$$

319. A matritsani a_{23} elementining minori va algebraik to'ldiruvchisini toping:

$$A=\begin{pmatrix} 1 & -3 & 5 & 1 \\ 2 & 2 & 3 & 1 \\ 3 & 1 & -2 & 1 \\ 2 & 3 & 3 & -1 \end{pmatrix}.$$

320. A matritsani a_{32} elementining minori va algebraik to’ldiruvchisini toping:

$$A=\begin{pmatrix} 1 & 2 & 3 & 0 \\ -1 & 2 & 0 & 1 \\ 3 & 5 & 2 & 0 \\ 2 & 0 & 3 & 1 \end{pmatrix}.$$

321. A matritsani a_{21} elementining minori va algebraik to’ldiruvchisini toping:

$$A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 0 \\ -2 & 1 & -1 \end{pmatrix}.$$

322. A matritsani a_{13} elementining minori va algebraik to’ldiruvchisini toping:

$$A=\begin{pmatrix} 1 & 2 & -1 \\ -2 & 0 & 2 \\ 1 & -1 & 0 \end{pmatrix}.$$

323. Quyidagi rekurrent formuladan $\{x_n\}$ ketma-ketlikni toping:

$x_1=3$

$x_2=5$ $\qquad x_n=7x_{n-1}-10x_{n-2}$

324. Quyidagi rekurrent formuladan $\{x_n\}$ ketma-ketlikni toping:

$x_1=2$

$x_2=3$ $\qquad x_n=6x_{n-1}-9x_{n-2}$

325. Quyidagi rekkurent formuladan $\{x_n\}$ ketma-ketlikni toping:

$x_1=0$

$x_2=1$ $\qquad x_n=-x_{n-1}+6x_{n-2}$

8§. Chiziqli tenglamalar sistemasi uchun Kramer qoidasi.

326. Quyidagi chiziqli tenglamalar sistamasini Kramer qoidasi yordamida yeching:

$$\begin{cases} x+y-z=1 \\ 2y+3z=0 \\ 2x+3y+z=0 \end{cases}$$

327. Quyidagi chiziqli tenglamalar sistamasini Kramer qoidasi yordamida yeching:

$$\begin{cases} x-4y+2z=0 \\ 2x+5y-z=0 \\ x-y+2z=0 \end{cases}$$

328. Quyidagi chiziqli tenglamalar sistamasi nechta yechimga ega?

$$\begin{cases} x+3y+5z=0 \\ x+2y+3z=0 \\ 3x+5y+7z=0 \end{cases}$$

329. λ ning qanday qiymatlarida quyidagi chiziqli tenglamalar sistamasi yagona yechimga ega?

$$\begin{cases} 4x-3y+2z=4 \\ 6x-2y+3z=1 \\ 5x-3y+\lambda z=3 \end{cases}$$

330. λ ning qanday qiymatlarida quyidagi chiziqli tenglamalar sistamasi yagona yechimga ega?

$$\begin{cases} 6x-4y+z=1 \\ 3x+2y=2 \\ 2x+y+\lambda z=3 \end{cases}$$

331. λ ning qanday qiymatlarida quyidagi chiziqli tenglamalar sistamasi yagona yechimga ega?

$$\begin{cases} x+2y-8z=3 \\ 2x+4y+7z=2 \\ 3x+6y+\lambda z=1 \end{cases}$$

332. λ ning qanday qiymatlarida quyidagi chiziqli tenglamalar sistamasi yagona yechimga ega?

$$\begin{cases} 2x+3y=-4 \\ 4x+6y=\lambda \end{cases}$$

333. λ ning qanday qiymatlarida bir jinsli chiziqli tenglamalar sistamasi noldan farqli yechimga ega?

$$\begin{cases} 4x-3y+2z=0 \\ 6x-2y+3z=0 \\ 5x-\lambda y+2z=0 \end{cases}$$

334. λ ning qanday qiymatlarida bir jinsli chiziqli tenglamalar sistamasi noldan farqli yechimga ega?

$$\begin{cases} x+2y+3z=0 \\ 4x+5y+6z=0 \\ 7x+8y+\lambda z=0 \end{cases}$$

9§. Matritsalarning xos qiymatlari va xos vektorlari.

335. Quyidagi matritsaning xos qiymatlarini toping

$$A=\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$$

336. Quyidagi matritsaning xos qiymatlarini toping

$$A=\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

337. Quyidagi matritsaning xos qiymatlarini va xos vektorlarini toping

$$A=\begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$$

338. Quyidagi matritsaning xos qiymatlarini va xos vektorlarini toping: $A=\begin{pmatrix} 4 & 1 & -15 \\ 1 & 4 & -5 \\ 1 & 1 & -4 \end{pmatrix}$

339. Quyidagi matritsaning xos qiymatlarini va xos vektorlarini toping

$$A=\begin{pmatrix} 1 & -3 & 3 \\ -2 & -6 & 13 \\ -1 & -4 & 8 \end{pmatrix}$$

10§. Kompleks sonlar va ular ustida amallar.

340. Hisoblang: $i^{98}+1$

341. Hisoblang: $\dfrac{2+3i}{2-3i}$

342. Hisoblang: $\left(\dfrac{1+i}{1-i}\right)^{40}$

343. Hisoblang: $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^2$

344. Hisoblang: $\left(-1+i\sqrt{3}\right)^3$

345. Hisoblang: $\left(\sqrt{3}-i\right)^{10}$

346. Hisoblang: $\left(\sqrt{3}+i\right)^6$

347. Hisoblang: $(1+i)^8$

348. Hisoblang: $(-1+i)^6$

349. Hisoblang: $i^{20}-2i^{40}$

350. *x* va *y* haqiqiy son ekanligi ma'lum. Quyidagi tenglamani yeching

$$(2+i)x+(2-i)y=4-i$$

351. *x* va *y* haqiqiy son ekanligi ma'lum. Quyidagi tenglamani yeching

$$(1-3i)x+(2-6i)y=-2+6i$$

352. Shunday *x*, *y* haqiqiy sonlar topingki, ular quyidagi tenglamani qanoatlantirsin

$$(2+3i)x-(4+6i)y=1+i$$

353. $z=1+i$ sonni ko'rsatkichli shaklda yozing.

354. $z=-1+i$ sonni ko'rsatkichli shaklda yozing.

355. $z=-1-i$ sonni ko'rsatkichli shaklda yozing.

356. $z=1-i$ sonni ko'rsatkichli shaklda yozing.

357. $z=-1$ sonni ko'rsatkichli shaklda yozing.

358. $z=i$ sonni ko'rsatkichli shaklda yozing.

359. $z=1+i\sqrt{3}$ sonni ko'rsatkichli shaklda yozing.

360. $z=\sqrt{i}$ ni hisoblang.

361. $z=\sqrt{-i}$ ni hisoblang.

362. $z=\sqrt{-1-i}$ ni hisoblang.

363. $z=\sqrt{1+i}$ ni hisoblang.

364. $z=\sqrt{-1+i}$ ni hisoblang.

365. $z=i$ kompleks son qanday eng kichik musbat darajada 1 ga teng?
366. $z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$ kompleks son qanday eng kichik musbat darajada 1 ga teng?
367. $z=\frac{1}{2}+i\frac{\sqrt{3}}{2}$ kompleks son qanday eng kichik musbat darajada 1 ga teng?

368. $z=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ kompleks son qanday eng kichik musbat darajada 1 ga teng?

369. $z=\frac{\sqrt{3}}{2}+i\frac{1}{2}$ kompleks son qanday eng kichik musbat darajada 1 ga teng?

11§. Kompleks sonlarning algebraik va trigonometrik shakli. Kompleks sonlardan ildiz chiqarish.

370-391. Quyida berilgan kompleks sonlarni trigonometrik shaklga keltiring.

370. $(2+2i\sqrt{3})^{10}$ **371.** $(5-12i)^{12}$ **372.** $(2+\sqrt{3}+i)^{5}$ **373.** $(-3-3i)^{6}$

374. $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{18}$ **375.** $(1+i\sqrt{2})^{14}$ **376.** $\left(\frac{2}{3}+\frac{2}{3}i\right)^{8}$ **377.** $\left(-\frac{1}{2}+\frac{1}{2}i\right)^{16}$

378. $(\sqrt{3}-i)^{10}$ **379.** $(12i)^{20}$ **380.** $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{8}$ **381.** $(-1+i\sqrt{3})^{8}$

382. $(12+5i)^6$ **383.** $(3i+4)^4$ **384.** $(-2-2i\sqrt{3})^9$ **385.** $\left(\frac{1}{3}-i\frac{1}{3}\right)^6$

386. $(8+6i)^5$ **387.** $(-15+8i)^7$ **388.** $(-3-4i)^{10}$ **389.** $(-1+i\sqrt{3})^{15}$

390. $(1+i)^{20}$ **391.** $(2-2i)^{15}$

392-413. Hisoblang.

392. $\sqrt{-2i}$ **393.** $\sqrt{8i}$ **394.** $\sqrt{3+4i}$ **395.** $\sqrt{15-8i}$

396. $\sqrt{-3+4i}$ **397.** $\sqrt{4+3i}$ **398.** $\sqrt{2-3i}$ **399.** $\sqrt{-8-6i}$

400. $\sqrt[4]{-1}$ **401.** $\sqrt{1-i}$ **402.** $\sqrt{1+i\sqrt{3}}$ **403.** $\sqrt{\sqrt{3}-i}$

404. $\sqrt[3]{1-i\sqrt{3}}$ **405.** $\sqrt[4]{1+i}$ **406.** $\sqrt[3]{-1-i\sqrt{3}}$ **407.** $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$

408. $\left(1-\frac{\sqrt{3}-i}{2}\right)^{22}$ **409.** $\frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}}+\frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}$

410. $\sqrt{\frac{1}{2}-i\frac{\sqrt{3}}{2}}$ **411.** $\sqrt{3+4i}$

412. $\sqrt{\frac{1}{3}+i\frac{1}{3}}$ **413.** $\sqrt{2-2i}$

414-434. Quyidagi kompleks sonlarni algebraik shaklda tasvirlang.

414. $12(\cos 330^0+i\sin 330^0)$ **415.** $21(\cos 750^0+i\sin 750^0)$

416. $15\left(\cos\left(-\frac{2\pi}{30}\right)+i\sin\left(-\frac{2\pi}{30}\right)\right)^5$ **417.** $3\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)^{15}$

418. $7\left(\cos\frac{19\pi}{24}+i\sin\frac{19\pi}{24}\right)^6$ **419.** $6\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)^3$

420. $4\left(\cos\frac{19\pi}{15}+i\sin\frac{19\pi}{15}\right)^5$ **421.** $16\left(\cos\frac{5\pi}{14}+i\sin\frac{5\pi}{14}\right)^7$

422. $2(\cos 945^0 + i\sin 945^0)^6$

423. $3(\cos(-1005^0) + i\sin(-1005^0))^3$

424. $4\left(\cos\left(-\frac{9\pi}{40}\right) + i\sin\left(-\frac{9\pi}{40}\right)\right)^5$

425. $5(\cos 1395^0 + i\sin 1395^0)^4$

426. $(\cos 675^0 + i\sin 675^0)^{10}$

427. $\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)^{100}$

428. $12\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)^{124}$

429. $6\left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right)^{90}$

430. $(\cos(-2295^0) + i\sin(-2295^0))^4$

431. $(\cos 765^0 + i\sin 765^0)^{10}$

432. $\left(3\left(\cos\frac{17\pi}{18} + i\sin\frac{17\pi}{18}\right)\right)^6$

433. $\left(2\left(\cos\left(-\frac{13\pi}{16}\right) + i\sin\left(-\frac{13\pi}{16}\right)\right)\right)^8$

434. $12\left(\cos\left(-\frac{211\pi}{144}\right) + i\sin\left(-\frac{211\pi}{144}\right)\right)^{24}$

12§. Ko'phadlar va ular ustida amallar.

435. $f(x)$ ko'phadni $g(x)$ ko'phadga qoldiqli bo'ling

$$f(x)=x^3+x^2+x-1; \qquad g(x)=x^2+1.$$

436. $f(x)$ ko'phadni $g(x)$ ko'phadga qoldiqli bo'ling

$$f(x)=x^4-x^3+x^2+x-1; \qquad g(x)=x^2+x+1.$$

437. $f(x)$ ko'phadni $g(x)$ ko'phadga qoldiqli bo'ling

$$f(x)=x^4-x^3-x^2-x-2; \qquad g(x)=x^2-x-1.$$

438. $f(x)$ ko'phadni $g(x)$ ko'phadga qoldiqli bo'lgan dagi qoldiqni toping

$$f(x)=x^4+2x^3-3x^2+5; \qquad g(x)=x^2-1$$

439. $f(x)$ ko'phadni $g(x)$ ko'phadga qoldiqli bo'lgan dagi qoldiqni toping

$$f(x)= x^3+1; \qquad g(x)=x^2-x+1.$$

440. $f(x)$ ko'phadni $g(x)$ ko'phadga qoldiqli bo'lgan dagi qoldiqni toping

$$f(x)=x^4+4; \qquad g(x)=x^2+2x+2.$$

441. $f(x)$ ko'phadni Gorner sxemasi yordamida $x-1$ bo'ling

$$f(x)=x^4+2x^3-x^2-3x+1$$

442. $f(x)$ ko'phadni Gorner sxemasi yordamida $x-2$ bo'ling

$$f(x)=2x^3+3x^2-5x+2$$

443. $f(x)$ ko'phadni Gorner sxemasi yordamida $x+1$ bo'ling

$$f(x)=x^3+x^2-x-1$$

444. $f(x)$ko'phadni Gorner sxemasi yordamida $x+2$ bo'ling

$$f(x)=x^3+2x^2+3x-2$$

445. Gorner sxemasi yordamida $f(3)$ ni hisoblang

$$f(x)=x^5+2x^4-3x^3+2x^2+x-2$$

446. Gorner sxemasi yordamida $f(2)$ ni hisoblang

$$f(x)=x^4+3x^3-5x^2-3x+2$$

447. Gorner sxemasi yordamida $f(4)$ ni hisoblang

$$f(x)=x^5-2x^4+3x^3-5x^2+2x-2$$

448. Gorner sxemasi yordamida $f(3)$ ni hisoblang

$$f(x)=x^4-3x^3+2x^2-x+1$$

449. Gorner sxemasi yordamida $f(x)$ ko'phadni $x-1$ ning darajalariga yoying

$$f(x)=2x^4+3x^3+x^2-x+1$$

450. Gorner sxemasi yordamida $f(x)$ ko'phadni $x+1$ ning darajalariga yoying

$$f(x)=x^3-2x^2+x-2$$

451. Gorner sxemasi yordamida $f(x)$ ko'phadni $x-2$ ning darajalariga yoying

$$f(x)=x^3+2x^2-x+1$$

452. Gorner sxemasi yordamida $f(x)$ ko'phadni $x+2$ ning darajalariga yoying

$$f(x)=x^4-2x^3+x^2-x+1$$

453. Evklid algoritmi yordamida $f(x)$ va $g(x)$ ko'phadlarning EKUBini toping

$$f(x)=x^4-2x^3+2x^2-2x+1; \quad g(x)=x^3-3x^2+3x-1.$$

454. Evklid algoritmi yordamida $f(x)$ va $g(x)$ ko'phadlarning EKUBini toping

$$f(x)=x^4+3x^3+4x^2+3x+1; \quad g(x)=x^3+3x^2+3x+1.$$

455. Evklid algoritmi yordamida $f(x)$ va $g(x)$ ko'phadlarning EKUBini toping

$$f(x)=4x^3-8x^2-3x+9; \quad g(x)=x^3+x^2-x-1.$$

456. Evklid algoritmi yordamida $f(x)$ va $g(x)$ ko'phadlarning EKUBini toping

$$f(x)=x^3+x+1; \quad g(x)=x^2+1.$$

457-477. Quyidagi ko'phadlarning EKUB ini toping.

457. $f(x)=x^4-1, \qquad g(x)=2x^3+x^2-2x-1.$

458. $f(x)=x^4+x^3-7x^2-x+6, \qquad g(x)=x^4-5x^2+4.$

459. $f(x)=x^4-2x^3-4x^2+4x-3, \qquad g(x)=2x^3-5x^2-4x+3.$

460. $f(x)=x^4+x^3+x^2+x+1, \qquad g(x)=3x^3+x^2+3x-1.$

461. $f(x)=x^4+2x^3-x^2-4x-2, \qquad g(x)=x^4+x^3-x^2-2x-2.$

462. $f(x)=x^5+3x^4+x^3+x^2+3x+1, \qquad g(x)=x^4+2x^3+x+2.$

463. $f(x)=3x^5+5x^4-16x^3-6x^2-5x-6, \qquad g(x)=3x^4-4x^3-x^2-x-2.$

464. $f(x)=4x^4-2x^3-16x^2+5x+9, \qquad g(x)=2x^3-x^2-5x+4.$

465. $f(x)=x^5-5x^4-2x^3+12x^2-2x+12, \qquad g(x)=x^3-5x^2-3x+17.$

466. $\begin{cases} f(x)=x^6-4x^5+11x^4-27x^3+37x^2-35x+35, \\ g(x)=x^5-3x^4+7x^3-20x^2+10x-25. \end{cases}$

467. $\begin{cases} f(x)=3x^7+6x^6-3x^5+4x^4+14x^3-6x^2-4x+4, \\ g(x)=3x^6-3x^4+7x^3-6x+2. \end{cases}$

468. $f(x)=x^5-7x^4-2x^3+2x^2-3x+4,\qquad g(x)=x^3-2x^2-3x+7.$

469. $f(x)=x^5+5x^4+9x^3+7x^2+5x+3,\qquad g(x)=x^4+2x^3+2x^2+x+1.$

470. $f(x)=x^4-x^3-4x^2+4x+1,\qquad g(x)=x^2-x-1.$

471. $f(x)=x^5+4x^4-10x^2-x+6,\qquad g(x)=5x^4+16x^3-20x^2-1.$

472. $f(x)=x^5+3x^4-x^3-7x^2+4,\qquad g(x)=5x^4+12x^3-3x^2-14x.$

473. $f(x)=x^5+x^4-x^3-2x-1,\qquad g(x)=3x^4+2x^3+x^2-2.$

474. $f(x)=x^6+2x^4-4x^3-3x^2+8x-5,\qquad g(x)=x^5+x^2-x+1.$

475. $f(x)=x^3+5x^2-x-5,\qquad g(x)=3x^2+10x-1.$

476. $f(x)=x^5-15x^3+16x^2+36x-48,\qquad g(x)=x^4+x^3-2x^2-3x-3.$

477. $f(x)=x^6+3x^5+x^4-8x^3-24x^2-x,\qquad g(x)=x^4-4x^3-11x^2-30x.$

478-488. Quyida berilgan ko'phadlarni karrali ko'paytuvchiga ajrating:

478. $f(x)=x^4+x^3-3x^2-5x-2$

479. $f(x)=x^5-3x^4+4x^3-4x^2+3x-1$

480. $f(x)=x^7-3x^6+5x^5+7x^4+7x^3-5x^2+3x-1$

481. $f(x)=x^6-4x^4-16x^2+16$

482. $f(x)=x^8+2x^7+5x^6+6x^5+8x^4+6x^3+5x^2+2x+1$

483. $f(x)=x^7-3x^6+5x^5-7x^4+7x^3-5x^2+3x-1$

484. $f(x)=x^7-3x^5-10x^4+16x^3+24x^2-32x-32$

485. $f(x)=x^5+3x^4-6x^3-10x^2+21x-9$

486. $f(x)=x^6-2x^5-x^4-2x^3+5x^2+4x+4$

487. $f(x)=x^5+8x^4+25x^3+38x^2+28x+8$

488. $f(x)=x^6-2x^5+4x^3-3x^2-2x+2$

489-498. Evklid algoritmidan foydalanib, $f_1(x)M_2(x)+f_2(x)M_1(x)=\delta(x)$ tenglikdan $M_1(x)$, $M_2(x)$ ko'phadlarni toping, bu erda $\delta(x)-f_1(x)$ va $f_2(x)$ ko'phadlarning EKUB sidir.

489. $f_1(x)=x^4+x^3-3x^2-4x-1,\qquad f_2(x)=x^3+x^2-x-1.$

490. $f_1(x)=x^5+x^4-x^3-2x-1,\qquad f_2(x)=3x^4+2x^3+x^2+2x-2.$

491. $f_1(x)=x^6-7x^4+8x^3-7x+7,\qquad f_2(x)=3x^5-7x^3+3x^2-7.$

492. $f_1(x)=x^5+3x^4-12x^3-52x^2-52x-12,\quad f_2(x)=x^4+3x^3-6x^2-22x-12.$

493. $f_1(x)=x^4-4x^3+1,\quad f_2(x)=x^3-3x^2+1.$

494. $f_1(x)=x^5-2x^4+x^3+7x^2-12x+10,\quad f_2(x)=3x^4-6x^3+5x^2+2x-2.$

495. $f_1(x)=x^5+x^4-x^3-3x^2-3x-1,\quad f_2(x)=x^4-2x^3-x^2-2x+1.$

496. $f_1(x)=x^4+7x^3+19x^2+23x+10,\quad f_2(x)=x^4+7x^3+18x^2+22x+12.$

497. $f_1(x)=x^4-2x^3-4x^2+6x+1,\quad f_2(x)=x^3-5x-3.$

498. $f_1(x)=x^4+2x^3+x+1,\quad f_2(x)=x^4+x^3-2x^2+2x-1.$

499-519. Quyida berilgan ko'phadlar qaysi sonlar maydonida keltiriladi va qaysi sonlar maydonida keltirilmaydi?

499. $f(x)=x^5+2x^3+x^2+x+1$

500. $f(x)=x^4+49x^3+14x^2$

501. $f(x)=x^5-10x^3-20x^2-15x-4$

502. $f(x)=x^3+3x^2-4$

503. $f(x)=x^4+2x^3+x+2$

504. $f(x)=x^4-2x^3+18x^2-27$

505. $f(x)=x^4-2x^3-x+2$

506. $f(x)=x^3+3x^2+3x-36$

507. $f(x)=27x^3-81x^2+297x+242$

508. $f(x)=x^3+9x^2+18x+28$

509. $f(x)=x^3+12x^2+45x+54$

510. $f(x)=x^4-4x^3-22x^2+100x-75$

511. $f(x)=x^4+8x^3+32x^2+80x+100$

512. $f(x)=x^4-8x^3+24x^2-9x-8$

513. $f(x)=16x^4-8x^2+64ix-65$

514. $f(x)=x^4+4x+3$

515. $f(x)=x^4+x^3+x^2+x+$

516. $f(x)=x^3-3x^2-3x+11$

517. $f(x)=8x^3+24x^2-81$

518. $f(x)=x^3+12x+63$

519. $f(x)=x^4-\dfrac{7}{2}x^2+x+\dfrac{21}{16}$

520. $f(x)=x^3+4x^2+5x+2$ ko'phad karrali ildizga egami?

521. $f(x)=x^3-4x^2+5x-2$ ko'phad karrali ildizga egami?

522. $f(x)=x^3+3x^2-4$ ko'phad karrali ildizga egami?

523. $f(x)=x^3-3x^2+4$ ko'phad karrali ildizga egami?

524. $f(x)=x^3+2x^2-4x-8$ ko'phad karrali ildizga egami?

525. $f(x)=x^3-3x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

526. $f(x)=x^3-12x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

527. $f(x)=2x^3-9x^2+12x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

528. $f(x)=2x^3-9x^2-12x+\lambda$ ko'phad λ ning qanday qiymatlarida karrali ildizga ega?

529. $a(x)u(x) + b(x)v(x) = c(x)$ Diofant tenglamasidan $u(x)$ va $v(x)$ ko'phadlarni toping
$a(x)=x^3-3x+2;\quad b(x)=x^3-3x;\quad c(x)=x^2-2.$

530. $a(x)u(x) + b(x)v(x) = c(x)$ Diofant tenglamasidan $u(x)$ va $v(x)$ ko'phadlarni toping
$a(x)=x^2+1;\quad b(x)=x^2-1;\quad c(x)=x.$

531. $a(x)u(x) + b(x)v(x) = c(x)$ Diofant tenglamasidan $u(x)$ va $v(x)$ ko'phadlarni toping
$a(x)=x^2+2;\quad b(x)=x^2;\quad c(x)=x+1.$

532. Viet teoremasidan foydalanib, ildizlari 1, 2, 3 bo'ladigan kubik ko'phad tuzing.

533. Viet teoremasidan foydalanib, ildizlari -1, 1, 2 bo'ladigan kubik ko'phad tuzing.

534. Viet teoremasidan foydalanib, ildizlari 1, -2, 1 bo'ladigan kubik ko'phad tuzing.

535. Viet teoremasidan foydalanib, ildizlari -1, -2, -3 bo'ladigan kubik ko'phad tuzing.

536. Viet teoremasidan foydalanib, ildizlari 1, 2, -1 bo'ladigan kubik ko'phad tuzing.

537. Quyidagi $f(x)$ ko'phadning barcha ildizlari kvadratlari yig'indisini toping

$$f(x)=x^3-5x^2+x-1;$$

538. Quyidagi $f(x)$ ko'phadning barcha ildizlari kvadratlari yig'indisini toping

$$f(x)=x^3+3x^2-2x+1;$$

539. Quyidagi $f(x)$ ko'phadning barcha ildizlari kvadratlari yig'indisini toping

$$f(x)=x^3-3x^2-x+1;$$

540. Quyidagi $f(x)$ ko'phadning barcha ildizlari kvadratlari yig'indisini toping

$$f(x)=x^3-2x^2-3x+2.$$

541. $f(x)=x^3+3x^2-5x+\lambda$ ko'phadni ikkita ildizini yig'indisi 1 ga teng bo'lsa, λ ni toping.

542. $f(x)=x^3-4x^2+5x+\lambda$ ko'phadni ikkita ildizini yig'indisi 2 ga teng bo'lsa, λ ni toping.

543. $f(x)=x^3-2x^2-3x-\lambda$ ko'phadni ikkita ildizini yig'indisi 2 ga teng bo'lsa, λ ni toping.

544. $f(x)=x^3-x^2+x-\lambda$ ko'phadni ikkita ildizini yig'indisi 3 ga teng bo'lsa, λ ni toping.

545. $f(x)=x^3-2x^2-2x+\lambda$ ko'phadni ikkita ildizini yig'indisi 1 ga teng bo'lsa, λ ni toping.

546. $f(x)=x^3-\lambda x^2+2x+4$ ko'phadni ikkita ildizini ko'paytmasi 2 ga teng bo'lsa, λ ni toping.

547. $f(x)=x^3-\lambda x^2-2x-3$ ko'phadni ikkita ildizini ko'paytmasi 1 ga teng bo'lsa, λ ni toping.

548. $f(x)=x^3-\lambda x^2+2x+4$ ko'phadni ikkita ildizini ko'paytmasi 2 ga teng bo'lsa, λ ni toping.

549. $f(x)=x^3-5x^2+\lambda x-3$ ko'phadni ikkita ildizini ko'paytmasi 1 ga teng bo'lsa, λ ni toping.

560. $f(x)=x^2+2x+\lambda$ ko'phad ildizlari kvadratlarining yig'indisi 1 ga teng bo'lsa, λ ni toping.

561. $f(x)=x^2-2x+\lambda$ ko'phad ildizlari kvadratlarining yig'indisi 2 ga teng bo'lsa, λ ni toping.

562. $f(x)=x^2-x+\lambda$ ko'phad ildizlari kvadratlarining yig'indisi 4 ga teng bo'lsa, λ ni toping.

563. $f(x)=x^2+4x-\lambda$ ko'phadning ildizlari kvadratlarining yig'indisi 4 ga teng bo'lsa, λ ni toping.

564. $f(x)=x^2-3x-\lambda$ ko'phadning ildizlari kvadratlarining yig'indisi 1 ga teng bo'lsa, λ ni toping.

565. x_1, x_2, x_3 lar $f(x)=x^3-3x^2+4x+5$ ning ildizlari bo'lsa,

$$q=\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_2}$$ ni hisoblang.

566. x_1, x_2, x_3 lar $f(x)=x^3-x+3$ ning ildizlari bo'lsa,

$$q=\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_2}$$ ni hisoblang.

567. x_1, x_2, x_3 lar $f(x)=x^3-x^2-x-2$ ning ildizlari bo'lsa,

$$q = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_2}$$ ni hisoblang.

568. x_1 , x_2 , x_3 lar $f(x)=x^3-5x^2+2x-7$ ning ildizlari bo'lsa,

$$q = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_2}$$ ni hisoblang.

569. Shunday $g(x)$ uchhad tuzingki, berilgan $f(x)$ uchhad ildizlarining kvadratlari uning ildizlari bo'lsin

$$f(x)=x^2-3x+4.$$

570. Shunday $g(x)$ uchhad tuzingki, berilgan $f(x)$ uchhad ildizlarining kvadratlari uning ildizlari bo'lsin

$$f(x)=x^2-4x-3.$$

571. Shunday $g(x)$ uchhad tuzingki, berilgan $f(x)$ uchhad ildizlarining kvadratlari uning ildizlari bo'lsin

$$f(x)=x^2-5x+8.$$

572. Shunday $g(x)$ uchhad tuzingki, berilgan $f(x)$ uchhad ildizlarining kvadratlari uning ildizlari bo'lsin

$$f(x)=x^2-6x-9.$$

573. Gorner sxemasi yordamida quyidagi $f(x)$ ko'phadni $x=1$ ildizining karrasini aniqlang

$$f(x)=x^4-2x^3+3x^2-4x+2$$

574. Gorner sxemasi yordamida quyidagi $f(x)$ ko'phadni $x=-1$ ildizining karrasini aniqlang

$$f(x)=x^4+5x^3+9x^2+7x+2$$

575. Gorner sxemasi yordamida quyidagi $f(x)$ ko'phadni $x=2$ ildizining karrasini aniqlang

$$f(x)=x^3-3x^2+4$$

576. Gorner sxemasi yordamida quyidagi $f(x)$ ko'phadni $x=-2$ ildizining karrasini aniqlang

$$f(x)=x^3+3x^2-4$$

577. Shunday darajasi eng kichik ko'phad tuzingki, 1, 0, -1 nuqtalarda mos ravishda 2, 3, 4 qiymatlarni qabul qilsin.
578. Shunday darajasi eng kichik ko'phad tuzingki, 1, -1, 2 nuqtalarda mos ravishda 3, 1, 1 qiymatlarni qabul qilsin.
579. Shunday darajasi eng kichik ko'phad tuzingki, -1, 1, 3 nuqtalarda mos ravishda 1, 2, 2 qiymatlarni qabul qilsin.
580. Shunday darajasi eng kichik ko'phad tuzingki, -2, 1, 3 nuqtalarda mos ravishda 1, 2, 3 qiymatlarni qabul qilsin.
581. Kompleks tekslikda $f(z)=5z^4-3z^3+2z^2-8z+7$ ko'phadning barcha ildizlari yotadigan doira toping.
582. Kompleks tekslikda $f(z)=4z^3-3z^2+2z+5$ ko'phadning barcha ildizlari yotadigan doira toping.
583. Kompleks tekslikda $f(z)=5z^4-4z^3+3z^2+z+2$ ko'phadning barcha ildizlari yotadigan doira toping.
584. Kompleks tekslikda $f(z)=z^4+z^3-2z^2+3z+1$ ko'phadning barcha ildizlari yotadigan doira toping.
585. $f(x)=3x^5-4x^4+2x^3-x^2+6x-3$ ko'phadning barcha haqiqiy ildizlari yotadigan interval toping.
586. $f(x)=x^4+x^3-2x^2-x+1$ ko'phadning barcha haqiqiy ildizlari yotadigan interval toping.
587. $f(x)=3x^4-4x^3+2x^2-4x-1$ ko'phadning barcha haqiqiy ildizlari yotadigan interval toping.
588. Quyidagi ko'phadning barcha koeffitsientlar yig'indisini toping

$$f(x)=(x^2-1)^{99}$$

589. Quyidagi ko'phadning barcha koeffitsientlar yig'indisini toping

$$f(x)=(x-1)^{40}(x^2-4)^{60}$$

590. Quyidagi ko'phadning barcha koeffitsientlar yig'indisini toping

$$f(x)=(x-1)(x^2+x+1)^{47}$$

591. Quyidagi ko'phadning barcha koeffitsientlar yig'indisini toping

$$f(x)=(x^3-1)^{69}$$

592. Quyidagi ko'phadning barcha koeffitsientlar yig'indisini toping

$$f(x)=(x^3-1)(x^2-x-5)^{17}$$

593. Quyidagi ko'phadning barcha koeffitsientlar yig'indisini toping

$$f(x)=(x^2+1)^{10}$$

594. Quyidagi ko'phadni ikkita ko'phad ko'paytmasiga ajrating

$$f(x)=x^4+4$$

595. Quyidagi ko'phadni ikkita ko'phad ko'paytmasiga ajrating

$$f(x)=x^{10}-1$$

596. Quyidagi ko'p o'zgaruvchili ko'phadni leksikografik ko'rinishda yozing

$$f(x)=2x_1x_2^2x_3^2+3x_1x_2^2-x_1^2x_2x_3^2+4x_1^2x_2x_3^4+2$$

597. Quyidagi ko'p o'zgaruvchili ko'phadni leksikografik ko'rinishda yozing

$$f(x)=3x_2^3-2x_3^2+4x_1^2x_2^2x_3^2+x_1^3x_2+5x_1^3x_2x_3+3$$

598. Quyidagi ko'p o'zgaruvchili ko'phadni leksikografik ko'rinishda yozing

$$f(x)=5x_1x_2^2x_3^2-2x_1x_2^2+3x_1^2x_3^2+2x_1^3x_2+4x_1^3x_2x_3-4$$

599. Quyidagi ko'p o'zgaruvchili ko'phadni leksikografik ko'rinishda yozing

$$f(x)=7x_3^2x_1+3x_1^3-2x_1^2x_2x_3+3x_1^2x_2x_3^2+6x_1^3x_3+1$$

600. Quyidagi bir jinsli simmetrik ko'phadni asosiy simmetrik ko'phadlar orqali ifodalang

$$f(x)=x_1^2x_2^2+x_1^2x_3^2+x_2^2x_3^3$$

601. Quyidagi bir jinsli simmetrik ko'phadni asosiy simmetrik ko'phadlar orqali ifodalang

$$f(x)=(x_1+x_2)(x_1+x_3)(x_2+x_3)$$

602. Quyidagi bir jinsli simmetrik ko'phadni asosiy simmetrik ko'phadlar orqali ifodalang

$$f(x)=(x_1+x_2-x_3)(x_1-x_2+x_3)(-x_1+x_2+x_3)$$

603. Quyidagi bir jinsli simmetrik ko'phadni asosiy simmetrik ko'phadlar orqali ifodalang

$$f(x)=(x_1+x_2+2x_3)(x_1+2x_2+x_3)(2x_1+x_2+x_3)$$

604. Quyidagi to'g'ri ratsional kasrni, sodda kasrlar yig'indisiga yoyib yozing

$$\frac{2}{x^2-1}$$

605. Quyidagi to'g'ri ratsional kasrni, sodda kasrlar yig'indisiga yoyib yozing

$$\frac{4}{(x-1)(x+3)}$$

606. Quyidagi to'g'ri ratsional kasrni, sodda kasrlar yig'indisiga yoyib yozing

$$\frac{2}{(x^2+1)(x-1)}$$

607. Quyidagi to'g'ri ratsional kasrni, sodda kasrlar yig'indisiga yoyib yozing

$$\frac{9}{(x+1)^2(x-2)}$$

13§. Uchinchi va to'rtinchi darajali tenglamalar.

Uchinchi darajali tenglamalar. Kompleks sonlar maydonidagi uchinchi darajali tenglamaning ikkala tomonini bosh koeffitsientga bo'lib, uni:

$$x^3+ax^2+bx+c=0 \tag{1}$$

ko'rinishga keltirish mumkin.

Bu tenglama quyidagi metod bilan echiladi.

(1) tenglamani yangi noma'lum y ga nisbatan ikkinchi darajali had ishtirok etmagan uchinchi darajali tenglamaga quyidagicha keltirish mumkin: α ni (1) tenglamada $x=y+\alpha$ almashtirishni bajargandan keyin yuqoridagi shartni qanoatlantiruvchi uchinchi darajali tenglama hosil bo'ladigan qilib tanlaymiz.

(1) da x o'rniga $y+\alpha$ ni qo'yib y^2 ning koeffitsientini nolga tenglashdan $3\alpha+a=0$ tenglama kelib chiqadi. Bu tenglamadan $\alpha=-\frac{a}{3}$ topiladi.

Aytilganlarga asosan (1) tenglamada

$$x=y-\frac{a}{3} \tag{2}$$

almashtirishni bajarsak,

$$y^3+py+q=0 \tag{3}$$

hosil bo'ladi, bunda:

$$p=b-\frac{a^2}{3}, \qquad q=\frac{2a^3}{27}-\frac{ab}{3}+c. \tag{4}$$

(3)-uchinchi darajali tenglamaning *normal* shakli deb ataladi.

(3) normal tenglamani yechish uchun

$$y=u+v \tag{5}$$

deymiz, bunda u va v-yangi noma'lumlar. Bu ifodani (3) tenglamaga qo'ysak, quyidagi kelib chiqadi:

$$(u+v)^3+p(u+v)+q=0,$$

bundan:

$$(u^3+v^3+q)+(3uv+p)(u+v)=0. \qquad (6)$$

Endi , u va v noma’lumlarni shunday aniqlaylikki ,

$$3uv+p=0 \quad \text{ëki} \quad uv=-\frac{p}{3} \qquad (7)$$

bajarilsin. Bu vaqtda (6) va (7) dan:

$$u^3+v^3=-q \quad , \qquad u^3v^3=-\frac{p^3}{27}$$

hosil bo’ladi. Ko’ramizki, u^3 va v^3 ushbu:

$$z^2+qz-\frac{p^3}{27}=0$$

kvadrat tenglamaning ildizlaridan iborat. Bu tenglamani yechib, quyidagini topamiz:

$$z_{1,2}=-\frac{q}{2}\pm\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}$$

yoki

$$u^3=z_1=-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}} \quad \text{va} \quad v^3=z_2=-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}},$$

bundan, (5) ga ko’ra:

$$y=u+v=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}. \qquad (8)$$

(8) tenglik, odatda, *Kardano formulasi* deb ataladi. Bu tenglik-ikkita ildizning yig’indisidan iborat bo’lib, har bir ildiz uchta qiymatga ega; u ning har bir qiymatini v ning har bir qiymati bilan olsak, $y=u+v$ uchun hammasi bo’lib to’qqizta qiymatni hosil qilamiz. Ammo (3) tenglama faqat uchta ildizga ega; shu sababli, yuqoridagi to’qqizta qiymatdan uchtasini, ya’ni $y=u+v$ yig’indining (7) shartni qanoatlantiruvchi qiymatlarini olishimiz kerak. Shu maqsadda avval:

$$u=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

ildizning uchta qiymatini topamiz. Buning uchun, ma’lumki, u ning bitta, masalan, u_1 ildizini 1 ning uchinchi darajali

$$\sqrt[3]{1}=\cos 0+i\sin 0=1,$$
$$\sqrt[3]{1}=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+i\frac{\sqrt{3}}{2}=\varepsilon,$$
$$\sqrt[3]{1}=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}=\varepsilon^2$$

ildizlariga ko’paytirishimiz lozim. Natijada u ning uchinchi darajali ildizlari $u_1, u_2=\varepsilon u_1, u_3=\varepsilon^2 u_1$ bo’ladi.

Endi v ning tegishli qiymatlarini (7) shartdan topamiz:

$$v_1=-\frac{p}{3u_1};\quad v_2=-\frac{p}{3u_2}=-\frac{p}{3\varepsilon u_1}=\varepsilon^2\left(-\frac{p}{3u_1}\right)=\varepsilon^2 v_1;$$
$$v_3=-\frac{p}{3u_3}=-\frac{p}{3\varepsilon^2 u_1}=\varepsilon\left(-\frac{p}{3u_1}\right)=\varepsilon v_1,$$

bunda $\varepsilon^3=1$ dan foydalandik. Shunday qilib, u ning har bir qiymatini v ning mos qiymatiga qo’shsak, y uchun quyidagi uchta qiymat kelib chiqadi:

$$y_1=u_1+v_1,\quad y_2=\varepsilon u_1+\varepsilon^2 v_1,\quad y_3=\varepsilon^2 u_1+\varepsilon v_1.$$

Agar bu tenglamalarga ε va ε^2 ning qiymatlarini qo’ysak, (3) normal tenglamaning ildizlari quyidagilarga teng bo’ladi:

$$\begin{aligned} y_1&=u_1+v_1,\\ y_2&=-\frac{1}{2}(u_1+v_1)+i\frac{\sqrt{3}}{2}(u_1-v_1),\\ y_3&=-\frac{1}{2}(u_1+v_1)-i\frac{\sqrt{3}}{2}(u_1-v_1). \end{aligned}\tag{9}$$

Endi, (2) tenglikdan foydalanib, (1) tenglamaningildizlarini topamiz:

$$\begin{aligned} x_1&=u_1+v_1-\frac{a}{3},\\ x_2&=-\frac{1}{2}(u_1+v_1)+i\frac{\sqrt{3}}{2}(u_1-v_1)-\frac{a}{3},\\ x_3&=-\frac{1}{2}(u_1+v_1)-i\frac{\sqrt{3}}{2}(u_1-v_1)-\frac{a}{3}. \end{aligned}\tag{10}$$

Misol. $x^3+3x^2+15x+13=0$ tenglamani yechaylik. Bunda $a=3,\ b=15,\ c=13$ bo’lgani uchun, (4) tengliklarga asosan, $p=15-\frac{9}{3}=12$ va $q=\frac{2\cdot 3^3}{27}-\frac{3\cdot 15}{3}+13=0$. Endi

$$u=\sqrt[3]{o+\sqrt{0+\frac{12^3}{27}}}=\sqrt[3]{\sqrt{64}}=\sqrt[3]{8}.$$

Agar $u_1=2$ desak, $v_1=-\frac{12}{3\cdot 2}=-2$ hosil bo'ladi.

Demak, (10) ga binoan, berilgan tenglamaning ildizlari quyidagilardan iborat:

$$x_1=2-2-1=-1,\quad x_2=i\frac{\sqrt{3}}{2}(2+2)-1=2\sqrt{3}i-1,\quad x_3=-i\frac{\sqrt{3}}{2}(2+2)-1=-2\sqrt{3}i-1.$$

To'rtinchi darajali tenglamalar. Kompleks sonlar maydonidagi to'rtinchi darajali tenglamaning ikkala tomonini bosh koeffitsientga bo'lib, uni:

$$x^4+ax^3+bx^2+cx+d=0 \tag{1}$$

ko'rinishga keltira olamiz.

To'rtinchi darajali tenglamani echishning usullari bor. Biz ularning ba'zilarini ko'rib o'tamiz.

1.Ferrari usuli. (1) tenglamaning keyingi uchta hadini o'ng tomonga o'tkazib, ikkala tomonga $\frac{a^2x^2}{4}$ ni qo'shamiz. Natijada:

$$\left(x^2+\frac{ax}{2}\right)^2=\left(\frac{a^2}{4}-b\right)x^2-cx-d$$

hosil bo'ladi. Endi, so'nggi tenglamaning ikkala tomoniga $\left(x^2+\frac{ax}{2}\right)y+\frac{y^2}{4}$ yig'indini qo'shib, ushbuga ega bo'lamiz:

$$\left(x^2+\frac{ax}{2}+\frac{y}{2}\right)^2=\left(\frac{a^2}{4}-b+y\right)x^2+\left(\frac{ay}{2}-c\right)x+\left(\frac{y^2}{4}-d\right). \tag{2}$$

Yangi y noma'lumni (2) tenglamaning o'ng tomoni to'liq kvadratdan iborat bo'lib qoladigan qilib tanlaymiz. Buning uchun:

$$\frac{a^2}{4}-b+y=A^2,\quad \frac{ay}{2}-c=2AB,\ \frac{y^2}{4}-d=B^2 \tag{3}$$

deb olishimiz kerak. Ushbu:

$$4A^2B^2=(2AB)^2$$

ayniyatga asosan, quyidagi natijaga kelamiz:

$$4\left(\frac{a^2}{4}-b+y\right)\left(\frac{y^2}{4}-d\right)=\left(\frac{ay}{2}-c\right)^2. \tag{4}$$

Qavslarni ochib, y ning darajalariga nisbatan o'xshash hadlarni yig'sak,

$$y^3-by^2+(ac-4d)y-[d(a^2-4b)+c^2]=0 \qquad (5)$$

shakldagi uchinchi darajali tenglamaga kelamiz. Ko'ramizki, y noma'lumning qiymatlari - shu (4) tenglamaning ildizlaridan iborat. Bu tenglamani *hal qiluvchi* tenglama yoki (1) tenglamaning *rezolventasi* deyiladi.

Agar (5) tenglamaning bironta ildizini y_0 bilan belgilasak, bu qiymatda (4) tenglik bajarilib, (3) tengliklarga asosan, (2) tenglama quyidagi:

$$\left(x^2+\frac{ax}{2}+\frac{y_0}{2}\right)^2=(Ax+B)^2$$

yoki

$$x^2+\frac{ax}{2}+\frac{y_0}{2}=\pm(Ax+B)$$

ko'rinishni oladim va, demak, berilgan to'rtinchi darajali tenglama ikkita:

$$x^2+\frac{ax}{2}+\frac{y_0}{2}=Ax+B,$$

$$x^2+\frac{ax}{2}+\frac{y_0}{2}=-Ax-B$$

kvadrat tenglamaning ko'paytmasiga yoyiladi. Bu tenglamalarni yechib, berilgan to'rtinchi darajali tenglamaning to'rtta ildizini topamiz.

Misol. $x^4+3x^3-5x-3=0$ tenglamani yechaylik. Bunda $a=3,\ \ b=0,\ \ c=-5,\ \ d=-3$.

Avval (5) tenglamani tuzamiz; a,b,c,d ning qiymatlarini (5) ga qo'yib, ushbuni topamiz:

$$y^3-3y+2=0.$$

Bu tenglamani yechamiz:

$$u=\sqrt[3]{-1+\sqrt{1-1}}=\sqrt[3]{-1},$$

bundan $u_1=-1;\quad v_1=-\dfrac{-3}{3(-1)}=-1;$ Demak, $y_0-1-1=-2.$

(3) tengliklardan A va B ni aniqlaymiz: $A^2=\frac{9}{4}-2=\frac{1}{4};\quad A=\pm\frac{1}{2};$ masalan, $A=\frac{1}{2}$ ni olsak, $2AB=\frac{3(-2)}{2}+5=2$ dan $B=2$ ni hosil qilamiz. Shunday qilib:

$$x^2+\frac{3x}{2}-1=\frac{1}{2}x+2,$$

$$x^2+\frac{3x}{2}-1=-\frac{1}{2}x-2.$$

Bu tenglamalarni yechib, berilgan tenglamaning ildizlarini topamiz:

$$x_1=\frac{-1+\sqrt{13}}{2},\qquad x_1=\frac{-1-\sqrt{13}}{2},\quad x_3=x_4=-1.$$

2.Lobachevskiy usuli.

$$x=y-\frac{a}{4} \tag{6}$$

almashtirish yordami bilan (1) tenglamani:

$$y^4+py^2+qy+r=0 \tag{7}$$

shaklga keltiramiz. Buni to'rtinchi darajali tenglamaning *normal* shakli deyiladi.

Bu normal tenglamaning istalgan ildizini y bilan belgilab, ushbu:

$$z^3-yz^2+az+\beta=0 \tag{8}$$

yordamchi tenglamani tuzamiz; shuni aytish kerakki, α ning qiymati bizga kerak bo'lmaydi, β ning qiymati esa keyinroq aniqlanadi. Agar (8) tenglamada z ni $-z$ bilan almashtirsak,

$$z^3+yz^2+az-\beta=0 \tag{9}$$

tenglama hosil bo'ladi. So'nggi (8) va (9) tenglamalarni o'zaro ko'paytirib,

$$u^3+lu^2+mu-n=0 \tag{10}$$

tenglamani hosil qilamiz, bunda $u=z^2$ va

$$l=2\alpha-y^2,\ m=\alpha^2+2\beta y,\ n=\beta^2.$$

Bu tengliklarning birinchi va uchinchisidan α va β ni aniqlab, ikkinchisiga qo'ysak,

$$y^4+2ly^3+8\sqrt{n}y+(l^2-4m)=0$$

tenglama kelib chiqadi. Bu tenglamaning xuddi yuqoridagi (7) tenglamadan iborat bo'lishini talab qilib,

$$2l=p,\ 8\sqrt{n}=q,\ l^2-4m=r$$

deymiz, bundan:

$$l=\frac{p}{2},\ n=\frac{q^2}{64},\ m=\frac{1}{4}\left(\frac{p^2}{4}-r\right) \tag{11}$$

hosil bo'ladi; $\beta^2=n$ va $n=\frac{q^2}{64}$ tengliklarga asosan, $\beta=\frac{q}{8}$ deb hisoblashimiz mumkin.

(11) tengliklardan foydalanib, (10) tenglamani:

$$u^3+\frac{p}{2}u^2+\frac{1}{4}\left(\frac{p^2}{4}-r\right)u-\frac{q^2}{64}=0 \tag{12}$$

shaklga keltiramiz. Bu-*hal qiluvchi* tenglama yoki (7) tenglamaning *rezolventasidir*.

Agar u_1, u_2, u_3 bilan (12) tenglamaning ildizlarini belgilasak, $z^2 = u$ ga asosan $z_1 = \sqrt{u_1},\ z_2 = \sqrt{u_2},\ z_3 = \sqrt{u_3}$ sonlar (8) yordamchi tenglamaning ildizlariniifoda laydi. Shu sababli:

$$z_1 + z_2 + z_3 = y, \quad z_1 z_2 z_3 = -\beta = -\frac{q}{8}$$

shartlar bajariladi.

Ko'ramizki, $z_1,\ z_2,\ z_3$ qiymatlar $z_1 z_2 z_3 = -\frac{q}{8}$ shartni qanoatlantiradigan bo'lsa,

$$\begin{aligned} &z_1, \quad -z_2, \quad -z_3; \\ &-z_1, \quad z_2, \quad -z_3; \\ &-z_1, \quad -z_2, \quad z_3 \end{aligned}$$

qiymatlar ham bu shartni qanoatlantiradi. Demak, (7) tenglamaning ildizlari quyidagilardan iborat bo'ladi:

$$\begin{aligned} y_1 &= z_1 + z_2 + z_3, \\ y_2 &= z_1 - z_2 - z_3, \\ y_3 &= -z_1 + z_2 - z_3, \\ y_4 &= -z_1 - z_2 + z_3. \end{aligned}$$

Bu qiymatlarni (6) ga qo'yib, (1) tenglamaning ildizlarini topamiz.

Misol. $x^4 + 4x^3 + \frac{5}{2}x^2 - 2x - \frac{3}{16} = 0$ tenglamani yechaylik. Avval $x = y - 1$ almashtirishni bajarib, normal tenglamaga o'tamiz:

$$y^4 - \frac{7}{2}y^2 + y + \frac{21}{16} = 0.$$

Bunda $p = -\frac{7}{2},\ q = 1,\ r = \frac{21}{16}$ ekanini e'tiborga olib, (12) rezolventani tuzamiz:

$$u^3 - \frac{7}{4}u^2 + \frac{7}{16}u - \frac{1}{64} = 0.$$

Bu tenglamani ko'paytuvchilarga ajratib yechish engildir.

$$\left(u^3 - \frac{1}{64}\right) - \frac{7}{4}u\left(u - \frac{1}{4}\right) = \left(u - \frac{1}{4}\right)\left(u^2 + \frac{1}{4}u + \frac{1}{16}\right) - \frac{7}{4}u\left(u - \frac{1}{4}\right) =$$

$$= \left(u - \frac{1}{4}\right)\left(u^2 - \frac{3}{2}u + \frac{1}{16}\right) = 0.$$

Demak, $u_1 = \frac{1}{4}, \quad u_{2,3} = \frac{3 \pm 2\sqrt{2}}{4}.$

Bunda $q = 1 > 0$ bo'lgani uchun, $z_1 z_2 z_3 = -\frac{q}{8} = -\frac{1}{8}$ shartni qanoatlantirish maqsadida: $z_1 = -\sqrt{\frac{1}{4}} = -\frac{1}{2}, \quad z_{2,3} = +\sqrt{\frac{3 \pm 2\sqrt{2}}{4}} = \frac{\sqrt{2} \pm 1}{2}$ deb olamiz. Natijada:

$$x_1=-\frac{1}{2}+\frac{\sqrt{2}+1}{2}+\frac{\sqrt{2}-1}{2}-1=\frac{-3+2\sqrt{2}}{2},$$
$$x_2=-\frac{1}{2}-\frac{\sqrt{2}+1}{2}-\frac{\sqrt{2}-1}{2}-1=\frac{-3-2\sqrt{2}}{2},$$
$$x_3=\frac{1}{2}+\frac{\sqrt{2}+1}{2}-\frac{\sqrt{2}-1}{2}-1=\frac{1}{2},$$
$$x_4=\frac{1}{2}-\frac{\sqrt{2}+1}{2}+\frac{\sqrt{2}-1}{2}-1=-\frac{3}{2}.$$

608-628. Kardano formulasidan foydalanib, quyidagi tenglamalarning yechimlarini toping.

608. $x^3+3x^2+15x+13=0$
609. $x^3-2x-4=0$
610. $x^3-3x+2=0$
611. $x^3-7x+6=0$
612. $x^3-6x+9=0$
613. $x^3+12x+63=0$
614. $x^3+3x^2-4=0$
615. $8x^3+24x^2-81=0$
616. $x^3+3x^2+3x-26=0$
617. $x^3+9x^2+18x+28=0$
618. $x^3+12x^2+45x+54=0$
619. $x^3-3x^2-3x+11=0$
620. $27x^3+108x^2+144x+118=0$
621. $27x^3-81x^2+297x+242=0$
622. $x^3+4x^2+x-6=0$
623. $x^3+8x^2+11x-20=0$
624. $x^3-2x^2+9x-18=0$
625. $x^3-x^2+4x-4=0$
626. $x^3+3x^2+3x-36=0$
627. $x^3-5x^2+x-5=0$
628. $x^3+2x^2+89x+18=0$

629-641. Ferrari usuli bilan quyidagi to’rtinchi darajali tenglamalarni yeching:

629. $x^4+4x+3=0$
630. $x^4+2x^3+x+2=0$
631. $x^4-4x+3=0$

632. $x^4-2x^3-x+2=0$
633. $x^4-8x^3+18x^2-27=0$
634. $x^4-2x^3+2x^2+4x-8=0$
635. $x^4-2x^3-8x^2+13x-24=0$
636. $x^4+2x^3-13x^2-38x-24=0$
637. $4x^4-7x^2-5x-1=0$
638. $x^4+4x^3-2x^2-12x+9=0$
639. $x^4-3x^3+2x^2+3x-9=0$
640. $x^4+x^3-10x^2-2x+4=0$
641. *a)* $x^4-2x^3-7x^2+8x+12=0$ *b)* $x^4+3x^3-5x-3=0$

642-649. Lobachevskiy usuli bilan quyidagi to'rtinchi darajali tenglamalarni yeching:

642. $x^4-4x^3-22x^2+100x-75=0$
643. $x^4+8x^3 32x^2+80x+100=0$
644. $x^4-8x^3+24x^2x-8=0$
645. $x^4+x^3-3x^2-4x-1=0$
646. $x^4-3x^3-6x^2+22x-12=0$
647. $x^4-4x^3+1=0$
648. $3x^4-4x^3-x^2-x-2=0$
649. $4x^4-2x^3-16x^2+5x+9=0$

14§. Shturm funktsiyalari. Shturm teoremasi.

650-670**.** Haqiqiy sonlar maydonidagi tenglamalarning haqiqiy ildizlari uchun chegaralar topilsin (Nyuton usuli):
650. $x^4-12x^2-x-4=0$
651. $x^7+2x^5-2x^4+4x^3+9x^2-x+13=0$
652. $x^{10}-x^3+5x^2-x+10=0$
653. $x^4+1{,}5x^3+8{,}5x^2-7=0$
654. $x^4-4x^3+7x^2-8x+3=0$
655. $x^5+7x^3-3=0$
656. $x^7-108x^5-445x^3+900x^2+801=0$
657. $x^4+4x^3-8x^2-10x+14=0$
658. $x^3-\sqrt{2}x^2+\sqrt{3}x-\sqrt{6}=0$
659. $2x^4+\sqrt{5}x^3-15x^2+x-\sqrt{5}=0$

660. $27x^3+108x^2+144x+118=0$
661. $x^3+2x^2+89x+18=0$
662. $x^3-2x^2+9x-18=0$
663. $x^4-4x^3-22x^2+100x-75=0$
664. $x^4+8x^3 32x^2+80x+100=0$
665. $x^4+2x^3-13x^2-38x-24=0$
666. $x^4-2x^3-8x^2+13x-24=0$
667. $x^6-2x^5-x^4+7x^3-5x+4=0$
668. $x^4-3x^3-6x^2+22x-12=0$
669. $x^4+x^3-10x^2-2x+4=0$
670. $4x^4-2x^3-16x^2+5x+9=0$

671-691. Shturm teoremasidan foydalanib, tenglamani ildizlari qaysi oraliqlarda yotishini ko'rsating:

671. $f(x)=x^4-12x^2-16x-4$
672. $f(x)=x^4-x-1$
673. $f(x)=2x^4-8x^3+8x^2+1$
674. $f(x)=x^4+x^2+1$
675. $f(x)=x^4+4x^3-12x+9$
676. $f(x)=x^4-2x^3-4x^2+5x+5$
677. $f(x)=x^4-2x^3+x^2-9x+1$
678. $f(x)=x^4-2x^3-3x^2+2x+1$
679. $f(x)=x^4-x^3+x^2-x-1$
680. $f(x)=x^4-4x^3-4x^2+4x+1$
681. $f(x)=x^4-2x^3-7x^2+8x+1$
682. $f(x)=x^4-4x^2+x+1$
683. $f(x)=x^4-x^3-x^2-x+1$
684. $f(x)=x^4-4x^3+8x^2-12x+8$
685. $f(x)=x^4-x^3-2x+1$
686. $f(x)=x^4-6x^2-4x+2$
687. $f(x)=4x^4-12x^2+8x-1$
688. $f(x)=x^4-x^3-4x^2+4x+1$
689. $f(x)=x^4-4x^3+x^2+6x+2$
690. $f(x)=x^4-6x^3+8x^2-6x+7$
691. $f(x)=x^4-2x^2+x-5$

15§. Halqa, maydon va gruppalar.

692. Барча бутун сонлар тўплами коммутатив ҳалқа бўладими?
693. Барча жуфт сонлар тўплами ҳалқа бўладими?
694. Барча тоқ сонлар тўплами ҳалқа бўладими?
695. Рационал сонлар тўплами ҳалқа ташкил қиладими?
696. Комплекс сонлар тўплами коммутатив ҳалқа бўладими?
697. **a)** a ва b бутун сонлар бўлганда $a+b\sqrt{p}$ ($p-$туб сон) кўринишдаги сонлар тўплами ҳалқа ташкил қиладими?

b) a ва b рационал сонлар бўлганда $a+b\sqrt{2}$ кўринишдаги сонлар тўплами ҳалқа ташкил қиладими?

c) a ва b рационал сонлар бўлганда $a+b\sqrt{3}$ кўринишдаги сонлар тўплами ҳалқа ташкил қиладими?

d) a ва b рационал сонлар бўлганда $a+b\sqrt{5}$ кўринишдаги сонлар тўплами ҳалқа ташкил қиладими?

698. Рационал сонлардан тузилган $n-$тартибли квадрат матрицалар тўплами(яъни Q тўплам) матрицаларни қўшиш ва кўпайтириш амалларига нисбатан ҳалқа ташкил қиладими?
699. Рационал сонлар тўплами майдон ташкил қиладими?
700. Ҳаққиқий сонлар тўплами майдон ташкил қиладими?
701. Комплекс сонлар тўплами майдон бўладими?
702. **a)** a ва b бутун сонлар бўлганда $a+b\sqrt{p}$ ($p-$туб сон) кўринишдаги сонлар тўплами майдон ташкил қиладими?

b) a ва b рационал сонлар бўлганда $a+b\sqrt{2}$ кўринишдаги сонлар тўплами майдон ташкил қиладими?

c) a ва b рационал сонлар бўлганда $a+b\sqrt{3}$ кўринишдаги сонлар тўплами майдон ташкил қиладими?

d) a ва b рационал сонлар бўлганда $a+b\sqrt{5}$ кўринишдаги сонлар тўплами майдон ташкил қиладими?

703. **a)** Determinanti 1 ga teng bo'lgan 2-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

b) Determinanti -1 ga teng bo'lgan 3-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

c) Determinanti 2 ga teng bo'lgan 3-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

d) Determinanti 0 ga teng bo'lgan 4-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

704. a) Rangi 2 ga teng bo'lgan 3-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

b) Rangi 4 ga teng bo'lgan 4-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

705. a) Dioganali faqat nollardan iborat bo'lgan 3-tartibli kvadrat matritsalar to'plami qo'shishga nisbatan gruppa hosil qiladimi?

b) Dioganali faqat birlardan iborat bo'lgan 3-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

c) Dioganali faqat nollardan iborat bo'lgan 3-tartibli kvadrat matritsalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

706. a) Nol nuqtada nol qiymat qabul qiluvchi funktsiyalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

b) Nol nuqtada nol qiymat qabul qiluvchi funktsiyalar to'plami qo'shishga nisbatan gruppa hosil qiladimi?

c) Nol nuqtada bir qiymat qabul qiluvchi funktsiyalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

707. Juft funktsiyalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

708. a) Toq funktsiyalar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

b) Toq funktsiyalar to'plami qo'shishga nisbatan gruppa hosil qiladimi?

709. a) $\{-1;1\}$ to'plam ko'paytirishga nisbatan gruppa hosil qiladimi?

b) $\{1;-1;i;-i\}$ to'plam ko'paytirishga nisbatan gruppa hosil qiladimi?

710. a) Moduli 1 ga teng bo'lgan barcha kompleks sonlar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

b)Moduli 2 ga teng bo'lgan barcha kompleks sonlar to'plami ko'paytirishga nisbatan gruppa hosil qiladimi?

711. Kompleks tekislikdagi birlik doira tashkarisida yotuvchi sonlar ko'paytirishga nisbatan gruppa hosil qiladimi?

712. Mavhum o'qda yotuvchi noldan farqli barcha sonlar ko'paytirishga nisbatan gruppa hosil qiladimi?

713. Mavhum o'qda yotuvchi barcha kompleks sonlar qo'shishga nisbatan gruppa hosil qiladimi?

714. Uchinchi tartibli simmetrik matritsalar to'plami qo'shishga nisbatan gruppa hosil qiladimi?

715. Uchinchi tartibli kososimmetrik matritsalar to'plami qo'shishga nisbatan gruppa hosil qiladimi?

716. 9 ta elementdan hosil qilingan toq o'rniga qo'yishlar ko'paytirishga nisbatan gruppa hosil qiladimi?

717. 8 ta elementdan hosil qilingan juft o'rniga qo'yishlar ko'paytirishga nisbatan gruppa hosil qiladimi?

718. 1 dan n-tartibli kompleks ildizlar ko’paytirishga nisbatan gruppa hosil qiladimi?

719. Musbat haqiqiy sonlar to’plami ko’paytirishga nisbatan gruppa hosil qiladimi?

720. Butun sonlar to’plami ayirishga nisbatan gruppa hosil qiladimi?

721. Tartibi 5 ga teng bo’lgan ko’phadlar to’plami qo’shishga nisbatan gruppa hosil qiladimi?

16§. Butun sonlarning bo'linish nazariyasi.

Evklid algoritmi. Sonlarning eng katta umumiy bo'luvchisi.

Ta'rif. $a, b \in C$ sonlarning har birini bo'ladigan songa shu sonlarning umumiy bo'luvchisi deyiladi.

Biz faqat natural bo'luvchilar bilan shug'ullanamiz. $a, b \in C$ sonlar bir nechta umumiy bo'luvchiga ega bo'lishi mumkin. Bu bo'luvchilar to'plamini biz $D_{a,b}$ orqali belgilaymiz. Masalan, $a = 24,\ b = 18$ bo'lsin, u holda $D_{24,\ 18} = \{1,\ 2,\ 3,\ 6\}$.

Ta'rif. $a, b \in N$ sonlarning har birini bo'luvchi sonlarning eng kattasi berilgan a va b sonlarning eng katta umumiy bo'luvchisi (EKUB) deyiladi. Ikkita sonning EKUBi odatda $(a,\ b)$ orqali belgilanadi.

Ta'rif. $(a,\ b) = 1$ bo'lsa, a va b sonlar o'zaro tub sonlar deyiladi.

Berilgan sonlarning EKUBini topish uchun avvalo har bir sonning bo'luvchilari to'plamini tuzamiz. Agar A to'plam $a \in N$ sonning bo'luvchilari to'plami, B esa $b \in N$ sonning bo'luvchilari to'plami bo'lsa, $D_{a,b} = A \bigcap B$ bo'lishi ravshan. $A \bigcap B$ kesishmaning eng katta elementi berilgan a, b sonlarning EKUBi bo'ladi. A va B to'plamlar chekli bo'lganligidan $D_{a,b}$ to'plam ham chekli bo'ladi. Natural sonlarning har qanday chekli to'plami doimo eng katta va eng kichik elementga ega.

Teorema. $(a / b) \Rightarrow [(D_{a,\ b} = D_b) \wedge ((a,\ b) = b)]$.

Isbot. a va b sonlarning har bir umumiy bo'luvchisi b ni ham bo'ladi. a / b bo'lgani uchun b ning har bir bo'luvchisi a ning ham bo'luvchisi bo'ladi. Shuning uchun $D_{a,\ b} = D_b$. Lekin b son bo'luvchilarining eng kattasi b ning o'zidir. Shuning uchun $(a,\ b) = b$. Teorema isbot qilindi.

Faraz qilaylik, $a \not/ b$ bo'lsin, u holda qoldiqli bo'lish haqidagi teoremaga asosan quyidagi tengliklar sistemasini yozish mumkin.

$$\begin{array}{ll} a = bq_1 + r_2, & 0 < r_2 < b, \\ b = r_2 q_2 + r_3, & 0 < r_3 < r_2, \\ \dots\dots\dots & \dots\dots \\ r_{n-2} = r_{n-1} q_{n-1} + r_n, & 0 < r_n < r_{n-1}, \\ r_{n-1} = r_n q_n. & \end{array} \qquad (1)$$

(1) sistemaning o'ng tomonidagi tengsizliklarga e'tibor bersak, quyidagi bog'lanish ko'zga tashlanadi:

$$b > r_2 > r_3 > r_4 > \ldots > r_n > 0,$$

bu yerda hamma $r_i (i=\overline{2,\, n})$ lar natural sonlardir. Lekin natural sonlar to'plami quyidan chegaralangan. Shuning uchun biror n nomerdan boshlab $r_{n+1}=0$ bo'ladi.
(1) tengliklar sistemasining birinchisiga asosan a va b ning ixtiyoriy umumiy bo'luvchisi r_2 ni bo'ladi va aksincha $a=r_2+bq_1$ ga asosan r_2 va b ning har qanday umumiy bo'luvchisi a sonni bo'ladi. Demak,

$$(D_{a,\, b}=D_{b,\, r_2}) \Rightarrow (a,\, b)=(b,\, r_2)$$

(1) sistemadagi ikkinchi, uchinchi va undan keyin keladigan tengliklarga asosan

$$D_{a,\, b}=D_{b,\, r_2}=D_{r_2,\, r_3}=\ldots=D_{r_{n-1}, r_n}=D_{r_n} \quad \text{va} \quad (a,\, b)=r_n. \qquad (2)$$

Ikki sonning EKUBini bu usulda topishni birinchi bo'lib, Evklid ko'rsatgani tufayli, bu usul odatda Evklid algoritmi deb yuritiladi. (2) ga asosan $D_{a,\, b}=D_{r_n}$ va va $r_n=(a,\, b)$ bo'lgani uchun quyidagi xulosani yoza olamiz: a va b sonlarning umumiy bo'luvchilari to'plami $D_{a,b}$ shu sonlar EKUBlarining bo'luvchilari to'plami D_{r_n} bilan ustma-ust tushadi va bu sonlarning EKUBi Evklid algoritmidagi noldan farqli eng oxirgi qoldiqqa teng bo'ladi. Bu xulosani qisqa qilib, quyidagicha yozish mumkin:

$$(D_{a,\, b}=D_{r_n}) \wedge ((a,\, b)=r_n).$$

Misol. 76501 va 29719 sonlarning eng katta umumiy bo'luvchisini toping.

Yechish:

$$76501=29719\cdot 2+17063,$$
$$29719=17063\cdot 1+12656,$$
$$17063=12656\cdot 1+4407,$$
$$12656=4407\cdot 2+3842,$$
$$4407=3842\cdot 1+565,$$
$$3842=565\cdot 6+452,$$
$$565=452\cdot 1+113,$$
$$452=113\cdot 4.$$

Demak, (76501, 29719)=113.

Sonli funktsiyalar.

Ta'rif. Natural sonar to'plamida aniqlangan funktsiya sonli funktsiya deyiladi.

Biz ba'zi sonli funktsiyalarni qarab o'tamiz.

1. Berilgan $n \in N$ sonning natural bo'luvchilari sonini $\tau(n)$ deb belgilaymiz. Ma'lumki, har qanday p natural sonni

$$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \cdot \ldots \cdot p_k^{\alpha_k} \tag{3}$$

shaklda yozish mumkin edi. (3) shakldagi sonning barcha natural bo'luvchilari

$$\alpha = p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot p_3^{\beta_3} \cdot \ldots \cdot p_k^{\beta_k} \tag{4}$$

ko'rinishga ega bo'lib, bu erda

$$0 \le \beta_1 \le \alpha_1; \quad 0 \le \beta_2 \le \alpha_2; \quad \ldots .0 \le \beta_k \le \alpha_k. \tag{5}$$

n sonning barcha bo'luvchilarini topish uchun (3) dagi β_i larning mumkin bo'lgan barcha kombinatsiyalarini qarab chiqish kerak. Har bir β_i (4) ga asosan $\alpha_i + 1$ marta o'zgaradi. β_i larning har xil qiymatlariga mos keluvchi kombinatsiyalar soni

$(\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1)$

ga teng. Demak, $\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1)$ ekan.

Misol. $n = 504$ bo'lsin. $504 = 2^3 \cdot 3^2 \cdot 7$ bo'lgani uchun

$\tau(504) = (3+1)(2+1)(1+1) = 24$

2. Berilgan sonning natural bo'luvchilari yig'indisi. Biz oldingi punktda p sonning barcha natural bo'luvchilari sonini ifodalovchi funktsiyani topdik. Endi shu natural bo'luvchilarning yig'indisi qaysi formula orqali berilishini tekshiramiz.

$n \in N$ sonning barcha natural bo'luvchilari yig'indisi odatda $S(n)$ yoki $\sum\limits_{n/d} d$ orqali belgilanadi. Quyidagi ko'paytmani qarab chiqamiz:

$$(1 + p_1 + p_1^2 + \ldots + p_1^{\alpha_1})(1 + p_2 + p_2^2 + \ldots + p_2^{\alpha_2}) \ldots (1 + p_k + p_k^2 + \ldots + p_k^{\alpha_k}) =$$

$$= \sum_{\beta_1, \beta_2, \ldots \beta_k} p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot \ldots \cdot p_k^{\beta_k} \tag{6}$$

Bu yerda har bir β_i mos ravishda 0 dan α_i gacha qiymatlarni qabul qiladi. Geometrik progressiya hadlari yig'indisini topish formulasidan foydalanib, (6) yig'indini quyidagicha yozamiz:

$$\sum_{\beta_1, \beta_2, \ldots \beta_k} p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot \ldots \cdot p_k^{\beta_k} = \frac{p_1^{\alpha_1 + 1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2 + 1} - 1}{p_2 - 1} \cdot \ldots \cdot \frac{p_k^{\alpha_k + 1} - 1}{p_k - 1}. \tag{7}$$

Ikkinchi tomondan, (7) ning chap tomonidagi har bir $p_i^{\beta_i} \, (i = \overline{1, k})(0 \le \beta_i \le \alpha_i)$ son n sonning bo'luvchisidir. Demak, (7) tenglik n sonning natural bo'luvchilari yig'indisini ifodalovchi formuladir, ya'ni

$$S(n)=\frac{p_1^{\alpha_1+1}-1}{p_1-1}\cdot\frac{p_2^{\alpha_2+1}-1}{p_2-1}\cdot\ldots\cdot\frac{p_k^{\alpha_k+1}-1}{p_k-1}\cdot$$

Misol. $$S(504)=S(2^3\cdot 3^2\cdot 7)=\frac{2^{3+1}-1}{2-1}\cdot\frac{3^{2+1}-1}{3-1}\cdot\frac{7^{1+1}-1}{7-1}=1560\cdot$$

722. Quyida berilgan sonlarni Evklid algoritmi yordamida EKUB ini toping:

1. 360 va 248
2. 1024 va 132
3. 1225 va 500
4. 2025 va 400
5. 3464 va 642
6. 876 va 300
7. 864 va 120
8. 720 va 144
9. 372 va 168
10. 144 va 1998

723. Quyida berilgan sonlarning bo'luvchilar soni nechaga teng:

1. 1224, 652
2. 284, 1125
3. 6025, 284
4. 7560, 372
5. 5008, 350
6. 400, 2072
7. 1332, 576
8. 640, 1296
9. 510, 6315
10. 216, 2250

724. Quyida berilgan sonlarning bo'luvchilar yig'indisini toping:

1. 504, 280
2. 180, 216
3. 375, 396
4. 735, 576
5. 480, 168
6. 1224, 1220
7. 400, 1512
8. 1036, 280
9. 325, 2072
10. 6315, 652

17§. Z halqada taqqoslamalar va ularning chegirmalar sinflari. Uzluksiz kasrlar

Uzluksiz(zanjirli) kasrlar. Oldingi laboratoriyadagi (1) tengliklar sistemasining birinchi tengligini b ga, ikkinchisini r_2 ga, uchinchisini r_3 ga va hokazo, eng oxirgisini esa r_n ga bo'lib, quyidagilarga ega bo'lamiz:

$$\frac{a}{b}=q_1+\frac{r_2}{b}=q_1+\frac{1}{\frac{b}{r_2}},$$

$$\frac{b}{r_2}=q_2+\frac{r_3}{r_2}=q_2+\frac{1}{\frac{r_2}{r_3}},$$

$$\frac{r_2}{r_3}=q_3+\frac{r_4}{r_3}=q_3+\frac{1}{\frac{r_3}{r_4}},$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

Bundan

$$\frac{a}{b}=q_1+\frac{1}{\frac{b}{r_2}}=q_1+\frac{1}{q_2+\frac{r_3}{r_2}}. \; . \; . \tag{1}$$

Agar $\frac{r_i}{r_{i+1}}=q_{i+1}+\frac{r_{i+2}}{r_{i+1}}$ nisbatlarni oldingi laboratoriyadagi (1) sistemadan topib (1) ga qo'ysak, $\frac{a}{b}$ nisbat quyidagi ko'rinishni oladi:

$$\frac{a}{b}=q_1+\cfrac{1}{q_2+\cfrac{1}{q_3+\cfrac{1}{q_4+\ddots+\frac{1}{q_n}}}} \tag{2}$$

$\frac{a}{b}$ nisbatning (2) ko'rinishiga uni uzluksiz(zanjirli) kasrga yoyish deyiladi,

$$q_1, \; q_1+\frac{1}{q_2}, \; q_1+\frac{1}{q_2+\frac{1}{q_3}}, \; . \; . \; .$$

lar esa uzluksiz kasrning qismlari (bo'laklari) deyiladi. Uzluksiz kasr quyidagicha belgilanadi:

$$\frac{a}{b}=(\overline{q_1,\ q_2,\ q_3,\ ...,\ q_n})$$

$q_1,\ q_2,\ q_3,\ ...,\ q_n$ lar uzluksiz kasrning qismiy maxrajlari, q_1 esa $\frac{a}{b}$ ratsional sonning butun qismi deyiladi.

Quyidagi uch hol bo'lishi mumkin:

a) $a>b$ bo'lsa, $q_1>0$ bo'ladi;

b) $0\le a<b$ bo'lganda esa $q_1=0$ bo'ladi;

c) $a<0$ bo'lsa, $\frac{a}{b}$ nisbatni $\frac{a}{b}=-k+\frac{r_2}{b},\ k>0$ shaklda yozib olamiz. Bu erda $\frac{r_2}{b}$ to'g'ri musbat kasr bo'ladi. Natijada quyidagi yoyilma hosil bo'ladi:

$$\frac{a}{b}=-k+\frac{r_2}{r}=(\overline{-k,\ q_2,\ q_3,\ ...,\ q_n})$$

1-eslatma. Har qanday butun sonni bir qismli(bo'lakli) uzluksiz kasr deb qarash mumkin.

Masalan, 5=(5).

$\frac{1}{a}$ shakldagi $(a>1)$ kasrni ikki qismli uzluksiz kasr deb qaraladi.

2-eslatma. Agar eng so'nggi qismiy maxrajga hech qanday shart qo'yilmagan bo'lsa, $\frac{a}{b}$ ratsional sonning uzluksiz kasrga yoyilmasi ikkita har xil ko'rinishga ega bo'ladi. $q_n>1$ bo'lsa, $\frac{a}{b}=(\overline{q_1,\ q_2,\ q_3,\ ...,\ q_n})$ bo'ladi.

Faraz qilaylik, $q_n>1$ shart qo'yilmagan bo'lsin. Unda $q_n=(q_n-1)+\frac{1}{1}$ tenglikka asosan $(\overline{q_1,\ q_2,\ ...,\ q_n})=(\overline{q_1,\ q_2,\ ...,\ q_n-1,\ 1})$ kabi yozish mumkin. Bu erda o'ng tomondagi yoyilmada qismlar soni chapdagi yoyilma qismlari sonidan bitta ortiqdir.

Misol.

$$\frac{95}{42}=2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{2}}}}=(\overline{2,\ 3,\ 1,\ 4,\ 2})$$

$(2,\ 3,\ 1,\ 4,\ 2)=(2,\ 3,\ 1,\ 4,\ 1,\ 1).$

Endi sonning butun va kasr qismlari ustida to'xtalib o'tamiz. Qoldiqli bo'lish teoremasiga asosan har qanday $a\in C$ va $m\in N$ uchun

$$a = mq + r, \qquad 0 \le r < m \tag{3}$$

bog’lanish mavjud va yagona edi. (3) ning ikkala tomoni m ga bo’lib, quyidagini hosil qilamiz:

$$\frac{a}{m} = q + \frac{r}{m}, \qquad 0 \le \frac{r}{m} < 1. \tag{4}$$

Demak, q son $\frac{a}{m}$ kasr sondan kichik bo’lgan butun sonlarning eng kattasi ekan. Bu usulda aniqlangan q son $\frac{a}{m}$ ratsional sonning butun qismi deyiladi va u $q = \left[\frac{a}{m}\right]$ orqali belgilanadi. $\frac{a}{m} - q = \frac{r}{m}$ son esa $\frac{a}{m}$ ratsional sonning kasr qismi deyiladi va u $\frac{r}{m} = \left\{\frac{a}{m}\right\}$ orqali belgilanadi.

Misollar.

$\left[\frac{147}{17}\right] = 8;$ $\left\{\frac{147}{17}\right\} = \frac{11}{17};$ $\left\{-\frac{79}{17}\right\} = \frac{6}{17};$ $\{-7{,}25\} = 0{,}75;$ $\{4\} = 0;$ $\left\{\frac{13}{17}\right\} = \frac{13}{17}.$

α sonning butun qismini (4) qoida asosida aniqlash sonning butun qismini ajratish deb ataladi.

Agar α haqiqiy son bo’lsa, uning butun qismi quyidagi shart asosida ajratiladi:

$k \le \alpha < k+1,$ bu yerda $k = [\alpha].$

Har qanday α haqiqiy son uchun quyidagi da’volar chin:

$\{\alpha\} = \alpha - [\alpha],$

$\alpha = [\alpha] + \{\alpha\}. \qquad 0 \le \{\alpha\} < 1.$

Munosib kasrlar. Biz yuqorida har bir ratsional sonni chekli uzluksiz kasrga yoyish mumkinligini ko’rib o’tdik. Endi masalani aksincha qo’yamiz. Har bir chekli uzluksiz kasr biror ratsional sonni ifodalaydimi? Bu masalani hal etishda $\frac{a}{b}$ ratsional sonning munosib kasrlari deb ataluvchi

$$\delta_1 = q_1, \ \delta_2 = q_1 + \frac{1}{q_2}, \ \delta_3 = q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3}}, \ . \ . \ . \tag{5}$$

kasrlar muhim rol o’ynaydi. Bu yerda

$$\delta_n = q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3 + \cfrac{1}{q_4 + \ddots + \frac{1}{q_n}}}}$$

bo'lganidan $n-$munosib kasr $\frac{a}{b}$ ratsional sonning o'zi bo'ladi. $k-$munosib kasr δ_k dan $(k+1)-$ munosib kasr δ_{k+1} ga o'tish uchun δ_k dagi q_k ni $q_k + \frac{1}{q_{k+1}}$ bilan almashtirish lozimligi (5) dan ko'rinib turibdi. Istalgan munosib kasrni hisoblash uchun $P_0 = 1,\ Q_0 = 0,\ P_1 = q_1,\ Q_1 = 1$ deb quyidagilarni yozib olamiz:

$$\delta_1 = \frac{q_1}{1} = \frac{P_1}{Q_1},$$

$$\delta_2 = q_1 + \frac{1}{q_2} = \frac{q_2 \cdot q_1 + 1}{q_2} = \frac{q_2 \cdot q_1 + 1}{q_2 \cdot 1 + 0} = \frac{q_2 \cdot P_1 + P_0}{q_2 \cdot Q_1 + Q_0} = \frac{P_2}{Q_2},$$

$$\delta_3 = \frac{\left(q_2 + \frac{1}{q_3}\right)P_1 + P_0}{\left(q_2 + \frac{1}{q_3}\right)Q_1 + Q_0} = \frac{q_3(q_2 \cdot P_1 + P_0) + P_1}{q_3(q_2 \cdot Q_1 + Q_0) + Q_1} = \frac{q_3 P_2 + P_1}{q_3 Q_2 + Q_1} = \frac{P_3}{Q_3}.$$

Matematik induktsiya prinsipiga asosan

$$\delta_k = \frac{P_k}{Q_k} = \frac{q_k P_{k-1} + P_{k-2}}{q_k Q_{k-1} + Q_{k-2}} \qquad (6)$$

ni yoza olamizyu Bu yerda

$$P_k = q_k P_{k-1} + P_{k-2},$$
$$Q_k = q_k Q_{k-1} + Q_{k-2}. \qquad (7)$$

(6) bog'lanish δ_k munosib kasrni hisoblash uchun xizmat qiladigan rekurrent formuladir. Quyidagi sxema istalgan P_k va Q_k sonlarni hisoblashga imkon beradi.

		q_1	q_2	q_3	q_4	...	q_{k-2}	q_{k-1}	q_k	...	q_n
P_k	$P_0 = 1$	$P_1 = q_1$	P_2	P_3	P_4	...	P_{k-2}	P_{k-1}	P_k	...	P_n
Q_k	$Q_0 = 0$	$Q_1 = 1$	Q_2	Q_3	Q_4	...	Q_{k-2}	Q_{k-1}	Q_k	...	Q_n

Misol. $\overline{(2,\ 3,\ 1,\ 4,\ 2)}$ ga mos ratsional son topilsin.

		$q_1=2$	$q_2=3$	1	4	2
P_k	1	2	$P_2=3\cdot2+1=7$	$P_3=7\cdot1+2=9$	$P_4=9\cdot4+7=$ $=43$	$P_5=43\cdot2+9=$ $=95$
Q_k	0	1	$Q_2=1\cdot3+0=3$	$Q_3=3\cdot1+1=4$	$Q_4=4\cdot4+3=$ $=19$	$Q_5=19\cdot2+4=$ $=42$

Demak, berilgan uzluksiz kasr uchun

$$\delta_1=\frac{1}{2},\quad \delta_2=\frac{7}{3},\quad \delta_3=\frac{9}{4},\quad \delta_4=\frac{43}{19},\quad \delta_5=\frac{95}{42}.$$

Taqqoslama va ularning xossalari. Qoldiqli bo'linish haqidagi teoremaga asosan har qanday ikkita natural son uchun yagona r va q_1 sonlar itopiladiki,

$$a=mq_1+r \tag{8}$$

tenglik bajariladi. Bu yerda $0\le r<m$ bo'lib, m bo'luvchi, q_1 chala bo'linma, r qoldiq deyiladi.

Shunday b sonni olaylikki,

$$b=mq_2+r \tag{9}$$

tenglik o'rinli bo'lsin.

Ta'rif. Agar ikkita butun a va b sonni $m\in N$ ga bo'lganda hosil bo'lgan qoldiqlar o'zaro teng bo'lsa, a va b sonlar m modul bo'yicha teng qoldiqli yoki taqqoslanuvchi deyiladi va

$$a\equiv b(\mathrm{mod}\, m) \tag{10}$$

orqali belgilanadi. Bu yozuv a va b sonlar m modul bo'yicha o'zaro taqqoslanadi deb o'qiladi. (8) dan (9) ni ayiramiz: $a-b=m(q_1-q_2)$ yoki

$$a-b=mt; \qquad t=0,\ \pm1,\ \pm2,\ ... \tag{11}$$

Endi taqqoslamalarning ta'rifidan kelib chiqadigan ba'zi bir sodda xossalar bilan tanishib o'tamiz.

1) m modul bo'yicha taqqoslanuvchi sonlarning ayirmasi shu modulga qoldiqsiz bo'linadi;

2) agar $a=b+mt$ bo'lib, b ni m ga bo'lganda qoldiq r ga teng bo'lsa, a ni ham m ga bo'lgandagi qoldiq r ga teng bo'ladi.

Haqiqatan, $b=mq_1+r$ ni $a=b+mt$ ga qo'yamiz:

$a=mq_1+r+mt=m(q_1+t)+r=mq_2+r.$

Demak, $a=mq_2+r$ bo'lib, a ni m ga bo'lgandagi qoldiq ham r ga teng ekan. Shunday qilib, $a\equiv b(\mathrm{mod}\, m)$ jumlani $a-b=mt$ va $a=b+mt$ jumlalar bilan ekvivalent deyish mumkin.

Agar $a = mq + r$ bo'lsa, $a \equiv r(\mathrm{mod}\, m)$ deb yozish mumkin.

3) agar a / m bo'lsa, $a \equiv 0(\mathrm{mod}\, m)$ bo'ladi.

Taqqoslama

a) refleksivlik;
b) simmetriklik;
c) tranzitivlik

xossalariga ega.

Isboti. a) $a \equiv a(\mathrm{mod}\, m)$, chunki $a - a = 0$ bo'lib, 0 son m ga bo'linadi;

b) $a \equiv b(\mathrm{mod}\, m)$ bo'lsin: $a - b = mt$, bundan $b - a = m(-t)$, demak, $b - a \equiv 0(\mathrm{mod}\, m)$ yoki $b \equiv a(\mathrm{mod}\, m)$;

v) $a \equiv b(\mathrm{mod}\, m)$ va $b \equiv c(\mathrm{mod}\, m)$ bo'lsa, u holda $a \equiv c(\mathrm{mod}\, m)$ bo'ladi. Haqiqatan ham,

$$a = b + mt_1; \qquad b = c + mt_2$$

tengliklarni hadlab qo'shsak, $a - c = mt$ hosil bo'ladi. Bu yerda $t = t_1 + t_2$. Demak, $a \equiv c(\mathrm{mod}\, m)$.

Endi taqqoslamalarning asosiy xossalarini bayon etamiz.

1-xossa. Bir xil modulli taqqoslamalarni hadlab qo'shish (ayirish) mumkin.

2-xossa. Bir xil modulli taqqoslamalarni hadlab ko'paytirish mumkin.

3-xossa. Agar $x \equiv y(\mathrm{mod}\, m)$ bo'lsa, u holda

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_n \equiv a_0 y^n + a_1 y^{n-1} + \ldots + a_n (\mathrm{mod}\, m)$$

bo'ladi.

4-xossa. Agar taqqoslamaning ikkala tomonidagi umumiy bo'luvchi modul bilan o'zaro tub bo'lsa, taqqoslamaning ikkala tomonini shu umumiy bo'luvchiga bo'lish mumkin.

5-xossa. Taqqoslamaning ikkala tomonini va modulni bir xil butun musbat songa ko'paytirish mumkin (agar bo'linsa, bo'lish mumkin).

6-xossa. Agar taqqoslama bir necha modul bo'yicha o'rinli bo'lsa, u shu modullarning eng kichik umumiy karralisi bo'yicha ham o'rinli bo'ladi.

7-xossa. Agar taqqoslama biror m modul bo'yicha o'rinli bo'lsa, u shu modulning ixtiyoriy bo'luvchisi m_1 modul bo'yicha ham o'rinli bo'ladi.

8-xossa. Taqqoslamaning bir tomoni va modulning eng katta umumiy bo'luvchisi bilan ikkinchi tomoni va modulning eng katta umumiy bo'luvchisi o'zaro teng bo'ladi.

Chegirmalar sistemalari. Barcha butun sonlarni biror natural m songa bo'lishdan 0, 1, 2, . . . , $(m-1)$ qoldiqlar hosil bo'ladi. Har bir qoldiqqa sonlarning biror sinfi mos keladi. m modulga bo'lganda bir xil qoldiq qoladigan sonlar to'plamini bitta sinf deb qaraymiz. U sinflarni mos ravishda

$$C_0,\, C_1,\, C_2,\, \ldots,\, C_{m-1} \qquad (12)$$

orqali belgilaymiz.

Bo'linma va qoldiqning mavjudligi va yagonaligi haqidagi teoremaga asosan chegirmalarning m modul bo'yicha har xil sinflari umumiy elementga ega bo'lmaydi. Demak, butun sonlar to'plami o'zaro kesishmaydigan sinflarga yoyiladi.

C_r sinfning elementlari $mq+r$ shaklga ega bo'lib, q ga har xil butun qiymatlar berish natijasida bu elementlarning barchasini hosil qilish mumkin. $m=10$ bo'lganda 3 qoldiq hosil qiladigan sonlar $10q+3$ ko'rinishga ega va $q=0;\ \pm 1;\ \pm 2; \ldots$ desak, . . . , -27, -17, -7, 3, 13, 23, . . . sinf hosil bo'ladi.

Ikkita butun son m modul bo'yicha taqqoslanuvchi bo'lishi uchun ular shu modul bo'yicha bitta sinfning vakili bo'lishi kerakligi o'z-o'zidan ravshan. Har bir sinfning ixtiyoriy elementi shu sinfning chegirmasi deyiladi. Ta'rif. m modul bo'yicha tuzilgan har bir sinfdan bittadan olib tuzilgan sonlar to'plami m modul bo'yicha chegirmalarning to'la sistemasi deyiladi.

$m=10$ modul bo'yicha $10q,\, 10q+1,\, \ldots,\, 10q+9$ sinflarni hosil qilish mumkin. Shularning har biridan bittadan olib tuzilgan 20, 31, 112, 13, 24, 135, 6, 147, -2, -31 sistema 10 modul tuzilgan chegirmalar sistemasidir.

Chegirmalarning m modul bo'yicha to'la sistemasi sifatida $\{0,\, 1,\, 2,\, \ldots,\, (m-1)\}$ to'plam olinadi. Ba'zi hollarda esa absolyut qiymati bo'yicha eng kichik chegirmalar olinadi. m juft bo'lsa, uning ko'rinishi $\left\{0,\, \pm 1,\, \pm 2,\, \ldots,\, \pm\frac{m-1}{2},\, \pm\frac{m}{2}\right\}$, m toq bo'lganda esa $\left\{0,\, \pm 1,\, \pm 2,\, \ldots,\, \pm\frac{m-1}{2}\right\}$ bo'ladi.

Yuqoridagi mulohazalarga asosan quyidagi xulosaga kelamiz. Berilgan sonlar to'plami biror m modul bo'yicha chegirmalarning to'la sistemasini hosil qilishi uchun bu to'plam elementlari quyidagi ikkita shartni qanoatlantirishi kerak:

1) Ular m modul bo'yicha har xil sinflarning vakillari bo'lishi;

2) Ularning soni xuddi modul m ga teng bo'lishi kerak.

1-teorema (chiziqli forma haqida). Agar $ax+b$ chiziqli formadagi x o'zgaruvchi har qanday butun b son va $(a,\, m)=1$ bo'lganda m modul bo'yicha chegirmalarning to'la sistemasini tashkil etsa, u holda $ax+b$ o'zgaruvchi ham m modul bo'yicha chegirmalarning to'la sistemasini tashkil etadi.

Chegirmalarning keltirilgan sistemasi. Taqqoslamalarning 8-xossasiga asosan m modul bo'yicha o'zaro taqqoslanuvchi sonlar m modul bilan bir xil bo'lgan eng katta umumiy bo'luvchiga ega edi. m modul bo'yicha taqqoslanuvchi sonlar bitta sinfning vakillaridan iboratligini biz yuqorida ko'rsatgandik. Demak, sinfning bitta chegirmasi modul bilan o'zaro tub bo'lsa, bu sinfning barcha elementlari ham modul bilan o'zaro tub bo'ladi. Shuning uchun m modul bilan o'zaro tub bo'lgan chegirmalar sinflari to'g'risida gapirish mumkin. Bu sinflar to'plami sonlar nazariyasida muhim rol o'ynaydi.

Ta'rif. m modul bilan o'zaro tub bo'lgan chegirmalar sinflaridan bittadan element olib tuzilgan to'plam chegirmalarning m modul bo'yicha keltirilgan sistemasi deyiladi.

Chegirmalarning keltirilgan sistemasini shu chegirmalarning to'la sistemasidan ham tuzish mumkin. Buning uchun to'la sistemadan m modul bilan o'zaro tub bo'lgan chegirmalarni ajratib olish kifoya. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} to'plam 10 modul bo'yicha chegirmalarning to'la sistemasi bo'lgani holda {1, 3, 7, 9} sistema 10 modul bo'yicha chegirmalarning keltirilgan sistemasidir. Xuddi shunday, {1, 3, -3, -1} ham 10 modul bo'yicha chegirmalarning keltirilgan sistemasi bo'ladi. Chegirmalarning keltirilgan sistemasidagi sonlar sonini aniqlash uchun *Eyler funktsiyasi* deb ataluvchi $\varphi(m)$ funktsiyadan foydalaniladi.

Eyler funktsiyasi har bir musbat butun m son uchun aniqlangan bo'lib, u 0, 1, 2, . . . , $m-1$ sonlar orasida m bilan o'zaro tub bo'lgan sonlar sonini bildiradi.

Berilgan sonlar to'plami m modul bo'yicha chegirmalarning keltirilgan sistemasi bo'lishi uchun quyidagi uchta shart bajarilishi kerak:

1) to'plam elementlari $\varphi(m)$ ta bo'lishi;

2) m modul bo'yicha o'zaro taqqoslanuvchi bo'lmasligi (ya'ni modul bo'yicha har xil sinf vakillari bo'lishi);

3) m modul bilan o'zaro tub bo'lishi zarur.

Misol. $a=15,\ m=14$ bo'lsin. $(15,14)=1,\ m=14$ modul bo'yicha chegirmalarning keltirilgan sistemasi {1, 3, 5, 9, 11, 13} dan iborat. Endi $5x$ ni ($m=14$ modul bo'yicha) hisoblaymiz.

$5 \cdot 1 \equiv 5 \pmod{14}$,
$5 \cdot 3 \equiv 15 \equiv 1 \pmod{14}$,
$5 \cdot 5 \equiv 25 \equiv 11 \pmod{14}$,
$5 \cdot 9 \equiv 45 \equiv 3 \pmod{14}$,
$5 \cdot 11 \equiv 55 \equiv 13 \pmod{14}$,
$5 \cdot 13 \equiv 65 \equiv 9 \pmod{14}$.

Demak, keltirilgan sistema {5, 1, 11, 3, 13, 9} bo'ladi, {1, 3, 5, 9, 11, 13} va {5, 1, 11, 3, 13, 9} sistemalar bir-biridan faqat elementlarining o'rni bilan farq qiladi, xolos. Bu elementlar ko'paytmalari esa o'zaro teng, ya'ni $1 \cdot 3 \cdot 5 \cdot 9 \cdot 11 \cdot 13 = 5 \cdot 1 \cdot 11 \cdot 3 \cdot 13 \cdot 9$.

Eyler funksiyasining xossalari.

Ta'rif. Agar 1) f(n) funksiya barcha natural n sonlar uchun aniqlangan bo'lib, n ning har qanday qiymatida f(n)≠0 bo'lsa va 2) ixtiyoriy o'zaro tub n_1 va n_2 sonlar uchun $f(n_1 \cdot n_2) = f(n_1) \cdot f(n_2)$ shartlar bajarilsa, f(n) funksiya multiplikativ funksiya deyiladi.

Teorema. Eyler funksiyasi multiplikativ funksiyadir.

φ(m) ni hisoblash formulalarini keltiramiz.

a) m=p bo'lsin. U holda φ(p)=p –1 ligi o'z-o'zidan aniq.

Misol. p = 7 bo'lsin. 1, 2, 3, 4, 5, 6 sonlarning har biri 7 bilan o'zaro tubdir.

b) $m = p^{\alpha}$ bo'lsin, $\varphi(p^{\alpha})$ ni hisoblash uchun 1 dan p^{α} gacha sonlarni quyidagicha yozib olamiz.

1, 2, 3, 4, . . . , p, . . . , 2p, . . . , 3p, . . . ,

$$p \cdot p \ldots p^{\alpha-1} p = p^{\alpha}. \qquad (13)$$

Bu qatordagi p, 2p, …, $p^{\alpha-1} \cdot p$ sonlarning barchasi p ga bo'lingani uchun p bilan o'zaro tub emas. p ga bo'linadigan sonlar soni $p^{\alpha-1}$ tadir. (13) qatorda esa p^{α} ta son bor. Demak, p bilan (13) dagi o'zaro tub sonlar soni

$$\varphi(p^{\alpha}) = p^{\alpha} - p^{\alpha-1} = p^{\alpha}\left(1 - \frac{1}{p}\right)$$

ta ekan.

c) $m = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \cdot \ldots \cdot p_k^{\alpha_k}$ bo'lsin. Eyler funksiyasi multiplikativ bo'lgani uchun

$$\varphi(m) = \varphi(p_1^{\alpha_1}) \cdot \varphi(p_2^{\alpha_2}) \cdot \ldots \cdot \varphi(p_k^{\alpha_k})$$

tenglik o'rinli. Har bir ko'paytuvchi uchun b) ni qo'llab, quyidagiga ega bo'lamiz:

$$\varphi(m) = p_1^{\alpha_1}\left(1-\frac{1}{p_1}\right)\cdot p_2^{\alpha_2}\left(1-\frac{1}{p_2}\right)\cdot\ldots\cdot p_k^{\alpha_k}\left(1-\frac{1}{p_k}\right),$$

$$\varphi(m) = p_1^{\alpha_1}\cdot p_2^{\alpha_2}\cdot\ldots\cdot p_k^{\alpha_k}\cdot\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_k}\right),$$

$$\varphi(m) = m\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_k}\right)$$

yoki

$$\varphi(m) = p_1^{\alpha_1-1}(p_1-1)\cdot p_2^{\alpha_2-1}(p_2-1)\ldots p_k^{\alpha_k-1}(p_k-1).$$

Misol. $m = 360 = 2^3\cdot 3^2\cdot 5.$

$$\varphi(m) = \varphi(360) = 2^2(2-1)\cdot 3(3-1)\cdot 5^0(5-1) = 4\cdot 6\cdot 4 = 96.$$

$$\varphi(360) = 96.$$

725. Quyida berilgan ratsional kasrni uzluksiz kasrga yoying:

1. $\frac{113}{47}$ 2. $\frac{57}{23}$ 3. $\frac{67}{23}$ 4. $\frac{95}{41}$

5. $\frac{371}{174}$ 6. $\frac{97}{50}$ 7. $\frac{73}{30}$ 8. $\frac{217}{112}$

9. $\frac{145}{53}$ 10. $\frac{65}{17}$ 11. $\frac{147}{53}$ 12. $\frac{97}{40}$

726. Quyida berilganlarga ko'ra chegirmalar sistemasini yozing:

1. $m=10$ 2. $m=15$
3. $m=13$ 4. $m=17$
5. $m=18$ 6. $m=12$
7. $m=14$ 8. $m=19$
9. $m=16$ 10. $m=21$

727.

1. 4^{25} ni 8 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.
2. 5^{32} ni 24 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

3. 33^{66} ni 5 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

4. 7^{54} ni 9 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

5. 222^{555} ni 7 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

6. $2^{60}+7^{30}$ ni 13 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

7. 3^{32} ni 5 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

8. 9^{40} ni 6 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

9. 12^{20} ni 7 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

10. 4^{35} ni 5 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

11. 17^{24} ni 7 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

12. 3^{2006} ni 5 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

13. 2^{172} ni 9 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

14. 4^{63} ni 3 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

15. 7^{45} ni 3 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

16. $3^{4n+2}+6^{2n+1}$ ni 5 ga bo'lgandagi qoldiqni taqqoslama yordamida toping.

728. Quyida berilgan sonlarning Eyler funksiyasini toping:

1) 375; 2) 720; 3)957; 4) 988; 5) 990; 6) 1200; 7) 1440; 8) 1500; 9)1890; 10) 4320.

729. Quyida berilgan tub sonlarning Eyler funksiyasini toping:

1) 17; 2) 31; 3) 43; 4) 71; 5) 83; 6) 109; 7) 179.

730. Quyida berilgan darajali tub sonlarning Eyler funksiyasini toping:

1) 3^5; 2) 5^4; 3) 11^3; 4) 17^2; 5) 23^2; 6) 31^2.

731. Quyida berilgan darajali tub sonlarning Eyler funksiyasini toping:

1) $5\cdot 11$; 2) $5\cdot 7\cdot 13$; 3) $17\cdot 23$; 4) $12\cdot 17$;
5) $14\cdot 15$; 6) $11\cdot 14\cdot 15$; 7) $32\cdot 81\cdot 49$; 8) $24\cdot 28\cdot 45$;
9) $720\cdot 957$; 10) $990\cdot 1890$.

732. a ni toping, agar $\varphi(a)=3600$ ga va $a=3^{\alpha}\cdot 5^{\beta}\cdot 7^{\gamma}$ ga teng bo'lsa.

733. a ni toping, agar $\varphi(a)=120$ ga va $a=pq$, $p-q=2$ ga teng hamda p, q- har xil tub sonlar bo'lsa.

18§. Bir noma'lumli birinchi darajali taqqoslamalar va ularni yechish

Bir noma'lumli birinchi darajali taqqoslamalar. Bunday taqqoslamalarning umumiy ko'rinishi quyidagicha:

$$ax \equiv b \pmod m), \qquad (1)$$

bu yerda $a,\ b-$butun sonlar. Taqqoslamani yechish deganda uni to'g'ri sonli taqqoslamaga aylantiruvchi sonlar sinfini tushunamiz. Chunki (1) taqqoslamani biror $x_1$son qanoatlantirsa, uni x_1+mt $(t-$butun son) sonlar sistemasi ham qanoatlantiradi. (1) taqqoslamaning yechimini topish uchun biz quyidagi ikkita holni qaraymiz. Bitta sinfdagi barcha yechimlarni bitta yechim deb qabul qilamiz.

1. $(a,\ m)=1.$ Agar (1) taqqoslama yechimga ega bo'lsa, u yechim m modul bo'yicha chegirmalarning birorta sinfidan iborat bo'ladi. Ma'lumki, chegirmalarning to'la sistemasidagi har bir chegirmaga bitta sinf mos kelardi. Demak, x o'zgaruvchi chegirmalarning to'la sistemasini qabul qilar ekan, u holda chiziqli forma haqidagi teoremaga asosan ax ham chegirmalarning to'la sistemasini qabul qiladi.

x o'zgaruvchining biror x_0 qiymatida ax_0 chegirma bilan b son bitta sinfga tegishli bo'ladi, ya'ni $ax_0 \equiv b \pmod m$ bo'lib, $x \equiv x_0 \pmod m$ (1) taqqoslamaning yagona yechimi bo'ladi.

2. $(a,\ m)=d>1.$ Ma'lumki, $ax \equiv b \pmod m$ taqqoslamani $ax-b=my$ deb yozish mumkin. Bu yerda $y-$butun son. Demak, $ax-b=my$ tenglikda $(m/d \wedge a/d) \Rightarrow b/d.$Bundan agar $b \nmid d$, ya'ni b son d ga bo'linmasa, (1) taqqoslama yechimga ega bo'lmaydi, degan natija kelib chiqadi.

Faraz qilaylik, $b=db_1$ bo'lsin. Taqqoslamalarning 5-xossasiga asosan (1) ning ikkala qismi va modulini d ga bo'lib, quyidagini hosil qilamiz:

$$a_1x \equiv b_1 \pmod{m_1}. \qquad (2)$$

Bu yerda $(a_1,\ m_1)=1$ bo'lganidan (1) holga asosan (2) taqqoslama m_1 modul bo'yicha yagona x_0 yechimga ega:

$$x \equiv x_0 \pmod{m_1} \Rightarrow x = x_0 + km_1, \qquad k \in C.$$

Bu yechim (1) ni ham qanoatlantiradi. Lekin (1) ning yechimlari shu bilan tugallanmaydi. Berilgan taqqoslamaning yechimlarini m modul bo'yicha topish uchun quyidagilarga e'tibor beramiz:

$$..., x_1 - m_1,\ x_1,\ x_1 + m_1,\ ...\ ,\ x_1 + (d-1)m_1,\ x_1 + dm_1,\ ... \qquad (3)$$

Bu chegirmalarning har biri m_1 modul bo'yicha teng qoldiqli bo'lib, m modul bo'yicha har xil sinfga tegishlidir. Shu har xil sinflarning vakillari

$$x_1,\ x_1 + m_1,\ x_1 + 2m_1,\ ...\ ,\ x_1 + (d-1)m_1 \qquad (4)$$

dan iborat. Haqiqatan, (4) ning har qanday ikkita elementi m modul bo'yicha taqqoslanuvchi emas. (3) sinfning (4) ga kirmagan har bir elementi uchun (4) dan shunday element topiladiki, ularning ayirmasi $m_1 d = m$ ga bo'linadi. Shuning uchun ular bitta sinfning vakillari hisoblanadi. Demak, $(a,\ m) = d$ va $(b,\ m) = d$ bo'lsa, (1) taqqoslama (4) orqali aniqlanuvchi d ta yechimga ega ekan. Yuqoridagilarga asosan quyidagi xulosalarni yoza olamiz:

Xulosalar. 1. Agar $(a,\ m) = 1$ bo'lsa, (1) ning yechimi mavjud va yagonadir.

2. $(a,\ m) = d > 0$ bo'lib,

$a)$ $a \nmid b$ chin bo'lsa, yechim mavjud emas;

$b)$ a / b chin bo'lsa, (1) taqqoslama d ta yechimga ega.

Misollar. 1. $3x \equiv 7 (\mathrm{mod}\, 11);\ (3, 11) = 1$ bo'lgani uchun yechim yagona bo'ladi. m modul bo'yicha chegirmalarning to'la sistemasi $\{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ dan iborat. Bevosita tekshirib ko'rish bilan $x \equiv -5 (\mathrm{mod}\, 11)$ yechim ekanligiga ishonch hosil qilamiz.

2. $5x \equiv 7 (\mathrm{mod}\, 15);\ (5, 15) = 5,$ lekin $7 \nmid 5$ bo'lgani uchun bu taqqoslama yechimga ega emas.

3. $9x \equiv 6 (\mathrm{mod}\, 15);\quad (9, 15) = 3;\quad (6,\ 15) = 3$ bo'lgani uchun taqqoslama uchta yechimga ega bo'ladi. Haqiqatan, taqqoslamani $3x \equiv 2 (\mathrm{mod}\, 5)$ shaklda yozib olamiz. $(3, 5) = 1$ bo'lgani uchun bu taqqoslama 5 modul bo'yicha yagona yechimga ega bo'ladi. Haqiqatan,

$$x \equiv -1 (\mathrm{mod}\, 5)$$

yechimdir. Berilgan taqqoslamaning yechimi –1, -1+5, -1+10$(\mathrm{mod}\, 15)$ yoki $x \equiv -1 (\mathrm{mod}\, 15).\ 4 (\mathrm{mod}\, 15),\ 9 (\mathrm{mod}\, 15)$ dan iborat.

Birinchi darajali taqqoslamalarni yechish usullari. Bir noma'lumli taqqoslamalarni yechishning bir qancha usullari mavjud. Biz hozir shulardan ba'zilarini bayon etamiz. Faraz qilaylik,

$$\mathrm{ax} \equiv \mathrm{b}\ (\mathrm{mod}\ \mathrm{m})$$

taqqoslama berilgan bo'lsin.

1.Tanlash usuli. Bu usulning mohiyati shundaki, (1) taqqoslamadagi x o'rniga chegirmalarning to'la sistemasidagi barcha chegirmalar ketma-ket qo'yib chiqiladi. Ulardan qaysi biri (1) ni to'g'ri taqqoslamaga aylantirsa, o'sha yechim bo'lib hisoblanadi. Biz yuqoridagi ikkita misolni shu usulda yechdik. Lekin modul ancha katta bo'lganda bu usul uncha samarali bo'lmay qoladi.

2.Koeffisentlarni o'zgartirish usuli. Amalda taqqoslamalarning xossalaridan foydalanib, noma'lum oldidagi koeffisentlarni va b ni shunday o'zgartirish kerakki, o'ng tomonda hosil bo'lgan son x ning koeffisentiga bo'linsin.
Agar $(b, m) = d > 0$ bo'lsa, yangi o'zgaruvchiga o'tish maqsadga muofiq bo'ladi.

Misollar.
1. $7x \equiv 5 (\mathrm{mod}\ 9)$,
$7x \equiv (5+9)(\mathrm{mod}\ 9)$,
$7x \equiv 14(\mathrm{mod}\ 9)$;
$(7,9) = 1$ bo'lganidan
$x \equiv 2(\mathrm{mod}\ 9)$.

2. $17x \equiv 25(\mathrm{mod}\ 28)$;
$17x + 28x \equiv 25(\mathrm{mod}\ 28)$,
$45x \equiv 25 (\mathrm{mod}\ 28)$, $9x \equiv 5(\mathrm{mod}\ 28)$,
$(9,28)= 1$ bo'lganidan $9x \equiv 5(\mathrm{mod}\ 28)$,
$9x \equiv 5 - 140 (\mathrm{mod}\ 28) \equiv - 135 (\mathrm{mod}\ 28)$.
$9x \equiv -135 (\mathrm{mod}\ 28)$,
$x \equiv -15 \equiv 13 (\mathrm{mod}\ 28)$.

3. ***Eyler teoremasidan foydalanish.*** Ma'mulki, $((a,m)=1) \Rightarrow (a^{\varphi(m)} \equiv 1(\mathrm{mod}\ m)$ edi. Bundan $a^{\varphi(m)}\ b \equiv b\ (\mathrm{mod}\ m)$ deb yozish mumkin. Bu taqqoslamalarni $ax \equiv b\ (\mathrm{mod}\ m)$ bilan solishtirib, $x \equiv a^{\varphi(m)-1} \cdot b(\mathrm{mod}\ m)$ ekanligiga ishonch hosil qilamiz. Misollar yechishda $a^{\varphi(m)-1}\ b$ ifodani m modul bo'yicha eng kichik chegirmaga keltirish lozim.

Misol. $3x \equiv 7(\mathrm{mod}\ 11)$
$x \equiv 3^{\varphi(11)-1}\ 7(\mathrm{mod}\ 11)$
$\varphi(11) \equiv 10$; $3^2 \equiv 9 \equiv - 2\ (\mathrm{mod}\ 11)$; $3^4 \equiv 4\ (\mathrm{mod}\ 11)$,

$$35 \equiv 12 \equiv 1 \pmod{11}; \quad 39 \equiv 4 \pmod{11}$$

bo’lganidan $x \equiv 39\ 7 \equiv 28 \equiv 6 \pmod{11}$ yoki $x \equiv 6 \pmod{11}$.
Lekin taqqoslamalarning moduli yetarlicha katta bo’lsa, yuqoridagi usullarning birortasi ham uncha samarali emas. Bunday hollarda quyidagi usul ancha foydalidir.

4. Uzluksiz kasrlardan foydalanish usuli. Ushbu

$$ax \equiv b \pmod{m}$$

taqqoslama berilgan bo’lib, $(a, m) = 1$ va $a > 0$ bo’lsin. m/a kasrni uzluksiz kasrga yoyib, uning munosib kasrlarini P_k / Q_k ($k = 1, 2, \dots, n$) deb belgilaymiz. P_k / Q_k qisqarmas kasr bo’lganidan
$P_n = m$, $Q_n = a$,
u holda oldindan ma’lum $P_n \cdot Q_{n-1}$-$1\ P_{n-1} Q_n = (-1)^n$ tenglik $mQ_{n-1} - P_{n-1}a = (-1)^n$ shaklni oladi. Oxirgi tenglikdan $a P_{n-1} = -(-1)^n + mQ_{n-1}$ yoki $aP_{n-1} = (-1)^{n-1}$ b ga ko’paytiramiz:

$$a\ (-1)^{n-1} \cdot b \cdot P_{n-1} \equiv b \pmod{m}. \qquad (5)$$

(5) va (1) ni solishtirib

$$x \equiv (-1)^{n-1} \cdot b \cdot P_{n-1} \pmod{m} \qquad (6)$$

ekan degan xulosaga kelamiz. Bu yerda P_{n-1} son m/a kasrning $(n-1)$ munosib kasrdagi suratdan iboratdir. (1) taqqoslama yagona yechimga ega bo’lgani uchun (6) yechim (1) ning yechimi b’ladi.

Misol. $285x \equiv 177 \pmod{924}$ taqqoslama yachilsin. $(285, 924) = 3$; $177 = 3 \cdot 59$ bo’lgani sababli taqqoslamaning moduli va ikkala qismini 3 ga bo’lamiz:

$$95x \equiv 59 \pmod{308}$$

Endi 308/59 kasrni munosib kasrga yoyamiz. Buning uchun ketma- kat bo’lishni quyidagicha bajaramiz:

$$308 = 95 \cdot 3 + 23,$$
$$95 = 23 \cdot 4 + 3,$$
$$23 = 3 \cdot 7 + 2,$$
$$3 = 2 \cdot 1 + 1,$$
$$2 = 2 \cdot 1$$

$q_1 = 3$, $q_2 = 4$, $q_3 = 7$, $q_4 = 1$, $q_5 = 1$. 110-§ da bayon qilingan usulga asosan quyidagi jadvalni tuzamiz:

q_k	0	3	4	7	1	2

P_k	1	3	13	94	107	308

Demak, $P_{n-1} = P_4 = 107$. Bundan

$$x \equiv (-1)4 \cdot 107 \cdot 59 \pmod{308},$$

$$x \equiv 153 \pmod{308}.$$

Berilgan taqqoslamalarning yechimlari esa quyidagilardan iborat bo'ladi:

$$x \equiv 153,\ 461,\ 769 \pmod{924}.$$

734. 1) 333^{6666} sonini 5 ga bo'lishdan hosil bo'lgan qoldiqni toping.

2) 12^{2n+1} sonini 133 ga bo'lishdan hosil bo'lgan qoldiqni toping.

3) $12^{2n+1}+11^{n+1}$ yig'indini 133 ga bo'lishdan hosil bo'lgan qoldiqni toping.

4) 15^{30} sonini 8 ga bo'lishdan hosil bo'lgan qoldiqni toping.

5) 25^{20} sonini 19 ga bo'lishdan hosil bo'lgan qoldiqni toping.

6) 22^{555} sonini 7 ga bo'lishdan hosil bo'lgan qoldiqni toping.

7) $3^{4n+2}+6^{2n+1}$ yig'indini 5 ga bo'lishdan hosil bo'lgan qoldiqni toping.

8) $103^{3n+2}+11^{6n+1}$ yig'indini 9 ga bo'lishdan hosil bo'lgan qoldiqni toping.

9) 13^{24} sonini 24 ga bo'lishdan hosil bo'lgan qoldiqni toping.

11) 103^{3n+2} sonini 9 ga bo'lishdan hosil bo'lgan qoldiqni toping.

12) 11^{6n+1} sonini 9 ga bo'lishdan hosil bo'lgan qoldiqni toping.

13) 2^{80} sonini 13 ga bo'lishdan hosil bo'lgan qoldiqni toping.

14) 13^{32} sonini 4 ga bo'lishdan hosil bo'lgan qoldiqni toping.

15) 25^{40} sonini 10 ga bo'lishdan hosil bo'lgan qoldiqni toping.

735. Quyida berilgan taqqoslamani tanlash usulida yeching:

1) $3x \equiv 8 \pmod{13}$

2) $25x \equiv 15 \pmod{17}$

3) $25x \equiv 15 \pmod{17}$

4) $5x \equiv 26 \pmod{12}$

5) $4x \equiv 7 \pmod{8}$

6) $4x \equiv 7 \pmod{8}$

7) $4x \equiv 7 \pmod{8}$

8) $14x \equiv 22 \pmod{36}$

736. Quyida berilgan taqqoslamani uzluksiz kasrlardan foydalanib yeching:

1) $23x \equiv 6678 \pmod{693}$

2) $25x \equiv 15 \pmod{17}$

3) $91x \equiv 143 \pmod{222}$

4) $271x \equiv 25 \pmod{119}$

5) $13x \equiv 178 \pmod{153}$

6) $12x \equiv 79 \pmod{18}$

737. Quyida berilgan taqqoslamani Eyler teoremasidan foydalanib yeching:

1) $5x \equiv 15 \pmod{20}$

2) $9x \equiv 3 \pmod{6}$

3) $9x \equiv 4 \pmod{7}$

4) $3x \equiv 2 \pmod{5}$

5) $5x \equiv 9 \pmod{7}$

19§. Chiziqli fazo va uning qism fazosi. Vektorlar sistemasining chiziqli bog'liqligi va chiziqli erkliligi. Bazis, vektorlarni bazis orqali ifodalash. Bir bazisdan ikkinchi bazisga o'tish.

738. a) $\mathbf{R}^3$ da $x_1 + x_2 + x_3 = 0$ tenglamani qanoatlantiruvchi vektorlar to'plami qism fazo bo'la oladimi?

b) $\mathbf{R}^3$ da $x_1 + x_2 + x_3 = 1$ tenglamani qanoatlantiruvchi vektorlar to'plami qism fazo bo'la oladimi?

c) $\mathbf{R}^3$ da birinchi va uchinchi koordinatalar ustma-ust tushuvchi to'plami qism fazo hosil qiladimi?

739. Tartibi 3 ga teng bo'lgan ko'phadlar to'plami qism fazo hosil qila oladimi?

740. Quyidagi vektorlar sistemasi chiziqli bog'liqmi?

a) $\vec{a}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{a}_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

b) $\vec{a}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

c) $\vec{a}_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

d) $\vec{a}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$

e) $\vec{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

f) $f_1 = \sin x$, $f_2 = \cos x$

j) $f_1 = 1$, $f_2 = \sin^2 x$, $f_3 = \cos^2 x$

741. a) $\bar{e}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ базисда $\bar{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ vektor koordinatalarini toping.

b) $\bar{e}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ базисда $\bar{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ vektor koordinatalarini toping.

c) $\bar{e}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ базисда $\bar{x} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ vektor koordinatalarini toping.

d) $\bar{e}_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ базисда $\bar{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ vektor koordinatalarini toping.

742. a) $\bar{e}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ базисдан $\bar{e}_1' = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ \bar{e}_2' = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ bazisga o'tish matritsasini toping.

b) $\bar{e}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ базисдан $\bar{e}_1' = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \ \bar{e}_2' = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ bazisga o'tish matritsasini toping.

c) $\bar{e}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \ \bar{e}_2 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ базисдан $\bar{e}_1' = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \ \bar{e}_2' = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ bazisga o'tish matritsasini toping.

d) $\bar{e}_1=\begin{pmatrix}3\\1\end{pmatrix},\ \bar{e}_2=\begin{pmatrix}5\\2\end{pmatrix}$ базисдан $\bar{e}_1'=\begin{pmatrix}2\\4\end{pmatrix},\ \bar{e}_2'=\begin{pmatrix}1\\3\end{pmatrix}$ bazisga o'tish matritsasini toping.

743. Quyidagi vektorlarga tortilgan qism fazo bazisini toping:

a) $a_1=\begin{pmatrix}1\\2\\3\end{pmatrix},\ a_2=\begin{pmatrix}4\\5\\6\end{pmatrix},\ a_3=\begin{pmatrix}7\\8\\9\end{pmatrix},\ a_4=\begin{pmatrix}1\\-1\\3\end{pmatrix}$

b) $a_1=\begin{pmatrix}1\\2\\3\end{pmatrix},\ a_2=\begin{pmatrix}3\\4\\5\end{pmatrix},\ a_3=\begin{pmatrix}5\\6\\7\end{pmatrix}$

c) $a_1=\begin{pmatrix}1\\2\\1\end{pmatrix},\ a_2=\begin{pmatrix}1\\0\\2\end{pmatrix},\ a_3=\begin{pmatrix}3\\4\\4\end{pmatrix}$

744. $\bar{x}$ vektorni $\bar{e}_1,\bar{e}_2$ bazisdagi koordinatalari x_1=1, x_2=3 bo'lsa, $\bar{e}_1',\bar{e}_2'$ bazisdagi koordinatalarini toping

$\bar{e}_1=\begin{pmatrix}2\\5\end{pmatrix},\ \bar{e}_2=\begin{pmatrix}1\\2\end{pmatrix}\qquad \bar{e}_1'=\begin{pmatrix}2\\1\end{pmatrix},\ \bar{e}_2'=\begin{pmatrix}5\\3\end{pmatrix}$

745. $\bar{x}$ vektorni $\bar{e}_1,\bar{e}_2$ bazisdagi koordinatalari x_1=2, x_2=2 bo'lsa $\bar{e}_1',\bar{e}_2'$ bazisdagi koordinatalarini toping

$\bar{e}_1=\begin{pmatrix}3\\1\end{pmatrix},\ \bar{e}_2=\begin{pmatrix}4\\1\end{pmatrix}\qquad \bar{e}_1'=\begin{pmatrix}1\\1\end{pmatrix},\ \bar{e}_2'=\begin{pmatrix}2\\3\end{pmatrix}$

746. L_1 qism fazo $\bar{a}_1,\bar{a}_2$ vektorlarga L_2 qism fazo esa $\bar{b}_1,\bar{b}_2$ vektorlarga tortilgan. $L_1\cup L_2$ qism fazoning biror bazisini toping

$$\bar{a}_1=\begin{pmatrix}1\\0\\1\end{pmatrix},\ \bar{a}_2=\begin{pmatrix}1\\0\\0\end{pmatrix},\qquad \bar{b}_1=\begin{pmatrix}1\\1\\1\end{pmatrix},\ \bar{b}_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$$

747. L_1 qism fazo $\bar{a}_1,\bar{a}_2$ vektorlarga L_2 qism fazo esa $\bar{b}_1,\bar{b}_2$ vektorlarga tortilgan. $L_1\cap L_2$ qism fazoning biror bazisini toping

$$\bar{a}_1=\begin{pmatrix}1\\1\\1\end{pmatrix},\ \bar{a}_2=\begin{pmatrix}1\\0\\0\end{pmatrix},\qquad \bar{b}_1=\begin{pmatrix}0\\1\\1\end{pmatrix},\ \bar{b}_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$$

748. L_1 qism fazo $\bar{a}_1, \bar{a}_2, \bar{a}_3$ vektorlarga L_2 qism fazo esa $\bar{b}_1, \bar{b}_2$ vektorlarga tortilgan. $L_1 \cup L_2$ ni o'lchamini aniqlang

$$\bar{a}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \bar{a}_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \quad \bar{a}_3 = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \qquad \bar{b}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \bar{b}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

749. L_1 qism fazo $\bar{a}_1, \bar{a}_2, \bar{a}_3$ vektorlarga L_2 qism fazo esa $\bar{b}_1, \bar{b}_2$ vektorlarga tortilgan. $L_1 \cap L_2$ ni o'lchamini aniqlang.

$$\bar{a}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \bar{a}_2 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \quad \bar{a}_3 = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \qquad \bar{b}_1 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \bar{b}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

750. $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, e_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ bazisdan e_1', e_2' bazisga o'tish matritsasi $C = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ bo'lsa, e_1', e_2' larni toping.

751. $e_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, e_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ bazisdan e_1', e_2' bazisga o'tish matritsasi $C = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$ bo'lsa, e_1', e_2' larni toping.

752. $e_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, e_2 = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ bazisdan e_1', e_2' bazisga o'tish matritsasi $C = \begin{pmatrix} -2 & -5 \\ -1 & -3 \end{pmatrix}$ bo'lsa, e_1', e_2' larni toping.

20§. Evklid fazosi. Skalyar ko'paytma. Ortogonallik. Vektorning o'zunligi. Vektorlarning ortogonal va ortonormal sistemasi. Gramm-Shmidt ortogonallashtirish jarayoni.

753. Quyidagi funktsiya skalyar ko'paytma bo'la oladimi?

$$\bar{x}, \bar{y} \in R^2, \quad \bar{x} = (x_1, x_2), \ \bar{y} = (y_1, y_2)$$

$$\bar{x} \cdot \bar{y} = x_1 y_1 + 2x_1 y_2 + 2x_2 y_2$$

754. Quyidagi ikkita vektor λ ning qanday qiymatlarida ortogonal bo'ladi?

$$\bar{a} = (\lambda,\ 2\lambda,\ 1), \quad \bar{b} = (-2,\ 1,\ 2)$$

755. Quyidagi ikkita vektor λ ning qanday qiymatlarida ortogonal bo'ladi?

$$\bar{a} = (1,\ \lambda, -1) \qquad \bar{b} = (-2,\ 3,\ 1)$$

756. Quyidagi ikkita vektor λ ning qanday qiymatlarida ortogonal bo'ladi?

$$\bar{a} = (\lambda,\ 1,\ -\lambda) \qquad \bar{b} = (1,\ 2,\ 3)$$

757. Quyidagi ikkita vektor λ ning qanday qiymatlarida ortogonal bo'ladi?

$$\bar{a} = (\lambda,\ -3,\ 1) \qquad \bar{b} = (\lambda,\ \lambda,\ 2)$$

758. Quyidagi ikkita vektor λ ning qanday qiymatlarida ortogonal bo'ladi?

$$\bar{a} = (3,\ \lambda,\ 2) \qquad \bar{b} = (-2,\ 0,\ 3)$$

759. λ ning qanday qiymatlarida quyidagi vektor o'zunligi 1 ga teng bo'ladi?

$$\bar{a} = \left(\lambda - 1,\ \frac{4}{5},\ \frac{3}{5}\right)$$

760. λ ning qanday qiymatlarida quyidagi vektor o'zunligi 1 ga teng bo'ladi?

$$\bar{a} = \left(\frac{-3}{5},\ \lambda + 1,\ \frac{4}{5}\right)$$

761. λ ning qanday qiymatlarida quyidagi vektor o'zunligi 1 ga teng bo'ladi?

$$\bar{a} = \left(\frac{5}{13},\ \frac{12}{13},\ \lambda\right)$$

762. Quyidagi vektorlar orasidagi burchak kosinusini toping

$$\bar{a}(3,4), \qquad \bar{b}(12,-5)\ .$$

763. Quyidagi vektorlar orasidagi burchak kosinusini toping

$$\bar{a}(8,15), \qquad \bar{b}(4,-3)\ .$$

764. Quyidagi vektorlar orasidagi burchak kosinusini toping

$$\bar{a}(5,12), \qquad \bar{b}(15,-8)\ .$$

765. Quyidagi vektorlar sistemasini Gramm-Shmidt usuli yordamida ortogonallashtiring

$\bar{a}(1,-2,\ 1), \quad \bar{b}(2,\ 1,-1)\ .$

766. Quyidagi vektorlar sistemasini Gramm-Shmidt usuli yordamida ortogonallashtiring
$\bar{a}(3,\ 1,\ 1),\quad \bar{b}(0,-1,2)$.

767. Quyidagi vektorlar sistemasini Gramm-Shmidt usuli yordamida ortogonallashtiring
$\bar{a}(1,1,\ 0,\ 0),\quad \bar{b}(1,\ 0,\ 1,1)$.

768. Quyidagi vektorlar sistemasini Gramm-Shmidt usuli yordamida ortogonallashtiring
$\bar{a}(0,-1,\ 0,\ 1),\quad \bar{b}(1,\ 1,\ 1,\ 0)$.

21§. Evklid fazosi. Ortogonal va ortonormal sistemalar. Ortogonallashtirish protsessi.

769-789. Quyida berilgan vektorlarni normal vektorga keltiring.

769. $\bar{b}=(2,\quad 0,\quad 1,\quad -7)$

770. $\bar{b}=(3,\quad 1,\quad 2,\quad 1)$

771. $\bar{b}=(1,\quad -1,\quad -1,\quad 1)$

772. $\bar{b}=(1,\quad -2,\quad 2,\quad -3)$

773. $\bar{b}=(-2,\quad 1,\quad 0,\quad -5)$

774. $\bar{b}=(0,\quad 1,\quad -3,\quad 4)$

775. $\bar{b}=(-3,\quad 0,\quad 4,\quad 0)$

776. $\bar{b}=(6,\quad 2,\quad -1,\quad 0)$

777. $\bar{b}=(1,\quad 0,\quad 1,\quad -2)$

778. $\bar{b}=(4,\quad -2,\quad 1,\quad -1)$

779. $\bar{b}=(1,\quad 1,\quad 3,\quad -5)$

780. $\bar{b}=(2,\quad 3,\quad -5,\quad 4)$

781. $\bar{b}=(2,\quad 1,\quad 5,\quad -1)$

782. $\bar{b}=(2,\quad 3,\quad -4,\quad -1)$

783. $\bar{b}=(3,\quad -1,\quad 3,\quad -3)$

784. $\bar{b}=(-12,\quad 9,\quad -8)$

785. $\bar{b}=(0,\quad -10,\quad 24)$

786. $\bar{b}=(12,\quad 0,\quad -5,\quad 0)$

787. $\bar{b}=(2,\quad 7,\quad -1,\quad 1)$

788. $\bar{b}=(1,\quad -3,\quad 4,\quad 2)$

789. $\bar{b}=(9,\quad -1,\quad 1,\quad -1)$

790-810. Ortogonallashtirish protsessi yordamida quyidagi vektorlarga ortogonal baza toping.

790. $\begin{cases}\bar{a}_1=(2,\quad 2,\quad 7,\quad -1),\\ \bar{a}_2=(3,\quad -1,\quad 2,\quad 4),\\ \bar{a}_3=(1,\quad 1,\quad 3,\quad 2).\end{cases}$

791. $\begin{cases}\bar{a}_1=(3,\quad 2,\quad -5,\quad 4),\\ \bar{a}_2=(3,\quad -1,\quad 3,\quad -3).\end{cases}$

792. $\begin{cases} \bar{a}_1 = (2, \quad 3, \quad -4, \quad -1), \\ \bar{a}_2 = (1, \quad -2, \quad 1, \quad 3). \end{cases}$

793. $\begin{cases} \bar{a}_1 = (2, \quad 1, \quad -1, \quad 1), \\ \bar{a}_2 = (1, \quad 2, \quad 1, \quad -1), \\ \bar{a}_3 = (1, \quad 1, \quad 2, \quad 1). \end{cases}$

794. $\begin{cases} \bar{a}_1 = (2, \quad 1, \quad -3), \\ \bar{a}_2 = (3, \quad 1, \quad -5), \\ \bar{a}_3 = (4, \quad 2, \quad -1). \end{cases}$

795. $\begin{cases} \bar{a}_1 = (2, \quad 3, \quad 5, \quad -4, \quad 1), \\ \bar{a}_2 = (1, \quad -1, \quad 2, \quad 3, \quad 5), \\ \bar{a}_3 = (1, \quad -1, \quad 1, \quad -2, \quad 3). \end{cases}$

796. $\begin{cases} \bar{a}_1 = (2, \quad -1, \quad 3, \quad 4, \quad -1), \\ \bar{a}_2 = (1, \quad 2, \quad -3, \quad 1, \quad 2). \end{cases}$

797. $\begin{cases} \bar{a}_1 = (4, \quad 3, \quad -1, \quad 1, \quad -1), \\ \bar{a}_2 = (2, \quad 1, \quad -3, \quad 2, \quad -5), \\ \bar{a}_3 = (1, \quad -3, \quad 0, \quad 1, \quad -2). \end{cases}$

798. $\begin{cases} \bar{a}_1 = (1, \quad 2, \quad 1), \\ \bar{a}_2 = (2, \quad 3, \quad 3), \\ \bar{a}_3 = (3, \quad 7, \quad 1). \end{cases}$

799. $\begin{cases} \bar{a}_1 = (3, \quad 1, \quad 4), \\ \bar{a}_2 = (5, \quad 2, \quad 1), \\ \bar{a}_3 = (1, \quad 1, \quad -6). \end{cases}$

800. $\begin{cases} \bar{a}_1 = (1, \quad 1, \quad 1, \quad 1), \\ \bar{a}_2 = (1, \quad 2, \quad 1, \quad 1), \\ \bar{a}_3 = (1, \quad 1, \quad 2, \quad 1), \\ \bar{a}_4 = (1, \quad 3, \quad 2, \quad 3). \end{cases}$

801. $\begin{cases} \bar{a}_1 = (1, \quad 1, \quad 1), \\ \bar{a}_2 = (1, \quad 1, \quad 2), \\ \bar{a}_3 = (1, \quad 2, \quad 3). \end{cases}$

802. $\begin{cases} \bar{a}_1 = (2, \quad 1, \quad -3), \\ \bar{a}_2 = (3, \quad 2, \quad -5), \\ \bar{a}_3 = (1, \quad -1, \quad 1). \end{cases}$

803. $\begin{cases} \bar{a}_1 = (1, \quad -1, \quad 1, \quad 0), \\ \bar{a}_2 = (1, \quad 1, \quad 0, \quad 1), \\ \bar{a}_3 = (2, \quad 0, \quad 1, \quad 1). \end{cases}$

804. $\begin{cases} \bar{a}_1 = (1, \quad 2, \quad -1, \quad -2), \\ \bar{a}_2 = (-2, \quad 3, \quad 0, \quad 1), \\ \bar{a}_3 = (-2, \quad 0, \quad 0, \quad 1), \\ \bar{a}_4 = (1, \quad 0, \quad 2, \quad 1). \end{cases}$

805. $\begin{cases} \bar{a}_1 = (3, \quad 1, \quad 0), \\ \bar{a}_2 = (0, \quad -2, \quad 1), \\ \bar{a}_3 = (1, \quad 0, \quad -2). \end{cases}$

806. $\begin{cases} \bar{a}_1 = (1, \quad 2, \quad 1, \quad 3), \\ \bar{a}_2 = (4, \quad 1, \quad 1, \quad 1), \\ \bar{a}_3 = (3, \quad 1, \quad 1, \quad 0). \end{cases}$

807. $\begin{cases} \bar{a}_1 = (1, \quad 1, \quad 1, \quad 1), \\ \bar{a}_2 = (1, \quad -1, \quad -1, \quad 1), \\ \bar{a}_3 = (2, \quad 1, \quad 1, \quad 3). \end{cases}$

808. $\begin{cases} \bar{a}_1 = (1, \quad 2, \quad 1, \quad 3), \\ \bar{a}_2 = (4, \quad 1, \quad 1, \quad 1), \\ \bar{a}_3 = (3, \quad 1, \quad 1, \quad 0). \end{cases}$

809. $\begin{cases} \bar{a}_1 = (1, \quad 0, \quad 1, \quad 0), \\ \bar{a}_2 = (1, \quad -1, \quad 0, \quad 1), \\ \bar{a}_3 = (-1, \quad 0, \quad 0, \quad 1). \end{cases}$

810. $\begin{cases} \bar{a}_1 = (2, \quad -1, \quad 1, \quad 0), \\ \bar{a}_2 = (0, \quad -1, \quad 0, \quad 1), \\ \bar{a}_3 = (3, \quad 0, \quad 1, \quad -1). \end{cases}$

811-817. Vektor fazoda $\bar{x}_1, \bar{x}_2, \bar{x}_3, ..., \bar{x}_n$ vektorlar berilgan. Uning bazisini va o'lchovini toping.

811. $\begin{cases} \bar{x}_1 = (1, \quad -1, \quad 1, \quad 0), \\ \bar{x}_2 = (1, \quad 1, \quad 0, \quad 1), \\ \bar{x}_3 = (2, \quad 0, \quad 1, \quad 1). \end{cases}$

812. $\begin{cases} \bar{x}_1 = (2, \quad 1, \quad 3, \quad 1), \\ \bar{x}_2 = (1, \quad 2, \quad 0, \quad 1), \\ \bar{x}_3 = (-1, \quad 1, \quad -3, \quad 0). \end{cases}$

813. $\begin{cases} \bar{x}_1 = (2, \quad 0, \quad 1, \quad 3, \quad -1), \\ \bar{x}_2 = (1, \quad 1, \quad 0, \quad -1, \quad 1), \\ \bar{x}_3 = (0, \quad -2, \quad 1, \quad 5, \quad -3), \\ \bar{x}_4 = (1, \quad -3, \quad 2, \quad 9, \quad -5). \end{cases}$

814. $\begin{cases} \bar{x}_1 = (2, \quad 1, \quad 3, \quad -1), \\ \bar{x}_2 = (-1, \quad 1, \quad -3, \quad 1), \\ \bar{x}_3 = (4, \quad 5, \quad 3, \quad -1), \\ \bar{x}_4 = (1, \quad 5, \quad -3, \quad 1). \end{cases}$

815. $\begin{cases} \bar{x}_1 = (1, \quad 0, \quad 0, \quad -1), \\ \bar{x}_2 = (2, \quad 1, \quad 1, \quad 0), \\ \bar{x}_3 = (1, \quad 1, \quad 1, \quad 1), \\ \bar{x}_4 = (1, \quad 2, \quad 3, \quad 4), \\ \bar{x}_5 = (0, \quad 1 \quad 2, \quad 3). \end{cases}$

816. $\begin{cases} \bar{x}_1 = (1, \quad 1, \quad 1, \quad 1, \quad 0), \\ \bar{x}_2 = (1, \quad 1, \quad -1, \quad -1, \quad -1), \\ \bar{x}_3 = (2, \quad 2, \quad 0, \quad 0, \quad -1), \\ \bar{x}_4 = (1, \quad 1, \quad 5, \quad 5, \quad 2), \\ \bar{x}_5 = (1, \quad -1, \quad -1, \quad 0, \quad 0). \end{cases}$

817. $\begin{cases} \bar{x}_1 = (1, \quad 2, \quad -1, \quad -2), \\ \bar{x}_2 = (-2, \quad 3, \quad 0, \quad 1), \\ \bar{x}_3 = (-2, \quad 0, \quad 0, \quad 1), \\ \bar{x}_4 = (1, \quad 0, \quad 2, \quad 1). \end{cases}$

818-823. $\bar{x}$ vektorni $\bar{e}_1, \bar{e}_2, \bar{e}_3, ..., \bar{e}_n$ bazislar orqali koordinatalarini toping.

818. $\begin{cases} \bar{x} = (1, \quad 2, \quad 1, \quad 1), \\ \bar{e}_1 = (1, \quad 1, \quad 1, \quad 1), \\ \bar{e}_2 = (1, \quad 1, \quad -1, \quad -1), \\ \bar{e}_3 = (1, \quad -1, \quad 1, \quad -1), \\ \bar{e}_4 = (1, \quad -1, \quad -1, \quad 1). \end{cases}$

819. $\begin{cases} \bar{x} = (0, \quad 0, \quad 0, \quad 1), \\ \bar{e}_1 = (1, \quad 1, \quad 0, \quad 1), \\ \bar{e}_2 = (2, \quad 1, \quad 3, \quad 1), \\ \bar{e}_3 = (1, \quad 1, \quad 0, \quad 0), \\ \bar{e}_4 = (0, \quad 1, \quad -1, \quad -1). \end{cases}$

820. $\begin{cases} \bar{x} = (7, \quad 14, \quad -1, \quad 2), \\ \bar{e}_1 = (1, \quad 2, \quad -1, \quad -2), \\ \bar{e}_2 = (2, \quad 3, \quad 0, \quad -1), \\ \bar{e}_3 = (1, \quad 2, \quad 1, \quad 4), \\ \bar{e}_4 = (1, \quad 3, \quad -1, \quad 0). \end{cases}$

821. $\begin{cases} \bar{x} = (6, \quad 9, \quad 14), \\ \bar{e}_1 = (1, \quad 1, \quad 1), \\ \bar{e}_2 = (1, \quad 1, \quad 2), \\ \bar{e}_3 = (1, \quad 2, \quad 3). \end{cases}$

822. $\begin{cases} \bar{x} = (6, \quad 2, \quad -7), \\ \bar{e}_1 = (2, \quad 1, \quad -3), \\ \bar{e}_2 = (3, \quad 2, \quad -5), \\ \bar{e}_3 = (1, \quad -1, \quad 1). \end{cases}$

823. $\begin{cases} \bar{x} = (3, \quad -2, \quad 5), \\ \bar{e}_1 = (1, \quad 0, \quad 2), \\ \bar{e}_2 = (1, \quad 0, \quad -2), \\ \bar{e}_3 = (1, \quad 1, \quad 1). \end{cases}$

824-827. $\bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4$ bazisdan $\bar{e}_1', \bar{e}_2', \bar{e}_3', \bar{e}_4'$ bazisga o'tish matritsasini toping.

824. $\begin{cases} \bar{e}_1 = (1, \quad 0, \quad 0, \quad 0), \\ \bar{e}_2 = (0, \quad 1, \quad 0, \quad 0), \\ \bar{e}_3 = (0, \quad 0, \quad 1, \quad 0), \\ \bar{e}_4 = (0, \quad 0, \quad 0, \quad 1). \end{cases}$ $\begin{cases} \bar{e}_1' = (1, \quad 1, \quad 0, \quad 0), \\ \bar{e}_2' = (1, \quad 0, \quad 1, \quad 0), \\ \bar{e}_3' = (1, \quad 0, \quad 0, \quad 1), \\ \bar{e}_4' = (1, \quad 1, \quad 1, \quad 1). \end{cases}$

825. $\begin{cases} \bar{e}_1 = (1, \quad 2, \quad -1, \quad 0), \\ \bar{e}_2 = (1, \quad -1, \quad 1, \quad 1), \\ \bar{e}_3 = (-1, \quad 2, \quad 1, \quad 1), \\ \bar{e}_4 = (-1, \quad -1, \quad 0, \quad 1). \end{cases}$ $\begin{cases} \bar{e}_1' = (2, \quad 1, \quad 0, \quad 1), \\ \bar{e}_2' = (0, \quad 1, \quad 2, \quad 2), \\ \bar{e}_3' = (-2, \quad 1, \quad 1, \quad 2), \\ \bar{e}_4' = (1, \quad 3, \quad 1, \quad 2). \end{cases}$

826. $\begin{cases} \bar{e}_1 = (1, \quad 1, \quad 1, \quad 1), \\ \bar{e}_2 = (1, \quad 2, \quad 1, \quad 1), \\ \bar{e}_3 = (1, \quad 1, \quad 2, \quad 1), \\ \bar{e}_4 = (1, \quad 3, \quad 2, \quad 3). \end{cases}$ $\begin{cases} \bar{e}_1' = (1, \quad 0, \quad 3, \quad 3), \\ \bar{e}_2' = (-2, \quad -3, \quad -5, \quad -4), \\ \bar{e}_3' = (2, \quad 2, \quad 5, \quad 4), \\ \bar{e}_4' = (-2, \quad -3, \quad -4, \quad -4). \end{cases}$

827. $\begin{cases} \bar{e}_1 = (1, \quad 2, \quad 1), \\ \bar{e}_2 = (2, \quad 3, \quad 3), \\ \bar{e}_3 = (9, \quad 7, \quad 1). \end{cases}$ $\begin{cases} \bar{e}_1' = (3, \quad 1, \quad 4), \\ \bar{e}_2' = (5, \quad 2, \quad 1), \\ \bar{e}_3' = (1, \quad 1, \quad 6). \end{cases}$

828-831. $\bar{e}_1', \bar{e}_2', \bar{e}_3', \bar{e}_4'$ bazisdan $\bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4$ bazisga o'tish matritsasini toping.

828. $\begin{cases} \bar{e}_1' = (1, \quad 0, \quad 3, \quad 3), \\ \bar{e}_2' = (-2, \quad -3, \quad -5, \quad -4), \\ \bar{e}_3' = (2, \quad 2, \quad 5, \quad 4), \\ \bar{e}_4' = (-2, \quad -3, \quad -4, \quad -4). \end{cases}$ $\begin{cases} \bar{e}_1 = (1, \quad 1, \quad 1, \quad 1), \\ \bar{e}_2 = (1, \quad 2, \quad 1, \quad 1), \\ \bar{e}_3 = (1, \quad 1, \quad 2, \quad 1), \\ \bar{e}_4 = (1, \quad 3, \quad 2, \quad 3). \end{cases}$

829. $\begin{cases} \bar{e}_1' = (1, & 1, & 0, & 0), \\ \bar{e}_2' = (1, & 0, & 1, & 0), \\ \bar{e}_3' = (1, & 0, & 0, & 1), \\ \bar{e}_4' = (1, & 1, & 1, & 1). \end{cases}$ $\begin{cases} \bar{e}_1 = (1, & 0, & 0, & 0), \\ \bar{e}_2 = (0, & 1, & 0, & 0), \\ \bar{e}_3 = (0, & 0, & 1, & 0), \\ \bar{e}_4 = (0, & 0, & 0, & 1). \end{cases}$

830. $\begin{cases} \bar{e}_1' = (3, & 1, & 4), \\ \bar{e}_2' = (5, & 2, & 1), \\ \bar{e}_3' = (1, & 1, & 6). \end{cases}$ $\begin{cases} \bar{e}_1 = (1, & 2, & 1), \\ \bar{e}_2 = (2, & 3, & 3), \\ \bar{e}_3 = (9, & 7, & 1). \end{cases}$

831. $\begin{cases} \bar{e}_1' = (2, & 1, & 0, & 1), \\ \bar{e}_2' = (0, & 1, & 2, & 2), \\ \bar{e}_3' = (-2, & 1, & 1, & 2), \\ \bar{e}_4' = (1, & 3, & 1, & 2). \end{cases}$ $\begin{cases} \bar{e}_1 = (1, & 2, & -1, & 0), \\ \bar{e}_2 = (1, & -1, & 1, & 1), \\ \bar{e}_3 = (-1, & 2, & 1, & 1), \\ \bar{e}_4 = (-1, & -1, & 0, & 1). \end{cases}$

22§. Chiziqli operatorlar. Chiziqli operatorning berilgan bazisdagi matritsasi. Chiziqli operatorning rangi, determinanti, izi, yadrosi, qiymatlar sohasi. Chiziqli operatorning invariant qism fazolari, xarakteristik ko'phadi, minimal ko'phadi, xos qiymatlari va xos vektorlari. Teskari operator. Chiziqli operatorlarni qo'shish, ayirish, songa ko'paytirish, o'zaro ko'paytirish.

832. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$\varphi : R^3 \to R^3, \quad \varphi(\bar{x}) = (x_1 + 2x_3, e^{x_3}, x_2 + x_3)$$

833. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$\varphi : R^3 \to R^3, \quad \varphi(\bar{x}) = (x_1 - x_2, x_2^2, x_1 + x_3)$$

834. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$\varphi : R^3 \to R^3, \quad \varphi(\bar{x}) = (\frac{x_1^2 + x_2^2}{x_1}, \frac{x_2^2}{x_1}, \frac{x_3^2}{x_1})$$

835. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$A : P^3 \to P^3 , \quad A\big(f(x)\big) = f'(x)$$

836. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$A : C[0,1] \to C[0,1], \quad A\big(f(x)\big) = \int_0^1 f(t)e^{xt}\,dt \ .$$

837. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$A : C[0,1] \to C[0,1], \quad A\big(f(x)\big) = x^2 f(0) \ .$$

838. Quyidagi akslantirish chiziqli operator bo'la oladimi?

$$\varphi : R^2 \to R^2 , \quad \varphi(\bar{x}) = (x_1 - x_2 \, , \, x_2^2 \,)$$

839. Quyidagi chiziqli operatorni $\bar{i}, \bar{j}$ bazisdagi matritsasini yozing

$$\varphi : R^2 \to R^2 , \quad \varphi(\bar{x}) = (x_1 + x_2 \, , - x_1 + 2x_2 \,)$$

840. Quyidagi chiziqli operatorni $\bar{i}, \bar{j}, \bar{k}$ bazisdagi matritsasini yozing.

$$\varphi : R^3 \to R^3 , \quad \varphi(\bar{x}) = (-x_3 , \, 2x_3 \, , - x_3 \,) \ .$$

841. Quyidagi chiziqli operatorni $\bar{i}, \bar{j}, \bar{k}$ bazisdagi matritsasini yozing.

$$\varphi : R^3 \to R^3 , \quad \varphi(\bar{x}) = (x_3 , \, - x_2 \, , x_1 \,)$$

842. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 2 & 0 \\ -1 & 2 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = e_1 + e_2 , \;\; e_2' = 2e_1 + e_2$ bazisdagi matritsasini toping.

843. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} -3 & 2 \\ 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = e_1 - e_2 , \;\; e_2' = -3e_1 + 4e_2$ bazisdagi matritsasini toping.

844. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = e_1 + e_2 , \;\; e_2' = -2e_1 + 3e_2$ bazisdagi matritsasini toping.

845. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ matritsa bilan berilgan. $\bar{x} = 2\bar{e}_1 - 3\bar{e}_2$ vektor obrazining $\bar{e}_1, \bar{e}_2$ bazisdagi koordinatalarini toping.

846. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ matritsa bilan berilgan. $\bar{x} = 2\bar{e} - 3\bar{e}_2$ vektor obrazining $\bar{e}_1, \bar{e}_2$ bazisdagi koordinatalarini toping.

847. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning rangini toping.

848. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning determinantini toping.

849. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning determinantini toping.

850. Chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisda $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning determinantini toping.

851. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining o'lchamini toping.

852. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining o'lchamini toping.

853. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining o'lchamini toping.

854. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator obrazining o'lchamini toping.

855. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator obrazining o'lchamini toping.

856. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining bazisini toping.

857. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining bazisini toping.

858. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining bazisini toping.

859. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator yadrosining bazisini toping.

860. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator obrazining bazisini toping.

861. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator obrazining bazisini toping.

862. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operator obrazining bazisini toping.

863. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$ matritsa bilan berilgan. U qaysi vektorni o'z joyida qoldiradi.

864. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$ matritsa bilan berilgan. U qaysi vektorni o'z joyida qoldiradi.

865. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$ matritsa bilan berilgan. U qaysi vektorni o'z joyida qoldiradi.

866. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ matritsa bilan berilgan. U qaysi to'g'ri chizikni o'z joyida qoldiradi.

867. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 0 & 2 \\ -4 & 1 \end{pmatrix}$ matritsa bilan berilgan. U qaysi to'g'ri chizikni o'z joyida qoldiradi.

868. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 1 & 2 \\ 5 & -1 \end{pmatrix}$ matritsa bilan berilgan. U qaysi to'g'ri chizikni o'z joyida qoldiradi.

869. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A=\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Uni xarakteristik ko'phadini toping.

870. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ matritsa bilan berilgan. U qaysi to'g'ri chiziqni o'z joyida qoldiradi.

871. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A=\begin{pmatrix} 2 & 1 & 3 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$ matritsa bilan berilgan. U qaysi to'g'ri chiziqni o'z joyida qoldiradi.

872. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A=\begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Uning xos vektor va xos qiymatlarini toping.

873. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. Uni xos vektor va xos qiymatlarini toping.

874. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 3 & 0 & 3 \\ 2 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. Uni xos vektor va xos qiymatlarini toping.

875. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$ matritsa bilan berilgan. Uni matritsaviy va spektral izlarini toping.

876. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 2 & 0 \\ -1 & 2 & 3 \end{pmatrix}$ matritsa bilan berilgan. Uni matritsaviy va spektral izlarini toping.

877. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \\ -3 & 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. Uni matritsaviy va spektral izlarini toping.

878. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ matritsa bilan berilgan. $\bar{y} = e_1 + 3e_2$ vektor qaysi vektorni obrazi bo'ladi.

879. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 0 & 1 & 0 \end{pmatrix}$ matritsa bilan berilgan. $\bar{y} = 2e_1 - e_2 + e_3$ vektor qaysi vektorni obrazi bo'ladi.

880. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ matritsa bilan berilgan. Bu operatorga teskari operator toping.

881. φ chiziqli operator $\bar{e}_1, \bar{e}_2, \bar{e}_3$ bazisdagi $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ -3 & 0 & -1 \end{pmatrix}$ matritsa bilan berilgan. Bu operatorga teskari operator toping.

882. φ chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisdagi $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ matritsa bilan berilgan. Bu operatorga teskari operator toping.

883. $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ bazisda φ chiziqli operator $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, e_2' = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ bazisda ψ chiziqli operator $A' = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ matritsa bilan berilgan. $\varphi + \psi$ ni $\bar{e}_1', \bar{e}_2'$ dagi matritsasini toping.

884. $e_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, e_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ bazisda φ chiziqli operator $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$ matritsa bilan berilgan. $e_1' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2' = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ bazisda ψ chiziqli operator $A' = \begin{pmatrix} 2 & 3 \\ 1 & 3 \end{pmatrix}$ matritsa bilan berilgan. $\varphi \cdot \psi$ ni $\bar{e}_1', \bar{e}_2'$ dagi matritsasini toping.

23§. Kvadratik formalar va ularni kanonik ko'rinishga keltirish(Lagranj usuli)

Quyidagi ifodaga kvadratik forma deyiladi.

$$f = \sum_{i,j=1}^{n} a_{ij} x_i x_j \qquad (a_{ij} = a_{ji}) \tag{1}$$

$$f = a_{11} x_1^2 + \ldots + a_{nn} x_n^2 + 2 \sum_{1 \le i < j \le n} a_{ij} x_i x_j \tag{2}$$

Quyidagi matritsaga

$$A=\begin{pmatrix} a_{11} & . & . & . & a_{1n} \\ . & . & . & . & . \\ a_{n1} & . & . & . & a_{nn1} \end{pmatrix}$$

(1) kvadrat formaning matritsasi deyiladi.

Quyidagi ko'paytmani ko'rib chiqamiz.

$$(x_1, \ \dots \ ,x_n)\begin{pmatrix} a_{11} & . & . & . & a_{1n} \\ . & . & . & . & . \\ a_{n1} & . & . & . & a_{nn1} \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}=(x_1, \ \dots \ ,x_n)\begin{pmatrix} \sum\limits_{j=1}^{n} a_{1j}x_j \\ \vdots \\ \sum\limits_{j=1}^{n} a_{nj}x_j \end{pmatrix}=\sum_{i=1}^{n} x_i(\sum_{j=1}^{n} a_{ij}x_j)=\sum_{i,j=1}^{n} a_{ij}x_ix_j=f$$

Demak,

$$f=\overline{X}^T\cdot A\cdot\overline{X} \tag{3}$$

Agar (3) kvadratik formada $\overline{X}=C\cdot\overline{X}$, $(\det C\neq 0)$ almashtirish bajarsak

$$f=(C\overline{Y})^T AC\overline{Y}=\overline{Y}^T\cdot C^TC\cdot\overline{Y}A=\overline{Y}^T B\overline{Y}$$

$$B=C^TAC \tag{4}$$

Izoh: $rangB=rangA$,chunki $B=C^TAC$ da $\det C\neq 0$

Ta'rif: $rangA$ ga f kvadrat formaning rangi deyiladi.

Izoh: $\det B$va $\det A$ ning ishorasi bir xil, chunki $\det B=\det A\cdot(\det C)^2$.

Teorema: Har qanday kvadrat formani xosmas $\overline{X}=C\cdot\overline{Y}$ almashtirish yordamida

$$f=\lambda_1y_1^2+\lambda_2y_2^2+\dots+\lambda_ny_n^2 \tag{5}$$

ushbu ko'rinishga keltirish mumkin.(bu ko'rinishga kanonik ko'rinishi deyiladi.

Isbot: Induksiya usulida isbotlaymiz. $n=1$bo'lsa $f=a_{11}x_1^2$ o'zi kanonik ko'rinishda erkin o'zgaruvchilar soni $n-1$ bo'lganda o'rinli deb olib, n uchun isbotlaymiz.

1-hol: $a_{11}\neq 0$ bo'lsin. Bu holda

$$f=a_{11}x_1^2+x_1^2(a_{12}x_2+\dots+a_{1n}x_n)+\varphi(x_2*\dots*x_n)=\frac{1}{a_{11}}\left[a_{11}{}^2x_1^2+2a_{11}x_1(a_{12}x_2+\dots+a_{1n}x_n)\right]+\varphi=$$

$$=\frac{1}{a_{11}}\left[(a_{11}x_1+\dots+a_{1n}x_n)^2-(a_{12}x_2+\dots+a_{1n}x_n)^2\right]+\varphi=\frac{1}{a_{11}}y^2+\varphi_1(x_2,\dots,x_n)$$

iduksiya faraziga ko'ra kvadrat formani xosmas almashtirish yordamida kanonik ko'rinishga keltirish mumkin.

$$f=\frac{1}{\mathrm{a}_{11}}y_1^2+\underbrace{\lambda_2y_2^2+\dots+\lambda_ny_n^2}_{\varphi_1}$$

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ \ldots\ldots \\ y_n = \qquad c_{n2}x_2 + \ldots + c_{nn}x_n \end{cases}$$

$$\widetilde{C} = \begin{cases} a_{11} & a_{12} & \ldots & a_{1n} \\ 0 & c_{22} & \ldots & c_{2n} \\ . & . & \ldots & . \\ 0 & c_{n1} & \ldots & c_{nn} \end{cases}$$

$$\det \widetilde{C} = a_{11} \cdot \det C \neq 0$$

2-hol: $a_{kk} \neq 0$ bo'lsa, bu holda qayta belgilash olib 1-holga keltiramiz.

3-hol: $a_{11} = \ldots = a_{nn} = 0$ va $a_{12} \neq 0$ bo'lsin.

$$x_1 = z_1 + z_2,\ \ x_2 = z_1 - z_2,\ \ x_3 = z_3,\ \ \ldots,\ x_n = z_n,$$

bu holda

$$f = 2a_{12}(z_1^2 - z_2^2) + \psi(z_1, \ldots, z_n)$$

bunga 1-holni qo'llaymiz.

Ta'rif: $A - r\ \overline{x} \neq 0$ bo'lganda $f = \sum_{i,j=1}^{n} a_{ij}x_i x_j > 0$ bo'lsa, f kvadrat formaga musbat aniqlangan kvadratik forma deyiladi.

$A - r\ \overline{x} \neq 0$ bo'lganda $f < 0$ bo'lsa, manfiy aniqlangan forma deyiladi.

Teorema: (Inertsiya qonuni). Kvadratik formani turli xil usulda xosmas almashtirish yordamida kanonik ko'rinishga keltirganda musbat (manfiy) koeffitsentlar soni bir xil bo'ladi.

Teorema: Musbat aniqlangan kvadratik formani xosmas almashtirish yordamida kanonik ko'rinishga keltirganda

$$f = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \ldots + \lambda_n y_n^2, \quad \lambda_1 > 0, \lambda_2, \ldots, \lambda_n > 0$$

bo'ladi.

Isbot: $\lambda_1 \leq 0$ deb faraz qilaylik, u holda $y_1 = 1, y_2 = 0, \ldots, y_n = 0$ desak, $f = \lambda_1 \leq 0$ bo'ladi.

$$\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n = 1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + \ldots + a_{nn}x_n = 0 \end{cases}$$

bu sistema noldan farqli $\overline{x_1} \neq 0$ yechimga ega, ziddiyat.

Izoh: $\lambda_1 > 0, \lambda_2, \ldots, \lambda_n > 0$ bo'lsa yig'indi musbat aniqlangan bo'ladi.

Teorema: (Silvestr kriteriyasi). $f=\sum_{i,j=1}^{n}a_{ij}x_ix_j$ ushbu kvadratik forma musbat aniqlangan bo'lishi uchun $A=(a_{ij})$ matritsaning barcha burchak minorlari musbat bo'lishi zarur va etarli, ya'ni

$$\Delta_1=a_{11}>0,\ \Delta_2=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{vmatrix}>0,\ldots,\ \Delta_n=\begin{vmatrix} a_{11} & . & . & . & a_{1n} \\ . & . & . & . & . \\ a_{n1} & . & . & . & a_{nn}\end{vmatrix}>0 \qquad (6)$$

Isbot: $n=1$ bo'lsin, u holda $f=a_{11}x_1^2$ bo'lgani uchun $\Delta_1=a_{11}$ bo'ladi. f musbat aniqlanganligidan $\Delta_1>0$ musbat ekani kelib chiqadi. Erkin o'zgaruvchilar soni $n-1$ bo'lganda o'rinli deb olib n uchun isbotlaymiz. Buning uchun berilgan kvadratik formani quyidagi ko'rinishda yozib olamiz:

$$f(x_1,x_2,...,x_n)=\sum_{i,j=1}^{n-1}a_{ij}x_ix_j+\sum_{i=1}^{n-1}a_{in}x_ix_n+a_{nn}x_n^2$$

$$\varphi(x_1,x_2,...,x_{n-1})=\sum_{i,j=1}^{n-1}a_{ij}x_ix_j$$

$\varphi(x_1,x_2,...,x_{n-1})=\varphi(x_1,x_2,...,x_{n-1},0)$ bo'lganidan f musbat aniqlangan bo'lsa, φ ham musbat aniqlangan bo'lishi kelib chiqadi.

Zarurligi. f musbat aniqlangan bo'lsin, u holda φ musbat aniqlangan ekanligidan induktsiya faraziga ko'ra $\Delta_1>0,\ \Delta_2>0,\ldots,\ \Delta_{n-1}>0$ kelib chiqadi. f musbat aniqlangan bo'lgani uchun uni $f=\lambda_1y_1^2+\lambda_2y_2^2+...+\lambda_ny_n^2,\quad \lambda_1>0,\lambda_2,\ldots,\lambda_n>0$ ushbu kanonik ko'rinishga keltirib bo'ladi. Bu holda uning matritsasi quyidagi ko'rinishda bo'ladi:

$$B=\begin{pmatrix} \lambda_1 & 0 & . & . & . & 0 \\ 0 & \lambda_2 & . & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & \lambda_n\end{pmatrix}$$

$\det B=\lambda_1\cdot\lambda_2\cdot...\cdot\lambda_n>0$ ekanligidan $\det A=\Delta_n=\det(C^TBC)=(\det C)^2\cdot\det B>0$.

Yetarliligi. $\Delta_1>0,\ \Delta_2>0,\ldots,\ \Delta_{n-1}>0,\ \Delta_n>0$ bo'lsin. Induktsiya faraziga ko'ra $\Delta_1>0,\ \Delta_2>0,\ldots,\ \Delta_{n-1}>0$ ekanidan φ musbat aniqlangan ekani kelib chiqadi. Demak, uni xosmas

$$\begin{pmatrix} x_1 \\ x_2 \\ . \\ x_{n-1}\end{pmatrix}=\begin{pmatrix} c_{11} & c_{12} & . & . & . & c_{1,n-1} \\ c_{21} & c_{22} & . & . & . & c_{2,n-1} \\ . & . & . & . & . & . \\ c_{n-1,1} & c_{n-1,2} & . & . & . & c_{n-1}\end{pmatrix}\cdot\begin{pmatrix} y_1 \\ y_2 \\ . \\ y_{n-1}\end{pmatrix} \qquad (7)$$

almashtirish yordamida quyidagi ko'rinishga keltirib bo'ladi:

$$\varphi=\lambda_1 y_1^2+\lambda_2 y_2^2+\ldots+\lambda_{n-1}y_{n-1}^2, \quad \lambda_1>0, \lambda_2, \ldots, \lambda_{n-1}>0.$$

(7) ni f dagi ifodasiga qo'yib, $x_n=y_n$ belgilash kiritsak,

$$f=\lambda_1 y_1^2+\lambda_2 y_2^2+\ldots+\lambda_{n-1}y_{n-1}^2+2\sum_{i=1}^{n-1}b_{in}y_i y_n+a_{nn}y_n^2 \tag{8}$$

$$f=\lambda_1\left[y_1^2+2\frac{b_{1n}}{\lambda_1}y_1 y_n\right]+\lambda_2\left[y_2^2+2\frac{b_{2n}}{\lambda_2}y_2 y_n\right]+\ldots+\lambda_{n-1}\left[y_{n-1}^2+2\frac{b_{n-1,n}}{\lambda_{n-1}}y_{n-1}y_n\right]+a_{nn}y_n^2$$

$$f=\lambda_1\left[y_1+\frac{b_{1n}}{\lambda_1}y_n\right]^2+\lambda_2\left[y_2+\frac{b_{2n}}{\lambda_2}y_n\right]^2+\ldots+\lambda_{n-1}\left[y_{n-1}+\frac{b_{n-1,n}}{\lambda_{n-1}}y_n\right]^2+$$

$$+\left[a_{nn}-\frac{b_{11}^2}{\lambda_1}-\frac{b_{12}^2}{\lambda_2}-\ldots-\frac{b_{n-1,1}^2}{\lambda_{n-1}}\right]y_n^2=\lambda_1 z_1^2+\lambda_2 z_2^2+\ldots+\lambda_{n-1}z_{n-1}^2+cz_n^2$$

$$\begin{cases} z_1=y_1+\dfrac{b_{1n}}{\lambda_1}y_n \\ \ldots\ldots\ldots\ldots \\ z_{n-1}=y_{n-1}+\dfrac{b_{n-1,n}}{\lambda_{n-1}}y_n \\ z_n=y_n \end{cases}$$

$$D=\begin{pmatrix} \lambda_1 & 0 & . & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & \lambda_n & 0 \\ 0 & 0 & . & . & 0 & c \end{pmatrix}$$

$\lambda_1\cdot\lambda_2\cdot\ldots\cdot\lambda_{n-1}\cdot c=\det D=\det(\widetilde{C}^T B\widetilde{C})=(\det\widetilde{C})^2\cdot c>0$. Bunga ko'ra $c>0$. Demak, f musbat aniqlangan ekan.

885. Quyida berilgan kvadratik formani kanonik shaklga Lagranj usulidan foydalanib keltirilsin.

1) $f=2x_1x_2+2x_1x_3-6x_2x_3$

2) $f=x_1x_3-4x_2x_3+x_2x_4$

3) $f=x_1x_3-4x_2x_3+x_2x_4$

4) $f=x_1^2+2x_1x_2+2x_2^2-2x_2x_3$

5) $f=x_1^2+2x_1x_2+2x_2^2+4x_2x_3+5x_3^2$

6) $f=x_1^2-4x_1x_2+2x_1x_3+4x_2^2+x_3^2$

7) $f = x_1x_2 + x_1x_3 + x_2x_3$

8) $f = x_1^2 + x_1x_2 + x_3x_4$

9) $f = 2x_1x_2 + 2x_1x_3 - 6x_2x_3$

10) $f = x_1^2 - 2x_1x_2 + 2x_1x_3 - 2x_1x_4 + x_2^2 + 2x_2x_3 - 4x_2x_4 + x_3^2 - 2x_4^2$

11) $f = 2x_1^2 + x_2^2 - 4x_1x_2 - 4x_2x_3$

12) $f = 2x_1^2 + 2x_2^2 + 3x_3^2 - 4x_1x_2 - 4x_2x_3$

13) $f = 3x_1^2 + 4x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_2x_3$

14) $f = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_2x_3$

15) $f = x_1^2 - 2x_2^2 - 2x_3^2 - 4x_1x_2 + 4x_1x_3 + 8x_2x_3$

16) $f = 5x_1^2 + 6x_2^2 + 4x_3^2 - 4x_1x_2 - 4x_1x_3$

17) $f = 3x_1^2 + 6x_2^2 + 3x_3^2 - 4x_1x_2 - 8x_1x_3 - 4x_2x_3$

18) $f = 7x_1^2 + 5x_2^2 + 3x_3^2 - 8x_1x_2 + 8x_2x_3$

19) $f = 2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_4^2 - 4x_1x_2 + 2x_1x_4 + 2x_2x_3 - 4x_3x_4$

20) $f = x_1^2 + x_2^2 + x_3^2 + x_4^2 + 2x_1x_2 - 2x_1x_4 - 2x_2x_3 + 2x_3x_4$

21) $f = 8x_1x_3 + 2x_1x_4 + 2x_2x_3 + 8x_2x_4$

22) $f = 2x_1x_2 + 2x_3x_4$

886. Quyidagi kvadratik formalarni musbat aniqlanganlikka tekshiring.

1) $f = 2x_1x_2 + 2x_1x_3 - 2x_1x_4 - 2x_2x_3 + 2x_2x_4 + 2x_3x_4$

2) $f = x_1^2 + x_2^2 + x_3^2 + x_4^2 - 2x_1x_2 + 6x_1x_3 - 4x_1x_4 - 4x_2x_3 + 6x_2x_4 - 2x_3x_4$

3) $f = 2x_1^2 + x_2^2 - 4x_1x_2 - 4x_2x_3$

4) $f = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_2x_3$

5) $f = (x_1 + x_2)(2x_1 - 4x_3) + 10x_1(x_1 - x_3)$

6) $f = x_1^2 + 2x_2^2 + 3x_3^2 - 4x_1x_2 - 4x_2x_3$

7) $f = x_1^2 - 2x_2^2 - 2x_3^2 - 4x_1x_2 + 4x_1x_3 + 8x_2x_3$

8) $f = 3x_1^2 + 4x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_2x_3$

9) $f = x_1^2 - 2x_1x_2 + x_2x_3 + 4x_3^2$

10) $f = 4x_1x_2 - 6x_1x_3 + 8x_2x_3$

11) $f = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 2x_1x_3 + 2x_2x_3 + 2x_3^2$

12) $f = -x_1^2 + 2x_1x_2 - 2x_2^2 + 2x_2x_3 - 2x_3^2$

13) $f = 4x_1^2 - 4x_1x_2 - x_3^2$
14) $f = 2x_1x_2 - 6x_2x_3 + 2x_3x_1$
15) $f = 4x_1^2 + x_2^2 + 5x_3^2 + 2x_1x_2 - 8x_1x_3 - 4x_2x_3$
16) $f = 2x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 2x_2x_3$
17) $f = 2x_1^2 + 3x_2^2 + 4x_3^2 - 2x_1x_2 + 4x_1x_3 - 3x_2x_3$
18) $f = x_1^2 - 2x_2^2 + x_3^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3$
19) $f = x_1^2 + x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$
20) $f = x_1^2 + 5x_2^2 - 4x_3^2 + 2x_1x_2 - 4x_1x_3$
21) $f = 4x_1^2 + x_2^2 + x_3^2 - 4x_1x_2 + 4x_1x_3 - 3x_2x_3$
22) $f = 3x_1^2 + 2x_2^2 - x_3^2 - 2x_4^2 + 2x_1x_2 - 4x_2x_3 + 2x_2x_4$

24§. Matritsali ko'phadlar. Kaninik λ-matritsalar. Ekvivalent λ-matritsalar.

887. Quyida berilgan λ-matritsalarni elementar almashtirishlar yordamida normal diagonal ko'rinishga keltiring:

1) $\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ 2) $\begin{pmatrix} \lambda^2 - 1 & \lambda + 1 \\ \lambda + 1 & \lambda^2 + 2\lambda + 1 \end{pmatrix}$

3) $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda + 5 \end{pmatrix}$ 4) $\begin{pmatrix} \lambda^2 - 1 & 0 \\ 0 & (\lambda - 1)^3 \end{pmatrix}$

5) $\begin{pmatrix} \lambda + 1 & \lambda^2 + 1 & \lambda^2 \\ 3\lambda - 1 & 3\lambda^2 - 1 & \lambda^2 + 2\lambda \\ \lambda - 1 & \lambda^2 - 1 & \lambda \end{pmatrix}$ 6) $\begin{pmatrix} \lambda^2 & \lambda^2 - \lambda & 3\lambda^2 \\ \lambda^2 - \lambda & 3\lambda^2 - \lambda & \lambda^3 + 4\lambda^2 - 3\lambda \\ \lambda^2 + \lambda & \lambda^2 + \lambda & 3\lambda^2 + 3\lambda \end{pmatrix}$

7) $\begin{pmatrix} \lambda - 2 & -1 & 0 \\ 0 & \lambda - 2 & -1 \\ 0 & 0 & \lambda - 2 \end{pmatrix}$ 8) $\begin{pmatrix} \lambda(\lambda + 1) & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & (\lambda + 1)^2 \end{pmatrix}$

9) $\begin{pmatrix} 1 - \lambda & \lambda^2 & \lambda \\ \lambda & \lambda & -\lambda \\ 1 + \lambda^2 & \lambda^2 & -\lambda^2 \end{pmatrix}$ 10) $\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$

11) $\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$ 12) $\begin{pmatrix} \lambda & -1 & 0 & 0 & 0 \\ 0 & \lambda & -1 & 0 & 0 \\ 0 & 0 & \lambda & -1 & 0 \\ 0 & 0 & 0 & \lambda & -1 \\ 1 & 2 & 3 & 4 & 5+\lambda \end{pmatrix}$

13) $\begin{pmatrix} \lambda+\alpha & \beta & 1 & 0 \\ -\beta & \lambda+\alpha & 0 & 1 \\ 0 & 0 & \lambda+\alpha & \beta \\ 0 & 0 & -\beta & \lambda+\alpha \end{pmatrix}$

14) $\begin{pmatrix} 2\lambda^2-12\lambda+16 & 2-\lambda & 2\lambda^2-12\lambda+17 \\ 0 & 3-\lambda & 0 \\ \lambda^2-6\lambda+7 & 2-\lambda & \lambda^2-6\lambda+8 \end{pmatrix}$

15) $\begin{pmatrix} 3\lambda^2-5\lambda+2 & 0 & 3\lambda^2-6\lambda+3 \\ 2\lambda^2-3\lambda+1 & \lambda-1 & 2\lambda^2-4\lambda+2 \\ 2\lambda^2-2\lambda & 0 & 2\lambda^2-4\lambda+2 \end{pmatrix}$

888. Quyida berilgan λ-matritsalarni invariant ko'paytuvchilar yordamida normal diagonal ko'rinishga keltiring:

1) $\begin{pmatrix} \lambda(\lambda-1) & 0 & 0 \\ 0 & \lambda(\lambda-2) & 0 \\ 0 & 0 & (\lambda-1)(\lambda-2) \end{pmatrix}$

2) $\begin{pmatrix} \lambda(\lambda-1) & 0 & 0 \\ 0 & \lambda(\lambda-2) & 0 \\ 0 & 0 & \lambda(\lambda-3) \end{pmatrix}$

3) $\begin{pmatrix} (\lambda-1)(\lambda-2)(\lambda-3) & 0 & 0 & 0 \\ 0 & (\lambda-1)(\lambda-2)(\lambda-4) & 0 & 0 \\ 0 & 0 & (\lambda-1)(\lambda-3)(\lambda-4) & 0 \\ 0 & 0 & 0 & (\lambda-2)(\lambda-3)(\lambda-4) \end{pmatrix}$

889. Quyida berilgan λ-matritsalarni invariant ko'paytuvchilarni toping:

1) $\begin{pmatrix} 3\lambda^2+2\lambda-3 & 2\lambda-1 & \lambda^2+2\lambda-3 \\ 4\lambda^2+3\lambda-5 & 3\lambda-2 & \lambda^2+3\lambda-4 \\ \lambda^2+\lambda-4 & \lambda-2 & \lambda-1 \end{pmatrix}$

2) $\begin{pmatrix} 3\lambda^3-2\lambda+1 & 2\lambda^2+\lambda-1 & 3\lambda^3+2\lambda^2-2\lambda-1 \\ 2\lambda^3-2\lambda & \lambda^2-1 & 2\lambda^3+\lambda^2-2\lambda-1 \\ 5\lambda^3-4\lambda+1 & 3\lambda^2+\lambda-2 & 5\lambda^3+3\lambda^2-4\lambda-2 \end{pmatrix}$

3) $\begin{pmatrix} 2\lambda^3-\lambda^2+2\lambda-1 & 2\lambda^3-3\lambda^2+2\lambda-3 & \lambda^3-2\lambda^2+\lambda-2 & 5\lambda^3-2\lambda^2+5\lambda-2 \\ \lambda^3+\lambda^2+\lambda+1 & \lambda^3-3\lambda^2+\lambda-3 & -\lambda^3-\lambda^2-\lambda-1 & 7\lambda^3-\lambda^2+7\lambda-1 \\ \lambda^3-2\lambda^2+\lambda-2 & \lambda^3+\lambda & 2\lambda^3-\lambda^2+2\lambda-1 & -2\lambda^3-\lambda^2-2\lambda-1 \\ 3\lambda^3-2\lambda^2+3\lambda-2 & 3\lambda^3-4\lambda^2+3\lambda-4 & 2\lambda^3-3\lambda^2+2\lambda-3 & 6\lambda^3-3\lambda^2+6\lambda-3 \end{pmatrix}$

4) $\begin{pmatrix} \lambda^3+\lambda^2-\lambda+3 & \lambda^3-\lambda^2+\lambda & 2\lambda^3+\lambda^2-\lambda+4 & \lambda^3+\lambda^2-\lambda+2 \\ \lambda^3+3\lambda^2-3\lambda+6 & \lambda^3-3\lambda^2+3\lambda-2 & 2\lambda^3+3\lambda^2-3\lambda+7 & \lambda^3+3\lambda^2-3\lambda+4 \\ \lambda^3+2\lambda^2-2\lambda+4 & \lambda^3-2\lambda^2+2\lambda-1 & 2\lambda^3+2\lambda^2-2\lambda+5 & \lambda^3+2\lambda^2-2\lambda+3 \\ 2\lambda^3+\lambda^2-\lambda+5 & 2\lambda^3-\lambda^2+\lambda+1 & 4\lambda^3+\lambda^2-\lambda+7 & 2\lambda^3+\lambda^2-\lambda+3 \end{pmatrix}$

25§. TEST SAVOLLARI:

1. n ta sondan xosil bo'lgan juft o'rin almashtirishlar soni:

A) $n!$ B) $\frac{n-1}{2}$ C) $\frac{n}{2}$ D) $\frac{n!}{2}$ E) n

2. 4, 5, 1, 3, 6, 2 o'rin almashtirishdagi inversiyalar soni:

A) 9 B) 7 C) 6 D) 8 E) 1

3. 3, 8, 5, 2, 4, 6, 7, 1 o'rin almashtirishdagi inversiyalar soni:
A) 13 B) 15 C) 16 D) 12 E) 1

4. O'rin almashtirishning bitta transpozitsiyasi:
A) elementlar sonini ko'paytiradi.
B) o'rin almashtirishning juftligini o'zgartiradi.
C) o'rin almashtirishni o'rniga qo'yishga o'zgartiradi.
D) inversiyalar sonini 3 ga ko'paytiradi.
E) o'rin almashtirishni o'zgartirmaydi.

5. O'rniga qo'yishlarning ko'paytmasi haqida aytilgan mulohazalarning qaysi biri to'g'ri?
A) kommutativ
B) assotsiativ
C) o'rniga qo'yishlarning ko'paytmasi doim toq bo'ladi
D) ko'paytmasi doim nol
E) natijada son chiqadi.

6. i va k ning qanday qiymatlarida 5-tartibli determinantda $a_{1i}a_{32}a_{4k}a_{25}a_{53}$ ko'paytma musbat ishora bilan qatnashadi?
A) $i=1,\ k=4$ B) $i=4,\ k=1$ C) $i=5,\ k=2$
D) $i=0,\ k=2$ E) $i=3,\ k=4$

7. 6-tartibli determinantda $a_{23}a_{31}a_{42}a_{56}a_{14}a_{65}$ ko'paytma
A) musbat ishora bilan qatnashadi
B) manfiy ishora bilan qatnashadi
C) katnashmaydi
D) ko'paytmaning ishorasini aniqlab bo'lmaydi

E) to’g’ri javob yo’q

8. Quyidagi matritsalardan qaysi birining determinanti noldan farqli:
A) ikkita bir xil satrlarga ega bo’lgan matritsa
B) proportsional ustunlarga ega bo’lgan matritsa
C) bitta ustuni nollalardan iborat matritsa
D) determinanti nolga teng bo’lgan matritsadan transponerlash yordamida hosil bo’lgan matritsa
E) birlik matritsa

9. n-tartibli determinantning nechta hadi bor?

A) n B) $\frac{n}{2}$ C) n^2 D) $n!$ E) $n+1$

10. λ ning qanday qiymatida $\begin{cases} \lambda x + y = 5 \\ 2x + y = 6 \end{cases}$ tenglamalar sistemasi echimga ega emas?

A) $\lambda = 3$ B) $\lambda = -3$ C) $\lambda = 2$ D) $\lambda \in R$ E) 0

11. λ ning qanday qiymatlarida $\begin{cases} \lambda x + y = 0 \\ x + \lambda y = 0 \end{cases}$ tenglamalar sistemasi noldan farqli echimga ega?

A) $\lambda = 1$ B) $\lambda = -1$ C) $\lambda = \pm 1$ D) $\lambda = 0$ E) $\lambda = 2$

12. Quyidagi yig’indi $a_{11}A_{12} + a_{21}A_{22} + ... + a_{n1}A_{n2}$ nimaga teng? Bu erda a_{ij} n-tartibli kvadrat matritsaning elementlari bo’lib, A_{ij} ularga mos keluvchi algebraik to’ldiruvchilarni bildiradi.
A) $\det A$ B) 0 C) 1 D) n E) $n!$

13. Matritsaning rangi quyidagilardan qaysi biriga teng emas?
A) chiziqli erkli satrlarning maksimal soniga
B) chiziqli erkli ustunlarning maksimal soniga
C) noldan farqli minorlarning maksimal tartibiga
D) matritsaning pog’onali ko’rinishidagi, noldan farqli satrlar soniga
E) matritsaning nolga teng elementlar soniga

14. $rankA = n,\ rankB = k,\ rank(A \cdot B) = m$ bo'lsa, quyidagi tengsizliklardan qaysi biri noto'g'ri?

A) $m \le n$ B) $m \le k$ C) $m \le n + k$

D) $k < m$ E) $m^2 \le n \cdot k$

15. $\det A = 3,\ \det B = 2,\ \det(A \cdot B)$ ni toping.

A) 5 B) 6 C) 3 D) 2 E) aniqlab bo'lmaydi.

16. $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}^{-1}$ matritsa quyidagilardan qaysi biriga teng?

A) $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$ C) $\begin{pmatrix} -1 & -2 \\ -2 & -5 \end{pmatrix}$

D) $\begin{pmatrix} 0{,}5 & 2 \\ 2 & 2{,}5 \end{pmatrix}$ E) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

17. $\begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$ matritsaga teskari matritsaning izini toping.

A) $2\cos\varphi$ B) $2\sin\varphi$ C) $\sin 2\varphi$ D) $\sin^2\varphi$ E) $\cos^2\varphi$

18. $\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ tenglama echimining determinantini toping.

A) 1 B) 2 C) 3 D) 0 E) ixtiyoriy.

19. i^{1998} ni hisoblang.

A) 1 B) –1 C) i D) $-i$ E) 0

20. i^{123} ni hisoblang.

A) 1 B) –1 C) i D) $-i$ E) 0

21. Agar $z = \cos\frac{\pi}{18} + i\sin\frac{\pi}{18}$ bo'lsa, z^{36} nimaga teng?

A) 1 B) –1 C) i D) $-i$ E) 0

22. Hisoblang: $(2+3i)(4-5i)+(2-3i)(4+5i)$

A) 43 B) 46 C) -5i D) 6i E) $i+5$

23. Hisoblang: $(1+2i)^6$

A) $117+44i$ B) $107-43i$ C) $101+42i$ D) i E) $5i$

24. Hisoblang: $\dfrac{(1+2i)^2-(2-i)^3}{(2+i)^2-(1-i)^3}$

A) $5-5i$ B) $5+5i$ C) 5 D) $1+i$ E) i

25. $z=-i$ ni trigonometrik shaklga keltiring.

A) $\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$ B) $cos\dfrac{3\pi}{2}+i\sin\dfrac{3\pi}{2}$

C) $\cos 2\pi+i\sin 2\pi$ D) $-\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$ E) $i\cos\dfrac{3\pi}{2}$

26. $z=-1$ ni trigonometrik shaklda yozing.

A) $\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$ B) $\cos\pi+i\sin\pi$ C) $\cos 2\pi+i\sin 2\pi$

D) $cos\dfrac{3\pi}{2}+i\sin\dfrac{3\pi}{2}$ E) $\cos\pi-i\sin\pi$

27. Hisoblang: $i^{20}-2i^{40}$.

A) i B) -i C) -1 D) 1 E) $2i$

28. $z=\dfrac{\sqrt{3}}{2}+i\cdot\dfrac{1}{2}$ son qanday eng kichik darajada 1 ga teng?

A) 6 B) 3 C) 0 D) 2 E) 12

29. $z = i$ son qanday eng kichik musbat darajada 1 ga teng?

A) 4 B) 3 C) 2 D) 1 E) 5

30. z_1 va z_2 kompleks sonlar bo'lsa, quyidagilardan qaysi biri noto'g'ri?

A) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ B) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

C) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}$ D) $\left|\overline{z_1}\right| = |z_1|$ E) $\overline{z_1} + z_2 = \overline{z_2} \cdot z_1$

31. z kompleks son bo'lsa, quyidagilardan qaysi biri noto'g'ri?

A) $\left|\overline{z}\right| = |z|$ B) $\overline{z} \cdot z = |z|^2$ C) $\arg \overline{z} = -\arg z$

D) $\overline{z} \cdot \overline{z} = |z|^2$ E) $z + \overline{z} = \operatorname{Re} z$

32. $z = -2 - 3i$ son kompleks tekisligining kaerida joylashgan?

A) I chorakda B) II chorakda C) III chorakda
D) IV chorakda E) koordinatalar boshida

33. $z = \sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$ kompleks sonining to'rtinchi darajasi nimaga teng?

A) 5 B) –4 C) $1+i$ D) $2(1+i)$ E) i

34. $3\left(\cos\dfrac{\pi}{5} + i\sin\dfrac{\pi}{5}\right)$ kompleks sonining (-3) darajasi kompleks tekislikning qaerida joylashgan?

A) I chorakda B) II chorakda C) III chorakda D) IV chorakda
E) koordinatalar boshida

35. Agar z_1 va z_2 kompleks sonlar bo'lsa, quyidagilarning qaysi biri noto'g'ri?

A) $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$ B) $\arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2$

C) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$ D) $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

E) $\arg(z_1 - z_2) = \arg z_1 \cdot \arg z_2$

36. Agar $z = \cos^2 \varphi + i \sin^2 \varphi$ bo'lsa, $\operatorname{Re} z + \operatorname{Im} z$ nimaga teng?

A) $2\cos^2 \varphi$ B) 1 C) $2\sin^2 \varphi$ D) $\sin 2\varphi$ E) $\cos 2\varphi$

37. $f(x) = x^4 + 3x^3 - x^2 - 4x - 3,\quad g(x) = 3x^3 + 10x^2 + 2x - 3$ ko'phadlarning EKUBini toping.

A) $x+5$ B) $x+3$ C) x^2+1 D) $2x+3$ E) 1

38. $f(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 6$ ko'phadni $g(x) = x^2 - 3x + 1$ ko'phadga bo'lgandagi qoldiq nimaga teng?

A) $x-5$ B) $25x+5$ C) $25x-5$ D) $25x$ E) 5

39. $4x^3 + x^2$ ko'phadni $x+1+i$ ko'phadga bo'lgandagi qoldiq nimaga teng.

A) $8-6i$ B) $8+6i$ C) 8 D) $6i$ E) $3+2i$

40. $x^4 + x^3 - 3x^2 - 4x - 1$ va $x^3 + x^2 - x - 1$ ko'phadlarning EKUBini toping.

A) $x+2$ B) $x+5$ C) $x+1$ D) x E) 2

41. m ning qanday qiymatlarida $f(x) = -(x-1)^2 - x^2 + m$ ko'phad $x^2 - x$ ko'phadga bo'linadi?

A) $m=1$ B) $m=2$ C) $m=-1$ D) $m=0$ E) $m \in R$

42. $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}^3$ matritsaning izini toping.

A) 30 B) 45 C) 0 D) 50 E) 3

43. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{10}$ matritsa elementlarining yig'indisini toping.

A) 10 B) 12 C) 11 D) 0 E) –1

44. $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}$ va $B = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}$ matritsalar ko'paytmasining determinantini hisoblang.

A) 3 B) –3 C) 0 D) –5 E) –1

45. $A = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ matritsalarni ko'paytirishdan hosil bo'lgan matritsa elementlarining yig'indisini hisoblang.

A) 10 B) 13 C) 18 D) 1 E) 3

46. Quyidagi o'rniga qo'yishlarning ko'paytmasini toping.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 1 & 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$$

A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 3 & 5 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 1 & 5 \end{pmatrix}$

E) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 2 & 1 \end{pmatrix}$

47. i va k larning qanday qiymatlarida $1,2,7,4,i,5,6,k,9$ o'rin almashtirish juft bo'ladi?

A) $i = 5, k = 3$ B) $i = 8, k = 3$ C) $i = 3, k = 8$

D) $i=3,\ k=3$ E) $i=9,\ k=2$

48. i va k larning qanday qiymatlarida $1, i, 2, 5, k, 4, 8, 9, 7$ o'rin almashtirish toq bo'ladi?

A) $i=3,\ k=6$ B) $i=6,\ k=3$ C) $i=2,\ k=2$

D) $i=7,\ k=5$ E) $i=3,\ k=5$

49. $n, n-1, n-2, ..., 2, 1$ o'rin almashtirishdagi inversiyalar sonini toping.

A) $n!$ B) $\frac{n(n+1)}{2}$ C) C_n^2 D) $(n+1)^2$ E) n^3

50. $\begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$ matritsa rangini hisoblang.

A) 3 B) 4 C) 2 D) 1 E) 5

51. Chiziqli tenglamalar sistemasi echimga ega bo'lsa, uning koeffitsientlaridan tuzilgan matritsa uchun quyidagilardan qaysi biri o'rinli?

A) uning rangi birga teng

B) uning rangi kengaytirilgan matritsaning rangiga teng

C) determinanti nolga teng

D) izi nolga teng

E) satrlari chiziqli erkli

52. Quyida ko'rsatilgan tengliklarning qaysi biri $f(x)$ va $g(x)$ ko'phadlarning o'zaro tubligini ko'rsatadi ($u(x)$ va $v(x)$ lar ko'phadlar).

A) $f(x)u(x)+g(x)v(x)=1$ B) $f^2(x)=g^2(x)$

C) $f^3(x)u(x)-g(x)=1$ D) $f(x)u(x)-g(x)-v(x)=0$

E) $f(x)+g(x)=u(x)+v(x)$

53. Agar $f(x)$ ko'phad o'zaro tub bo'lgan $\varphi(x)$ va $\psi(x)$ ko'phadlarning har biriga bo'linsa …

A) $f(x)$ ularning yig'indisiga bo'linadi

B) $f(x)$ ularning ayirmasiga bo'linadi

C) $f(x)$ ularning ko'paytmasiga bo'linadi

D) $f(x)$ ularning kvadratlarining yig'indisiga bo'linadi
E) $f(x)$ ularning ko'paytmasiga teng

54. Agar $f(x)$ ko'phad $\varphi(x)$ va $\psi(x)$ ko'phadlar bilan o'zaro tub bo'lsa:
A) $f(x)$ ularning yig'indisi bilan o'zaro tub
V) $f(x)$ ularning ayirmasi bilan o'zaro tub
S) $f(x)$ ularning ko'paytmasi bilan o'zaro tub
D) $f(x)$ ularning kvadratlariga teng
E) $f(x)$ ularning ko'paytmasiga teng

55. $f(x)$ ko'phad x-c ga bo'linsa:
A) $f(s)=1$ *B)* $f(c)=0$ *C)* $f(s)=-1$ *D)* $f(c)=x$ *E)* $f(c)=-c$

56. $f(x)$ ko'phadni x-c ga bo'lgandagi qoldiqni toping.
A) $f(0)$ B) $f(-c)$ C) $f(-1)$ D) $f(c)$ E) 0

57. Uchinchi darajali $f(x)$ va $g(x)$ ko'phadlar to'rtta turli nuqtada bir xil qiymatlarga ega bo'lsa, quyidagilarning qaysi biri to'g'ri.
A) $f(x)=g(x)$ *B)* $\deg f(x)+1<g(x)$ *C)* $f(x)>g(x)$
D) $f(x)<g(x)$ *E)* $f(x)=g^2(x)+1$

58. $f(x)=(x^5+1)^{100}$ ko'phad barcha koeffitsientlarining yig'indisini toping.
A) -1 B)2 C) 2^{100} D) 0 E) 500

59. $f(x)=(x^2+x)^{100}$ ko'phad barcha koeffitsientlarining yig'indisini toping.

A) 2 B) 2^{100} C) 3 D) 200 E) 300

60. $f(x)=x^5-4x^4+6x^3-1$ ko'phad barcha ildizlarining yig'indisini toping.

A) -5 B) 5 C) 6 D) -1 E) – 8

61. $f(x)=x^4-9x^3+176x-90$ ko'phad barcha ildizlarining ko'paytmasini toping.

A) 90 B) -9 C) -90 D) -1 E) 17

62. $f(x)=x^3-3x+\lambda$ ko’phad λ ning qanday qiymatlarida karrali ildizga ega?

$A)\ \lambda=3$ $B)\ \lambda=\pm 2$ $C)\ \lambda=2$
$D)\ \lambda=1$ $E)\ \lambda=0$

63. $f(x)=x^3-12x+\lambda$ ko’phad λ ning qanday qiymatlarida karrali ildizga ega?
$A)\ \lambda=\pm 12$ $B)\ \lambda=12$ $C)\ \lambda=\pm 16$
$D)\ \lambda=0$ $E)\ \lambda=16$

64. Ildizlari 1; 2; 3 bo’ladigan kubik ko’phad tuzing.
$A)\ f(x)=x^3-2x+5$ $B)\ f(x)=2x^3-4x+7$
$C)\ f(x)=x^3-1997x-196$ $D)\ f(x)=x^3-6x^2+11x-6$
$E)\ f(x)=x^3-27$

65. Ildizlari -1; 1; 2 bo’ladigan kubik ko’phad tuzing.
$A)\ f(x)=x^3+1$ $B)\ f(x)=x^3-5x$ $C)\ f(x)=x^3-7x+5$
$D)\ f(x)=2x^3-7x^2+6$ $E)\ f(x)=x^3-2x^2-x+2$

66. $f(x)=x^3-2x^2+x$ ko’phadni *x-1* ga bo’lgan dagi qoldiqni toping.

A) 2 B) -1 C) 0 D) -4 E) 1

67. $f(x)=x^6-2x^2+1$ ko’phadni *x+1* ga bo’lgan dagi qoldiqni toping.

A) 0 B) -2 C) 1 D) 4 E) -1

68. $\frac{1}{x^2-1}$ to’g’ri ratsional kasrni, sodda ratsional kasrlar yig’indisiga yoyib yozganda yig’indida nechta had qatnashadi?

A) 1 B) 3 C) 2 D) 5 E) 6

69. $\frac{2}{(x^2+1)(x-1)}$ to’g’ri ratsional kasrni kompleks sonlar maydoni ustida sodda kasrlar yig’indisi shaklida yozganda yig’indida nechta had qatnashadi?

A) 1 B) 2 C) 4 D) 3 E) 6

70. $1, 3, -2, 1, -4, -8, -3, 4, 1$ sonlar ketma-ketligida nechta ishora almashtirish bor?

A) 3 B) 3 C) 10 D) 4 E) 5

71. $1, 3, -2, 1, -4, -8, -3, 4, 1$ sonlar ketma-ketligida nechta ishora almashtirish bor?

A) 3 B) 2 C) 10 D) 4 E) 5

72. 1, -5, 6, 3, -2, 1 sonlar ketma-ketligida nechta ishora almashtirish bor?

A) 4 B) 2 C) 6 D) 7 E) 8

73. Agar f_i va f_{i+1} Shturm sistemasining qo'shni hadlari bo'lsa:
A) Ularning ildizlari bir xil
B) Ularning ildizlari ko'paytmasi teng
C) Ular umumiy ildizga ega emas
D) Ikkalasining ildizlar soni teng
E) Nol soni ikkalasining ham ildizi bo'ladi.

74. Agar $f_s(x)$ Shturm sistemasining oxirgi ko'phadi bo'lsa, uning ildizlar soni qancha?

A) 1 B) ∞ C) 0 D) 2 E) n

75. $f(x) = x^3 + 3x^2 - 1$ ko'phadning haqiqiy ildizlar soni nechta?

A) 2 B) 0 C) 3 D) 1 E) 4

76. $f(x) = x^4 - x - 1$ ko'phadning haqiqiy ildizlar soni nechta?

A) 4 B) 3 C) 1 D) 0 E) 2

77. $f(x) = x^4 + 4x^3 - 12x + 9$ ko'phadning haqiqiy ildizlar soni nechta?

A) 0 B) 3 C) 4 D) 2 E) 1

78. Agar sonlar sistemasi o'z elementlarining yig'indisini, ayirmasini va ko'paytmasini o'z ichiga olsa, u nima bo'ladi?
A) maydon B) halqa C) gruppa
D) yarim gruppa E) Abel gruppasi

79. Agar sonlar sistemasi o'zining ixtiyoriy ikkita elementining yig'indisini, ayirmasini ko'paytmasini va (noldan farqli elementga) bulinmasini o'z ichiga olsa, u quyidagilardan qaysi biri bo'ladi?

A) halqa B) gruppa C) maydon
D) Abel gruppasi E) faktor gruppa

80. *G* - ko'paytirishga nisbatan Abel gruppasi bo'lsa, uning *x*, *y* elementlari uchun quyidagilarning qaysi biri hamisha bajariladi?

A) $x+u=u+x$ B) $x\,u=u\,x$ C) $x^2=u^2+u$ D) $x+u=xu$ E) $xu=u-x$

81. Quyidagilarning qaysi biri tsiklik gruppa bo'ladi?

A) 1 sonining *n* chi darajali ildizlari gruppasi *B) N* *C) Q* *D) R* *E) Z*

82. *G* *n*-tartibli chekli gruppa bo'lib, *A* uning *k*-tartibli qism gruppasi bo'lsa, quyidagilardan qaysi biri to'g'ri bo'ladi?
A) $n^2=k$ *B)* $n<k$ *C)* $n=km,\ m\in N$
D) $nm=k,\ m\in N$ *E)* $n=k=5$

83. Ko'paytmaga nisbatan 3-darajali juft o'rniga qo'yishlar gruppasining tartibi nimaga teng?
A) 6 *B) 5* *C) 3* *D) 2* *E)* gruppa bulmaydi.

84. Ko'paytmaga nisbatan 4-darajali juft o'rniga qo'yishlar gruppasining tartibi nimaga teng?
A) 6 *B) 12* *C) 3* *D) 2* *E)* gruppa emas

85. Agar *G* va G' gruppalar orasida o'zaro bir qiymatli moslik mavjud bo'lib, ixtiyoriy $a,b\in G$ elementlarining ko'paytmasiga ular akslarining ko'paytmasi mos kelsa:

A) G va G' larning noli umumiy deyiladi.
B) G va G' gruppalar izomorf deyiladi.
S) G gruppa G' ning faktor gruppasi deyiladi.
D) G gruppa G' ning normal gruppasi deyiladi.
E) G gruppa G' ning qism gruppasi deyiladi.

86. $A=\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix}$ matritsaning determinantini hisoblang.

A) –3 *B) -5* *C) 4* *D) 8* *E)* aniqlab bo'lmaydi.

87. Determinanti nolga teng bo'lgan matritsaga teskari matritsa nechta?

A) cheksiz kup *B) 1* *C) 2* *D) 0* *E) 3*

88. Agar bir jinsli chiziqli tenglamalar sistemasining koeffitsientlaridan tuzilgan matritsaning determinanti nolga teng bo'lsa, uning nechta echimi mavjud?
A) 1 *B)* cheksiz kup *C) 0* *D) 2* *E) 3*.

89. Agar $A=A^T$ bo'lsa,
A) A-kososimmetrik
B) A simmetrik
C) det A=0
D) rank A=0
E) A=-A

90. Birlik kvadrat matritsaning determinanti nimaga teng?
A) n *B)3* *C) 2* *D) 1* *E)* mavjud emas.

91. $\begin{pmatrix} 1 & 1 & 1 \\ 0 & \sin\varphi & \cos\varphi \\ 0 & -\cos\varphi & \sin\varphi \end{pmatrix}$ matritsaning determinanti nimaga teng?
A) $\sin^2\varphi$ *B)* $\cos^2\varphi$ *C)* $2\sin\varphi\cos\varphi$ *D) 1* *E) 0*

92. Quyidagilardan qaysi biri 3-tartibli determinantni yoyganda musbat ishora bilan qatnashadi?

1) $a_{11}a_{22}a_{33}$ *2)* $a_{13}a_{21}a_{32}$ *3)* $a_{11}a_{23}a_{32}$

A) 1,2 *B)* faqat *1* *S) 2,3* *D) 1,3* *E)3*

93. (0,0) turdagi tenzor bu:

A) bichiziqli funktsiya B) skalyar
S) chiziqli funktsiya D) vektor E) chiziqli operator

94. CHiziqli operator R^2 fazoning qandaydir bazisida $A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$ matritsa bilan berilgan. Bu operator xos qiymatlarining yig'indisi quyidagilardan qaysi biriga teng?

A) 2 *B) 5* *C) 10* *D) 6* *E) 25*

95. $\bar{e}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\bar{e}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ bazisda $\bar{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ vektorning koordinatasi quyidagilardan qaysi biriga teng?

A) $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ B) $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ C) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ D) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ E) $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

96. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 2 & 0 \\ -1 & 2 \end{pmatrix}$ matritsa bilan berilgan, $\bar{e}_1' = \bar{e}_1 + \bar{e}_2'$, $\bar{e}_2' = 2\bar{e}_1 + \bar{e}_2'$ bazisdagi matritsasini toping.

A) 4 B) 2 C) 3 D) 5 E) 0

97. $f = x_1^2 + 2x_1x_2$ kvadratik formaning rangini toping.
A) 1 B) 2 C) 0 D) -1 E) 3

98. Chiziqli operator $\bar{e}_1, \bar{e}_2$ bazisda $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ matritsa bilan berilgan. Qiymatlar sohasining o'lchami quyidagilardan qaysi biriga teng?
A) 3 B) -3 C) 2 D) 0 E) 1

99. λ ning qanday qiymatlarida $f = x_1^2 + 4x_1x_2 + \lambda x_2^2$ kvadratik forma musbat aniqlangan?
A) $\lambda > 0$ *B)* $\lambda < 0$ *C)* $\lambda = 0$ *D)* $\lambda > 5$ *E)* $\lambda > 4$

100. Quyidagilardan qaysi biri λ-matritsalar uchun kiritilgan elementar almashtirish bo'lmaydi?
A) $A(\lambda)$ ning ixtiyoriy satrini songa kupaytirish
B) $A(\lambda)$ ning ixtiyoriy ustunini songa kupaytirish
S) $A(\lambda)$ ning ixtiyoriy satriga boshqa bir satrini ko'phadga ko'paytirib qo'shish.
D) $A(\lambda)$ ning ixtiyoriy ustuniga boshqa bir ustunini ko'phadga ko'paytirib qo'shish.
E) $A(\lambda)$ ning ixtiyoriy satrini boshqa satriga ko'paytirish

101. Unimodulyar matritsaga teskari matritsa:

A) simmetrik B) unitar S) unimodulyar
D) musbat aniqlangan E) manfiy aniqlangan

102. O'z-o'ziga qo'shma operatorning turli xos qiymatlariga mos keladigan xos vektorlari

A) ortogonal emas B) ortogonal S) bir biriga teng
D) nol bo'ladi E) proportsional bo'ladi

26§. Oraliq va yakuniy nazorat uchun savollar

1. Chiziqli tenglamalar sistemasi. Determinantlar.

1. Chiziqli tenglamalar sistemasi. Sistemaning birgalikdaligi. Gauss usulida chiziqli tenglamalar sistemasini yechish.
2. Ikkinchi tartibli chiziqli tenglamalar sistemasi. Ikkinchi tartibli determinant. Ikkinchi tartibli chiziqli tenglamalar sistemasi uchun Kramer usuli.
3. Uchinchi tartibli chiziqli tenglamalar sistemasi. Uchinchi tartibli determinant. Uchinchi tartibli chiziqli tenglamalar sistemasi uchun Kramer qoidasi.
4. O'rin almashtirishlar soni haqidagi teorema. Transpozitsiya. O'rin almashtirish va transpozitsiya haqida teorema.
5. Inversiya. Juft va toq o'rin almashtirishlar. Transpozitsiya va o'rin almashtirishning juft-toqligi haqidagi teorema.
6. O'rniga qo'yishlar, ularning soni. Juft va toq o'rniga qo'yishlarning soni. O'rniga qo'yishlarning ko'paytmasi.
7. *n*-tartibli determinant ta'rifi. Transponerlashda determinant o'zgarmasligi.
8. Determinantlarda satrlarni o'rnini almashtirish. Ikkita bir xil satrga ega bo'lgan determinant.
9. Satrdan ko'paytuvchini chiqarish xossasi. Proportsional satrlarga ega bo'lgan determinant.
10. Bir satri yig'indidan iborat bo'lgan determinant.
11. Bitta satri boshqa satrlarining chiziqli kombinatsiyasi bo'lgan determinant.
12. Determinantning minori, uning algebraik to'ldiruvchisi. Minorni uning algebraik to'ldiruvchisiga ko'paytmasi haqidagi teorema.
13. Determinantni satr yoki ustun bo'yicha yoyish.
14. Laplas teoremasi.
15. Burchagi nol bo'lgan determinant haqidagi teorema.
16. Vandermond determinanti.
17. *n*-tartibli chiziqli tenglamalar sistemasi uchun Kramer qoidasi.

2. Chiziqli tenglamalar sistemalari (umumiy nazariya).

18. n o'lchamli chiziqli fazo (R^n). Vektorlarning chiziqli bog'liqligi va chiziqli erkliligi. *n* o'lchamli vektor fazoda *n* ta chiziqli erkli vektorga misol. Chiziqli bog'liqlikning ikkita ta'rifi va ularning ekvivalentligi.
19. Ekvivalent vektorlar sistemalari va ularning oddiy xossalari.

20. Vektorlar algebrasining asosiy teoremasi va uning natijalari.
21. Vektorlar sistemasining rangi. Ekvivalent vektorlar sistemasining rangi.
22. Matritsaning rangi. Matritsaning rangi haqida teorema.
23. Matritsaning maksimal chiziqli erkli satrlar soni. n -tartibli determinantning nolga tenglik sharti.
24. Chiziqli tenglamalar sistemasi. Kronekker-Kapelli teoremasi.
25. Birgalikda bo’lgan chiziqli tenglamalar sistemasini echish usuli.
26. Bir jinsli chiziqli tenglamalar sistemasi. Echimlar to’plamining tuzilishi.
27. Echimlar fazosining o’lchami. Echimlar fundamental sistemasi.

3. Matritsalar algebrasi.

28. Matritsalarni qo’shish, songa ko’paytirish va matritsalarni ko’paytirish amali. Matritsalar ko’paytmasining kommutativ emasligi. Qo’shish va ko’paytirish amallarining assotsiativligi.
29. Matritsalar ko’paytmasining determinanti haqida teorema.
30. Teskari matritsa. Uning mavjudligi va yagonaligi.
31. Matritsalar ko’paytmasining rangi haqida teorema.

4. Kompleks sonlar.

32. Kompleks sonlar sistemasini kiritish.
33. Kompleks sonlarning trigonometrik formasi.
34. Muavr formulasi.
35. Kompleks sonning qo’shmasi va uning xossalari.
36. Kompleks sondan kvadrat ildiz chiqarish.
37. Kompleks sondan n-darajali ildiz chiqarish.
38. 1 sonidan chiqarilgan n-darajali ildizlar va ularning xossalari.

5. Halqa va maydonlar.

39. Halqa va maydon tushunchasi. Misollar.
40. Halqa va maydonning xossalari.
41. Nolning bo’luvchilari.
42. Halqalarning izomorfligi.
43. Maydonning xarakteristikasi.

6. Ko’phadlar.

44. Bir o’zgaruvchili ko’phadlar va ular ustida amallar. Qoldiqli bo’lish haqidagi teorema.
45. Ko’phadlarning bo’luvchilari va ularning oddiy xossalari.
46. Eng katta umumiy bo’luvchi. Evklid algoritmi.
47. Diofant tenglamasining echimi haqidagi teorema (ko’phadlar uchun).
48. O’zaro tub ko’phadlar haqidagi teorema va uning natijalari.
49. Ko’phadning ildizi va uni birinchi darajali bo’luvchilar bilan bog’lanishi. Bezu teoremasi. Gorner sxemasi.
50. Ko’phadning karrali ildizi haqidagi teorema.
51. 3-darajali ko’phad ildizlarini topish uchun Kardano formulalari.
52. 3-darajali ko’phad diskriminanti va uning xossalari.
53. 4-darajali ko’phad ildizlarini topish uchun Ferrari formulalari.
54. Ozod hadsiz ko’phadning o’zluksizligi haqidagi lemma.
55.Teylor formulasi va ko’phadning o’zluksizligi haqidagi lemma.
56. Ko’phad bosh hadi ning moduli haqidagi lemma.
57. Ko’phad modulining o’sishi haqidagi lemma va uning natijalari.
58. Dalamber lemmasi.
59. Algebraning asosiy teoremasi.
60. Algebraning asosiy teoremasidan kelib chiqadigan natijalar.
61.Viet formulalari.
62. Ko’phadlar uchun yagonalik teoremasi va Lagranjning interpolyatsion formulasi.
63. Koeffitsientlari haqiqiy bo’lgan ko’phadlar haqida teoremalar.
64. Maxraji o’zaro tub ko’paytuvchilarga ajratilgan qisqarmas to’g’ri kasrni yoyib yozish.
65. Maxraji keltirilmaydigan ko’phadning darajasidan iborat bo’lgan qisqarmas to’g’ri ratsional kasrni sodda kasrlar yig’indisi ko’rinishida ifodalash.
66. Shturm funktsiyalar sistemasi.
67. Shturm teoremasi.
68. Haqiqiy ildizlarni ajratish.
69. Ko’phad ildizlarini taqribiy hisoblash usullari.
70. Ko’p o’zgaruvchili ko’phadlar va ularning leksikografik yozuvi.
71. Simmetrik ko’phadlar. Simmetrik ko’phadlar haqidagi asosiy teorema (mavjudligi).
72. Simmetrik ko’phadlar haqidagi asosiy teorema(yagonaligi).

7. Gruppa va halqalar.

73. Gruppa ta’rifi. Tartibi. Kommutativ va nokommutativ gruppalar. Misollar.
74. Gruppalarning izomorfligi.
75. Elementning tartibi. TSiklik gruppa va u haqidagi teoremalar.
76. Qism gruppa. Qo’shma sinflar. Gruppani kesishmaydigan qo’shma sinflar birlashmasiga yoyish.
77. Lagranj teoremasi.
78. Tartibi tub son bo’lgan gruppa haqidagi teorema.
79. Gruppaning normal bo’luvchilari va ularning xossalari.
80. Faktor-gruppa. Misollar.
81. Abel va tsiklik gruppalarning faktor-gruppalari.
82. Gruppalarning gomomorfizmi. Gomomorfizmning yadrosi haqidagi teorema.
83. Gomomorfizm haqidagi teorema.
84. Gruppaning markazi. Tartibi tub sonning darajasidan iborat bo’lgan gruppaning markazi.
85. Gruppaning to’plamdagi ta’siri. Orbitalar tushunchasi.
86. Silov teoremalari.
87. Chekli Abel gruppalari haqida teoremalar.
88. Halqa, qism halqa va ideallar.
89. Halqalar gomomorfligi.
90. Faktor-halqa.

8. Chiziqli fazolar.

91. Chiziqli fazo ta’rifi. Chiziqli fazo aksiomalaridan kelib chiqadigan natijalar. Misollar.
92. Chiziqli fazoning o’lchami va bazisi. Vektorlarning koordinatasi.
93. Bazis o’zgarganda vektor koordinatalarining o’zgarish qonuni.
94. Chiziqli fazolarning izomorfligi haqidagi teorema.
95. Qism fazolar va unga doir misollar.
96. Vektorlarning chiziqli kombinatsiyasidan tashkil topgan fazo va uning o’lchami.
97. Qism fazolarning kesishmasi, algebraik yig’indisi va to’g’ri yig’indisi. Grassman tengligi.
98. Evklid fazosining ta’rifi, Koshi-Bunyakovskiy tengsizligi va uning natijalari.
99. Ortogonal va ortonormal bazislar. Gramm-Shmidtning ortogonallashtirish jarayoni.

100. Evklid fazolarining izomorfligi.

9. Chiziqli operatorlar.

101. Chiziqli operatorlar va ular ustida amallar. Misollar.
102. Chiziqli operator matritsasi va u haqidagi teorema.
103. Chiziqli operatorning turli bazislardagi matritsalari orasidagi bog'lanish. Chiziqli operatorning rangi va determinanti.
104. Chiziqli operatorning yadrosi va aksi haqidagi teorema.
105. Chiziqli operatorning teskarisi va uning mavjudlik sharti.
106. Chiziqli operatorning invariant qism fazolari. Xos vektor va xos qiymatlar. Xarakteristik ko'phad.
107. Xarakteristik ko'phad va uning invariantligi. Chiziqli operatorning izi. Xos qiymatning mavjudligi haqida lemma.
108. Chiziqli operatorlar uchun Gamilton-Keli teoremasi.
109. Chiziqli forma va uning skalyar ko'paytma orqali tasviri.
110. Bichiziqli formalar va ularning matritsasi.
111. Bazis o'zgarganda bichiziqli forma matritsasining o'zgarish qonuni.
112. Simmetrik bichiziqli formalar va ularning kanonik ko'rinishi.
113. Kvadratik formalar. Kvadratik formani kanonik ko'rinishga keltirish haqidagi teorema (Lagranj usuli).
114. Kvadratik formani YAkobi usulida kanonik ko'rinishga keltirish.
115. Kvadratik formaning rangi haqidagi teorema.
116. Inertsiya qonuni haqidagi teorema.
117. Musbat va manfiy aniqlangan kvadratik formalar. Ikkita ta'rifning ekvivalentligi haqida teorema.
118. Silvestr kriteriyasi.
119. Bichiziqli formalar va chiziqli operatorlar orasidagi bog'lanish.
120. Chiziqli operatorning qo'shmasi va uning xossalari. Chiziqli operator qo'shmasining ortogonal bazisdagi matritsasi.
121. O'z-o'ziga qo'shma chiziqli operatorlar va ularning oddiy xossalari. (Xos qiymatlarning haqiqiyligi, har xil xos qiymatlarga mos keluvchi xos vektorlarning ortogonalligi, xos vektoriga ortogonal bo'lgan qism fazoning invariantligi).
122. O'z-o'ziga qo'shma chiziqli operatorning xos vektorlaridan to'zilgan bazis haqidagi teorema.
123. Kvadratik formalarni bosh o'qlarga keltirish.
124. Bir juft kvadratik formani kanonik ko'rinishga keltirish.

125. Unitar va ortogonal chiziqli operatorlar va ularning oddiy xossalari. (Unitar bo'lishning zaruriy va etarli sharti, xos qiymatlarining moduli, har xil xos qiymatlarga mos keluvchi xos vektorlarning ortogonalligi, xos vektoriga ortogonal bo'lgan qism fazoning invariantligi).

126. Unitar operatorning xos vektorlaridan to'zilgan bazis haqidagi teorema.

127. O'rin almashuvchi chiziqli operatorlar. Invariant qism fazo va umumiy xos vektor haqidagi lemmalar.

128. O'rin almashuvchi o'z-o'ziga qo'shma chiziqli operatorlar uchun umumiy kanonik bazisning mavjudligi kriteriyasi.

129. Normal chiziqli operatorlar va ularning xossalari.

130. Musbat aniqlangan chiziqli operatorlar. Ulardan kvadrat ildiz chiqarish haqidagi teorema.

131. Chiziqli operatorni simmetrik va ortogonal chiziqli operatorlar ko'paytmasiga yoyish haqidagi teorema.

132. n-o'lchamli Evklid fazosida berilgan ortogonal operatorning invariant qism fazolari haqida lemmalar.

133. n-o'lchamli Evklid fazosida berilgan ortogonal operator matritsasi uchun sodda ko'rinishni beradigan ortonormal bazisning mavjudligi haqidagi teorema.

10. Matritsaning Jordan normal formasi.

134. λ-matritsalar. λ-matritsalar uchun elementar almashtirishlar.

135. λ-matritsa normal formasining mavjudligi va yagonaligi haqida teorema.

136. Unimodulyar λ-matritsalar va ularning xossalari.

137. λ-matritsalarning ekvivalentligi. λ-matritsalar ekvivalentligining zaruriy sharti.

138. λ-matritsalar ekvivalentligining etarli sharti.

139. Matritsaviy ko'phadlar. Qoldiqli bo'lish algoritmi.

140. Matritsalar o'xshashligi haqidagi asosiy teorema.

141. Jordan normal formasini topish algoritmi.

142. Jordan normal formalarining o'xshashligi.

11. Tenzorlar haqida tushuncha.

143. Chiziqli fazoga qo'shma bo'lgan fazo ta'rifi. Biortogonal bazislar.
144. Chiziqli fazo va unga qo'shma bo'lgan fazoda vektor koordinatalarining o'zgarish konuni.
145. Ko'pchiziqli formalar va ularning berilgan bazisdagi ko'rinishi.
146. Bir bazisdan ikkinchi bazisga o'tishda ko'pchiziqli forma koeffitsientlarining o'zgarish qonuniyati.
147. Tenzorning ta'rifi. Misollar.
148. Bir xil turdagi tenzorlarning tenglik alomati.
149. Tenzorlarni qo'shish va ko'paytirish amali.
150. Tenzorlarni yig'ishtirish amali. Misollar.
151. Simmetrik va antisimmetrik tenzorlar.
152. Berilgan tenzordan foydalanib simmetrik va antisimmetrik tenzor to'zish.

M U N D A R I J A

1§.Chiziqli tenglamalar sistemasi. Gauss usuli……………………………....
2§. Matritsani pog'onali ko'rinishga keltirish. Matritsaning rangi va asosiy ustunlari………………………………………………...............................
3§. Chiziqli tenglamalar sistemasi. Kroneker-Kapelli teoremasi. ..…………
4§. Determinantni hisoblash………………………………………………
5§. Matritsalarni transponerlash, qo'shish, ko'paytirish, darajaga ko'tarish va teskarisini topish. Determinantlarni ko'paytirish……………………………
6§. O'rin almashtirishlar, o'rniga qo'yishlar va ular ustida amallar…………
7§. Matritsalarning determinanti. Minor va uning algebraik to'ldiruvchisi…
8§. Chiziqli tenglamalar sistemasi uchun Kramer qoidasi…………………….
9§. Matritsalarning xos qiymatlari va xos vektorlari………………………….
10§. Kompleks sonlar va ular ustida amallar………………………………….
11§. Kompleks sonlarning algebraik va trigonometrik shakli. Kompleks sonlardan ildiz chiqarish……………………………………………………
12§. Ko'phadlar va ular ustida amallar………………………………….....
13§. Uchinchi va to'rtinchi darajali tenglamalar……………………………
14§. Shturm funktsiyalari. Shturm teoremasi…………………………………
15§. Halqa, maydon va gruppalar……………………………………………..
16§. Butun sonlarning bo'linish nazariyasi.
17§. Z halqada taqqoslamalar va ularning chegirmalar sinflari. Uzluksiz kasrlar……………………………………………………………………
18§. Bir noma'lumli birinchi darajali taqqoslamalar va ularni yechish………
19§. Chiziqli fazo va uning qism fazosi. Vektorlar sistemasining chiziqli bog'liqligi va chiziqli erkliligi. Bazis, vektorlarni bazis orqali ifodalash.
Bir bazisdan ikkinchi bazisga o'tish……………………………………..
20§. Evklid fazosi. Skalyar ko'paytma. Ortogonallik. Vektorning o'zunligi. Vektorlarning ortogonal va ortonormal sistemasi. Gramm-Shmidt ortogonallashtirish jarayoni……………………………………………...
21§.Evklid fazosi. Ortogonal va ortonormal sistemalar. Ortogonallashtirish protsessi…………………………………………………………………..
22§. Chiziqli operatorlar. Chiziqli operatorning berilgan bazisdagi matritsasi. Chiziqli operatorning rangi, determinanti, izi, yadrosi, qiymatlar sohasi. Chiziqli operatorning invariant qism fazolari, xarakteristik ko'phadi, minimal ko'phadi, xos qiymatlari va xos vektorlari. Teskari operator. Chiziqli operatorlarni qo'shish, ayirish, songa ko'paytirish, o'zaro ko'paytirish. ……………………………………………………………..

23§. Kvadratik formalar va ularni kanonik ko'rinishga keltirish(Lagranj usuli)……………………………………………………………………..
24§.Matritsali ko'phadlar. Kaninik λ-matritsalar. Ekvivalent λ-matritsalar. Unimodulyar λ-matritsalar.
25§. Test savollari…………………………………………………………….
26§. Oraliq va yakuniy nazorat uchun savollar……………………………….
Adabiyot…………………………………………………………………

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