

Xranda and Tree

IEEE Xtreme 13

Xranda and Tree

Time limit: 2500 ms

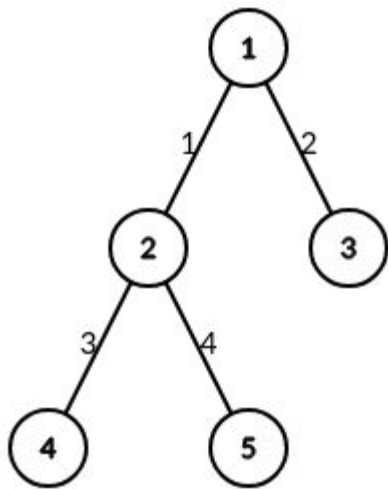
Memory limit: 256 MB

Xranda and her boyfriend, a man who does not chew are living in an **tree** with N nodes, with numerical labels on the edges. Their passion for treeology determined them to answer some seemingly impossible problems like "how many labeled trees lie in the isomorphism class of a given tree?" or "what is the generalised rotation distance from their tree and some arbitrary other one?". Exhausted by these tasks, they ask you to solve an easier one.

We define the distance $d(u, v)$ between two nodes u and v as the largest label of an edge which belongs to the unique path between them. Compute the sum of the distances across each possible pair of nodes, that is:

$$\sum_{a=1}^n \sum_{b=1}^{a-1} d(a, b)$$

Given that this number can be quite large, we are only interested of its remainder when divided by $10^9 + 7$.



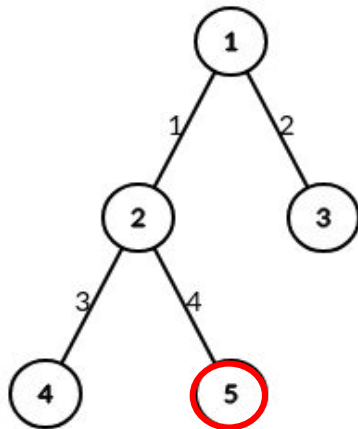
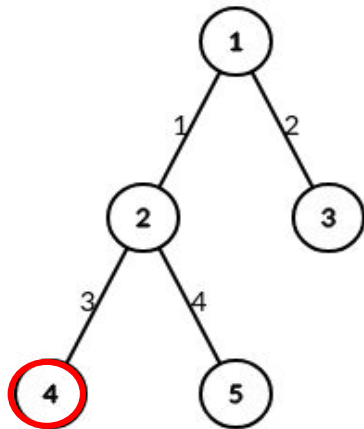
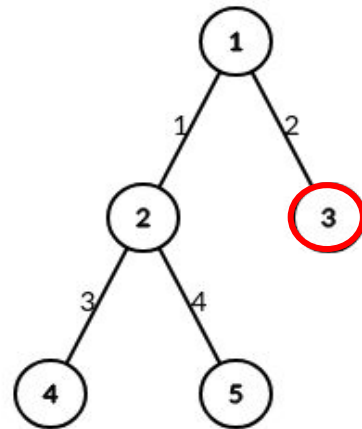
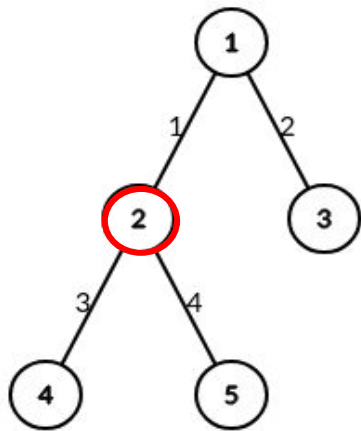
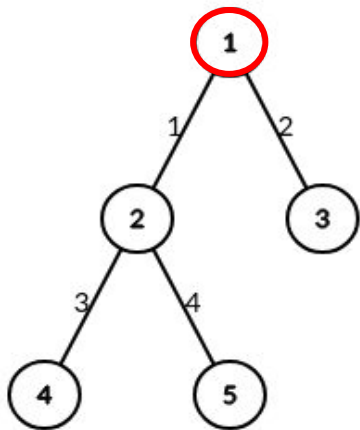
$$\sum_{a=1}^n \sum_{b=1}^{a-1} d(a, b)$$

We have the following distances:

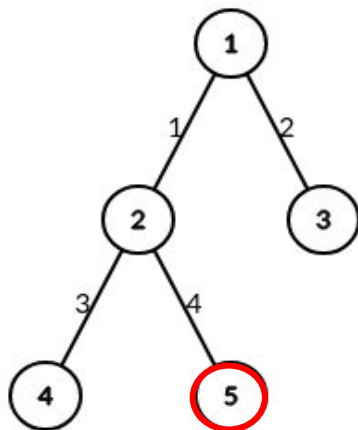
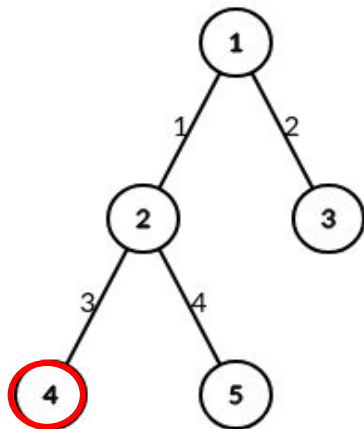
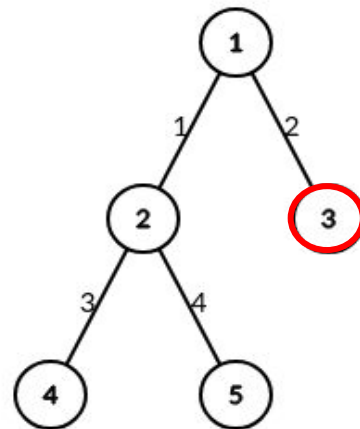
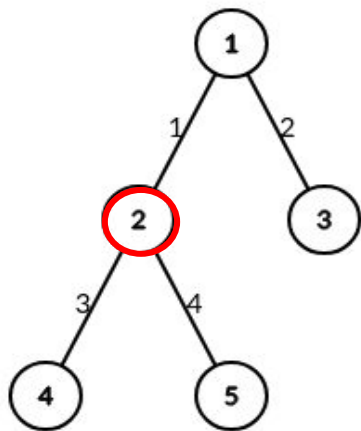
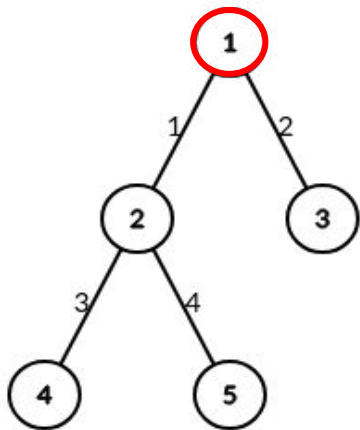
$$\begin{aligned} d(1, 2) &= 1; d(1, 3) = 2; d(1, 4) = 3; \\ d(1, 5) &= 4; d(2, 3) = 2; d(2, 4) = 3; \\ d(2, 5) &= 4; d(3, 4) = 3; d(3, 5) = 4; \\ d(4, 5) &= 4 \end{aligned}$$

The total sum is 30.

$d(a, b)$: Max edge value from a to b



n DFS



n DFS
 $O(n^2)$
(bad)

Score: 25/100 (2564 ms - 41.3 MB)

⬆	Test Number ⬆	CPU Usage ⬆	Memory Usage ⬆	Result ⬆
	20	2560 ms	41.3 MB	Time limit exceeded
	19	2559 ms	41.3 MB	Time limit exceeded
	18	2558 ms	41.2 MB	Time limit exceeded
	17	2558 ms	41.3 MB	Time limit exceeded
	16	2557 ms	41.2 MB	Time limit exceeded
	15	2560 ms	41.2 MB	Time limit exceeded
	14	2563 ms	41.3 MB	Time limit exceeded
	13	2564 ms	41.2 MB	Time limit exceeded
	12	2559 ms	41.2 MB	Time limit exceeded
	11	2558 ms	41.3 MB	Time limit exceeded
	10	2564 ms	41.3 MB	Time limit exceeded
	9	2558 ms	41.2 MB	Time limit exceeded
	8	2559 ms	41.3 MB	Time limit exceeded
	7	2558 ms	41.3 MB	Time limit exceeded
	6	2557 ms	41.3 MB	Time limit exceeded
	5	337 ms	4392 KB	OK
	4	387 ms	4424 KB	OK
	3	346 ms	4400 KB	OK
	2	340 ms	4384 KB	OK
	1	352 ms	4392 KB	OK
➤	0	21 ms	3968 KB	OK

Load

~~Nodes~~



Edges

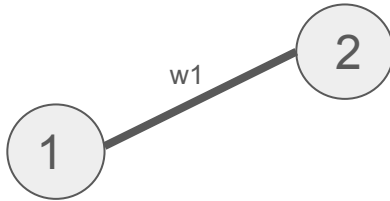
Πόσες φορές
πρέπει να μετρηθεί

~~Nodes~~

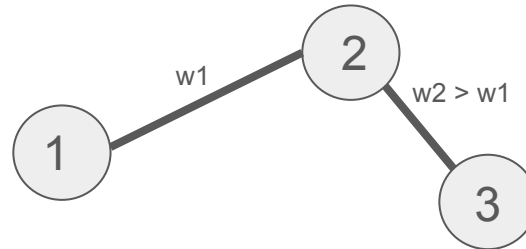


Edges

Πόσες φορές
πρέπει να μετρηθεί

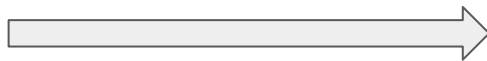


1 φορά
 $\text{total} += w1$



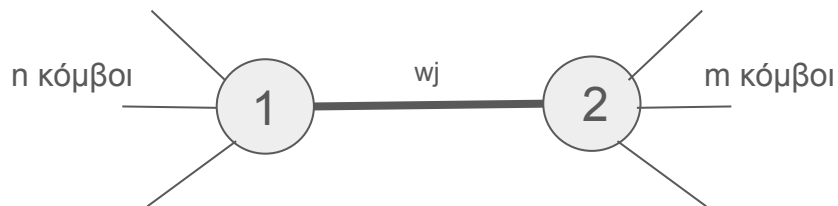
2 φορές
 $\text{total} += 2 * w2$

~~Nodes~~



Edges

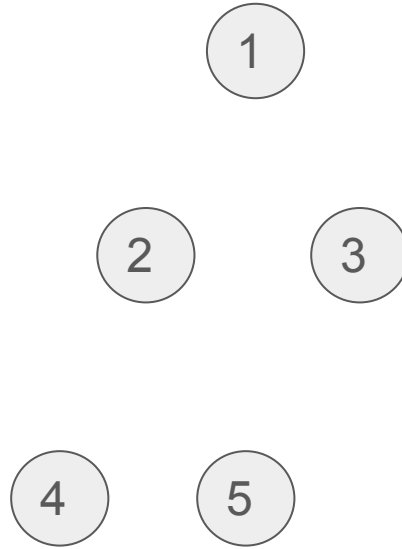
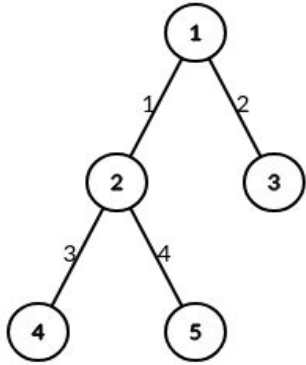
Πόσες φορές
πρέπει να μετρηθεί



$w_j >$ όλες τις άλλες ακμές

Συνολικά
μονοπάτια που
περνάνε από w_j :
 $n * m$

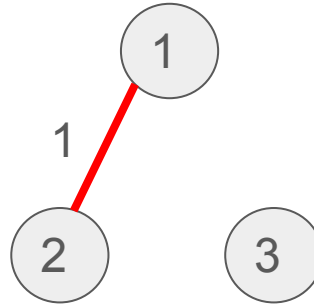
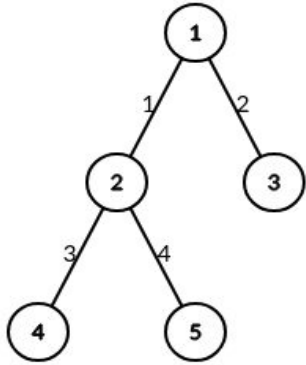
$$\text{total} += w_j * n * m$$



total = 0

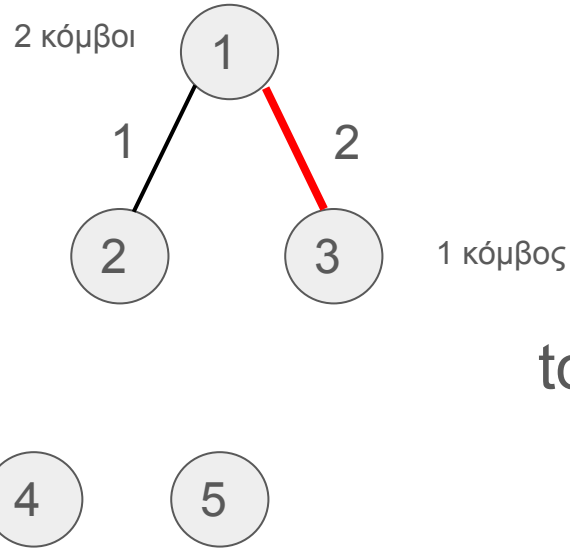
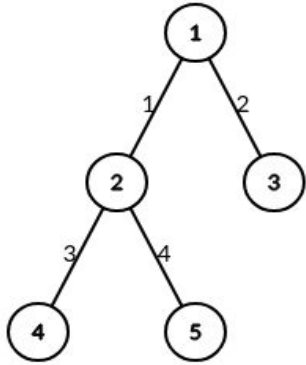
Sorted

Edges = [(1, 2, w=1), (1, 3, w=2), (2, 4, w=3), (2, 5, w=4)]



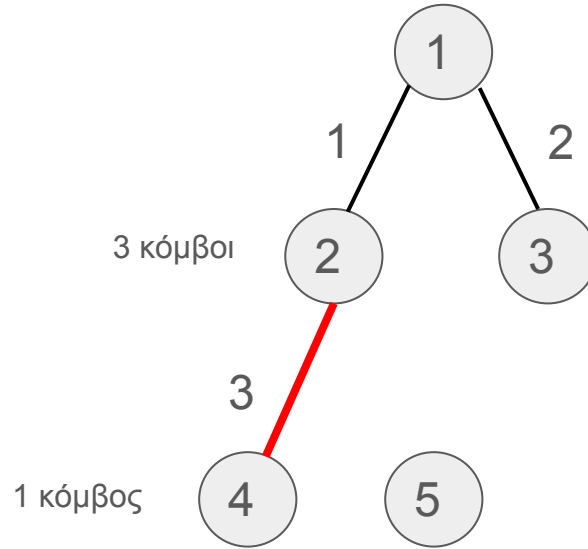
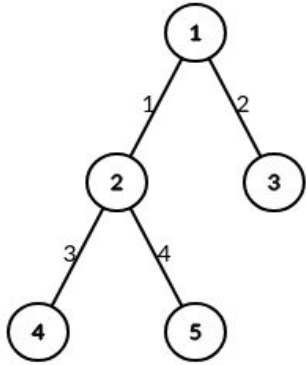
total = 1

Edges = [(1, 3, w=2), (2, 4, w=3), (2, 5, w=4)]



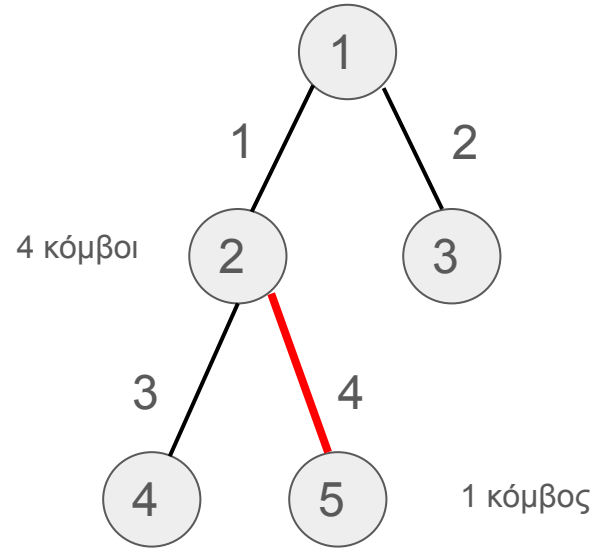
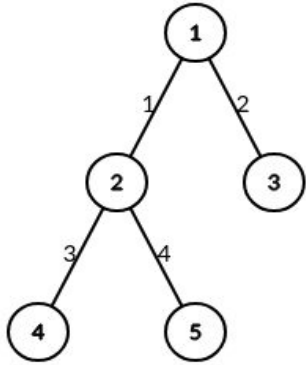
$$\text{total} = 1 + 2 * 2 * 1 = 5$$

Edges = [(2, 4, w=3), (2, 5, w=4)]



$$\text{total} = 5 + 3 * 3 * 1 = 14$$

Edges = [(2, 5, w=4)]



$$\text{total} = 14 + 4 * 4 * 1 = 30$$

Edges = []

Sort: $n \log n$

Disjoint Set

Find: $\log n$ (find which set a node belongs to)

Union: $\log n$ (merge 2 sets)

Total Complexity: $O(n \log n) + n * O(\log n) = O(n \log n)$