MELT SPINNING OF POLYMER FIBER PROCESS

Project Seminar

Guided By:
Prof. Dr. Thomas Götz

Mathematical Modeling, Simulation, and Optimization

Akshaybhai Shiyani (222100494)
Anu Rani Thomas(221100607)
Utsav Vaghani(222100493)

August 31, 2023

0.1 Declaration

We declare that the project seminar report titled "Melt Spinning of Polymer Fiber Process" is an authentic work completed by us, with the guidance of Prof. Dr. Thomas Götz. The content of this report reflects our ideas expressed in our own words. Any external ideas or phrases included have been appropriately cited and referenced. This report has been submitted to meet academic requirements.

Date: 31 August 2023

0.2 Acknowledgment

The success and completion of this project seminar required a lot of direction and assistance from various people. We are very lucky to have received this throughout the course of our project seminar. All that we have accomplished is owed only to their supervision and help, for which we are grateful.

First and foremost, we appreciate God Almighty for his blessings throughout our Seminar project effort. We are extremely grateful to our project guide Prof.Dr. Thomas Götz, Professor, Department of Mathematical Modelling, Simulation, and Optimization, took an active interest in our project seminar and guided us all the way through till we completed our project work by providing all of the required information for constructing a good system.

0.3 Abstract

Melt spinning is a widely used process in the production of polymer Fibers. This process involves different parameters. Our study model involved a numerical analysis of the mathematical model governing the melt-spinning process of Fibers, based on averaging the mass and momentum equation. The primary objective is to explore the impact of gravity on the system. Finally, we compare the numerical simulation of with and without gravity equations by the variation of different variables. These findings, highlight the behavior of our Fiber model to change in both spin line length and velocity, providing valuable insights into systems behavior.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Nowadays, we use polymer Fibers in different ways and in different manufacturing industries. There are so many procedures to produce polymer Fibers. Here we used the melt-spinning process in the production of polymer Fibers. Melt spinning offers several advantages such as high production rates, ease of processing, and the ability to produce a wide range of Fiber types. It will depend upon various factors like density, viscosity, diameter of the tube, velocity, and length[1].
In our Fiber spinning model, we used two major equations. The conservation of mass equation and conservation of momentum equation and we changed the major equations to our steady state model. By applying the boundary conditions, we obtained our steady-state model. Then our study continues with our obtained steady-state model. Our study model involves a numerical analysis of the mathematical model governing the melt-spinning process of Fibers, based on the averaging of the mass and momentum equation. The primary objective is to explore the impact of gravity on the system. For this, we used the numerical approximation method and finally, we compared the numerical simulations of with and without gravity equations by the variation of different variables.

Chapter 2

Fiber spinning process

2.1 Model Description

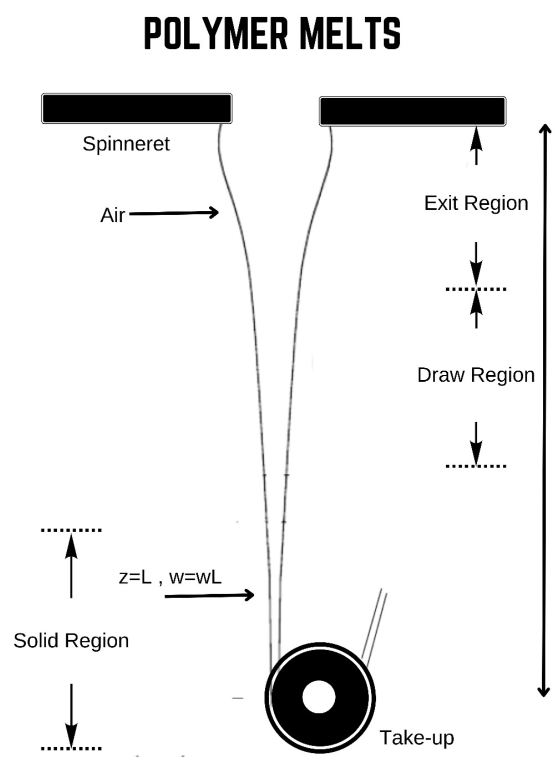


Figure 2.1: Melt Spinning Process [2]

The figure shows a sketch of the Fiber-spinning process[2]. The spinning is in the downward direction. Here, we have one hole with a radius R, and

basically, we put the molten polymer to it, and it automatically goes in the downward direction, this represents the parameter z and will get a solid output. This output is collected through a drum which is rotated at a certain take-up velocity, which represents w_L. Here T represents the Temperature and w represents the velocity of the fluid.
When the fluid goes down, it will get solidified because of the low temperature, and the winding at constant speed to build thin strips. Also, the tube is getting thinner to it comes in down. Throughout the process, we will fix the feeding speed(w_0) and take up speed(w_L) of the fluid. This parameter plays an important role in identifying the property of the fluid and the quality measure and the efficiency check[2]. The take-up velocity is considerably higher than the feeding speed and because of this the Fiber becomes stretched in length and reduces its diameter. The primary objective is to explore the impact of gravity on the system. This process depends upon the various parameters and the boundary conditions. By this, we must analyze the relation between the parameters like velocity, length, viscosity, and density.

2.2 Basic Equations

First, we are taking the equations, parameters, and boundary conditions. There are two different main equations used to build a model. We used the basic balance laws for mass and momentum[1].

Conservation of mass

From this basic law, we will get the behavior of fluid and it is important for understanding and analyzing the flow of fluid in various contexts. The continuity equation is given by[1]:

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial z}(Aw) = 0 \tag{2.1}$$

Here, w represents the velocity, A is the cross-sectional area of the Fiber, and z denotes the coordinates along the spin line with respect to time t.

Conservation of momentum

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} - \frac{1}{A\rho}\frac{\partial}{\partial z}(A\sigma) - g = 0 \tag{2.2}$$

Here, g is the gravity, ρ represents the density of the fluid and σ is the axial stress. Axial stress acts along with the longitudinal direction and helps to find the fluid viscosity effect and pressure variations in the axial directions. It will help to find flow behavior in the system. The below equation denotes the axial stress η [1]:

$$\sigma = 3\eta * \frac{\partial w}{\partial z} \tag{2.3}$$

where η is the viscosity of the polymer.
Both the equations are along with the boundary conditions[4]:

A(z=0) $=A_0$, w(z=0) $=w_0$, w(z=L)$=w_L$

Chapter 3

Dimensionless Equations

Now we convert our equations to dimensionless form for the easiness of calculation and simplification of our equation, we used a dimensionless equation by eliminating the measurement units. First, we have to identify the physical parameters present in our equation assign the dimensions of different units, and construct the dimensionless variable. By using the following dimensionless scale, we created the dimensionless form of the equations[3]:

$$w^* = \frac{w}{w_0}\ , \quad z^* = \frac{z}{L}\ , \quad A^* = \frac{A}{A_0}\ , \quad \sigma^* = \frac{\sigma L}{\eta w_0}\ , \quad t^* = \frac{t w_0}{L}$$

While we replace and simplify the equations we will get the dimensionless equations like:

$$\frac{\partial A^*}{\partial t^*} + \frac{\partial}{\partial z}(A^* w^*) = 0 \tag{3.1}$$

$$\frac{\partial w^*}{\partial t^*} + w^* \frac{\partial w^*}{\partial z^*} - \frac{3\eta}{A^* \rho L w_0} \frac{\partial}{\partial z^*}(A^* \frac{\partial w^*}{\partial z^*}) - \frac{gL}{w_0^2} = 0 \tag{3.2}$$

along with the boundary values

A(z=0)=1, w(z=0)=1, w(z=L)=w_L

Where g is the gravity, L is the length, w is the velocity along with z direction. For a better understanding, we consider two different dimensionless parameters Reynolds number(R_e) and Fraud Number(F_r)

Reynolds number and Fraud Number

The parameter Re represents the Reynolds number and it is used to predict the flow of a liquid over the surface[3].

$$R_e = \frac{\rho L w_0}{\eta}$$

Where ρ is the fluid density, w_0 is the initial velocity, L is the characteristics length, η is the viscosity. Also, d $= \frac{wl}{w0} > 1$. It denotes the draw ratio, which means the ratio between take-up speed and feeding speed.

The parameter Fr represents the dimensionless quantity called Fraud Number.The equation is given by[3]:

$$F_r = \frac{w_0^2}{gL}$$

Where, w_0 is the initial velocity

Chapter 4

Steady State Model And Analytical Calculations

4.1 Our Steady State Model

We consider our model to be a steady state and Isothermal and it is a combination of a couple of non-linear first and second-order ordinary differential equations with the help of a numerical approach[2]. Under our conditions and requirements, we made our steady-state model to analyze the behavior of the system. For the steady-state model, we must eliminate the time derivatives. So in our steady-state model, our model looks like this:

$$\frac{\partial}{\partial z}(Aw) = 0 \tag{4.1}$$

$$w\frac{\partial w}{\partial z} - \frac{3}{R_e A}\frac{\partial}{\partial z}(A\frac{\partial w}{\partial z}) - \frac{1}{F_r} = 0 \tag{4.2}$$

4.2 Analytical Calculation

By Solving equation (4.1)

$$A = \frac{1}{w} \tag{4.3}$$

Use this relation in equation (4.2)

$$w\frac{\partial w}{\partial z} - \frac{3w}{R_e}\frac{\partial}{\partial z}(\frac{1}{w}\frac{\partial w}{\partial z}) - \frac{1}{F_r} = 0 \tag{4.4}$$

$$\frac{\partial^2 w}{\partial z^2} = w\frac{R_e}{3}\frac{\partial w}{\partial z} + \frac{1}{w}(\frac{\partial w}{\partial z})^2 + \frac{R_e}{3F_r} \tag{4.5}$$

This equation is a second-order ODE, which means we need to convert it into a first-order ODE to obtain a numerical solution

4.3 First Order ODE for Numerical Approach

Now, we have our equation in the form of a secondary ordinary differential equation. By taking

$$\frac{\partial w}{\partial z} = y \tag{4.6}$$

along with the boundary conditions like:

A(z=0)=1 ,w(z=0)=1 , w(z=1)=w_d

By this we obtained our first order Ordinary differential equation. Using this equation we will get the numerical solution.

$$\frac{\partial y}{\partial z} = \frac{1}{w}(y)^2 + \frac{wR_e}{3}(y) - \frac{R_e}{3F_r} \tag{4.7}$$

4.4 Equations for Numerical method

Our main aim is to identify the impacts of gravity in our model. For that, we write the with-gravity and without-gravity equations according to our model to do the numerical simulations.

With-gravity Equation

$$\frac{\partial y}{\partial z} = \frac{1}{w}(y)^2 + \frac{wR_e}{3}(y) - \frac{R_e}{3F_r} \tag{4.8}$$

Without-gravity Equation

$$\frac{\partial y}{\partial z} = \frac{1}{w}(y)^2 + \frac{wR_e}{3}(y) \tag{4.9}$$

Chapter 5

Numerical Approximation Method

5.1 Shooting method

For the boundary value problems, there are several analytical methods available which we may use to solve boundary value issues. These approaches result in a solution with integration constants that are easily calculated by boundary conditions.
However, sometimes we are unable to identify integration constants under the given boundary conditions. In this situation, we may use a numerical technique that turns the boundary value problem into an initial value problem.[4] Such a case occurs in our model so we use the shooting method approach to simulate our Fiber model to find out the relationship between the z coordinate and the velocity w [2]. The shooting method is used to convert our boundary value problem to the initial value problem. The boundary value problem is given by:

$$\frac{\partial^2 w}{\partial z^2} = f(z, w, \frac{\partial w}{\partial z}), w(0) = 1, w(1) = w_L \tag{5.1}$$

Into an initial value problem

$$\frac{\partial^2 w}{\partial z^2} = f(z, w, \frac{\partial w}{\partial z}), w(0) = 1, \frac{\partial w}{\partial z} = \alpha \tag{5.2}$$

Where the number α is simply a guess

After that we applied numerical techniques like Runge Kutta, Euler's method, or PYTHON ODE solver to find out approximation α until it satisfies conditions α - $w_L = 0$.

Once we can determine the best prediction for α, we can solve the initial value problem.

Parameters Used

The several parameters and values depend upon the results. It will show the below table. We consider honey as the fluid. we take the density and viscosity of the honey to apply in the model[2].

Parameter	Values
Density of Honey	1420 kg/m^3
Viscosity of Honey	10 kg/ms
Feeding Speed	1 m/s
Take Up speed	5 m/s
Length	1 m

Table 5.1: Parameters used in the Model

Chapter 6

Numerical Approximation Results

6.1 Simulation Results

We obtained our steady-state model equations with different situations called with and without gravity. Now, we are going to present the outputs that we obtained in the numerical approximation. Which helps to identify and to explore the impact of gravity on our model.
First, we take the length as L=1m and velocity w=5 m/s,then we get this result. As illustrated in the graph below, there are no significant fluctuations observed between the curve with the influence of gravity and the curve without it.

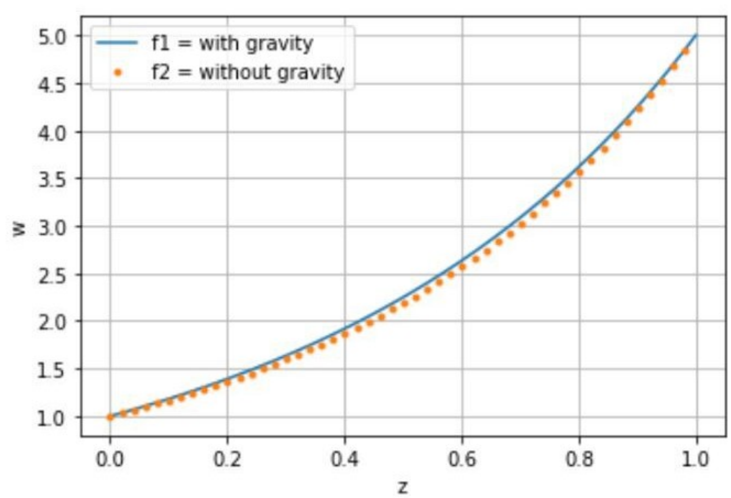


Figure 6.1: Numerical Approximation Result

Then we simulate our solution to keep the velocity w=5 m/s by varying the value of length 2 m. A moderate change is observable in both curves. According to the curve, it can be inferred that the velocity increases as the length increases in the presence of gravity.

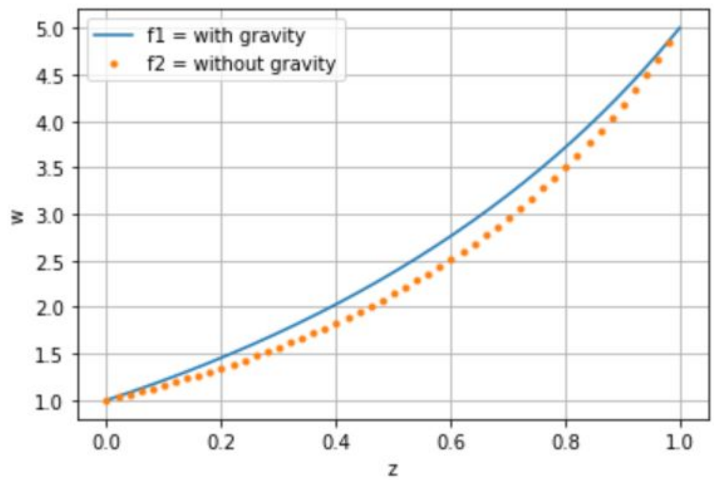


Figure 6.2: Simulation Results L=2 m, w=5 m/s

For the sake of enhanced comprehension, we will now modify the velocity to 10 m/s while maintaining the length at 1 m to identify the behavior of the curve, which shows the negligible difference between both curves same as 6.1

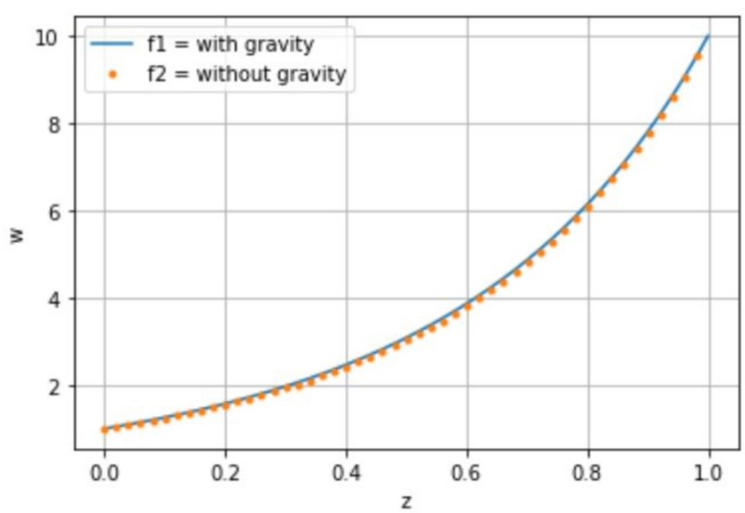
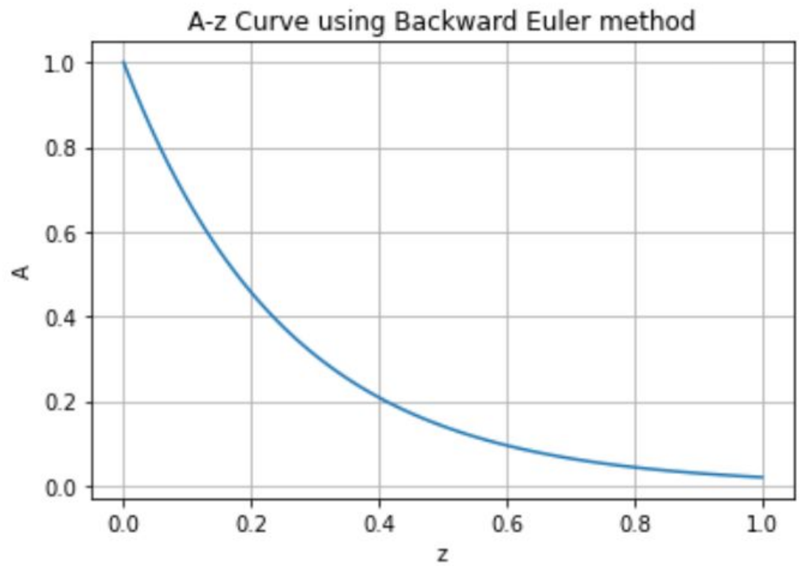


Figure 6.3: Simulation Results L=1m,w=10 m/s

Based on the preceding simulation, it is evident that the velocity of the fiber model is dependent upon varying lengths. It is apparent that an increase in the length of the spine line results in a faster achievable velocity in the presence of gravity. while in terms of without gravity, it is the same.

6.2 A-z Curve

This curve shows the relationship between area and length, from which we can determine the final area of the polymer filament.



The value of A at z = 1 is: 0.019800040113920236

Figure 6.4: A-z Curve

Chapter 7

Conclusion and future scope

Our study involved a numerical analysis of the mathematical model governing the melt-spinning process of Fibers, with a particular focus on mass and momentum equations. The primary objective was to explore the impact of gravity on the system. Based on the simulation results, we noticed distinct variations in the steady-state melt spinning Fiber model graph when manipulating the spin line length, while maintaining consistent velocity. Additionally, we observed further variations in the graph when adjusting the velocity while keeping the spin line length constant. These findings, highlight the behavior of our Fiber model to changes in both spin line length and velocity, providing valuable insights into the system's behavior.
The examination of mass and momentum equations will encompass the conservation of energy equation in order to assess the influence of temperature on the system. The incorporation of the Energy equation will enable the simulation of the correlation between temperature and length, thereby yielding a more comprehensive understanding of the system.

Chapter 8

References

.

1. *A Mathematical Model for the Melt Spinning of Polymer Fibers* (SINTEF and University of South-Eastern Norway PhD,January 1997)

2. *Numerical Based Approach to Develop Analytical Solution of a Steady-state Isothermal Melt Spinning Model*(Department of Mathematics and Statistics, King Faisal University, KSA,2016)

3. *H.P.: Derivation of a Mathematical Model for Fibre Spinning*(Department of Mathematics, Mechanics Division, University of Oslo, Oslo ,1997)

4. *Numerical simulation of a two-phase melt spinning model.* (Applied mathematical modelling, 35(6), 2959-2971.)