# Estudo Probabilidades

## Cap. I

(1) a) ~~$\Omega = \{1,2,3,4,5,6\}$~~

~~$\Omega = \{$ Cara~~

| | Cara 1 | Coroa 1 |
|---|---|---|
| Cara 2 | Cara 1 Cara 2 | Cara 2 Coroa 1 |
| Coroa 2 | Coroa, Cara | Coroa, Coroa |

b)

A → Sair Cara 1 Coroa 2
B → Sair Cara 1 Cara 2
C → Sair Cara 2 Coroa 1
D → Sair Cara 2 Cara 1

c) $P(\text{Cara 1 Cara 2} \cup \text{Coroa 1 Coroa 2})$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

(2) Turma
- 16 rapazes — 8 cp
- 24 raparigas — 12 cp

$P(\text{Rapaz}) = \frac{16}{40}$ $\quad P(cp) = \frac{20}{40}$ $\quad P(R \cup cp) = P(R) + P(cp) - P(R \cap cp)$

$P(\text{rapaz} \cup cp) = \frac{16}{40} + \frac{20}{40} - \frac{8}{40} = \frac{28}{40} = 0.7$

(3) 180 professores
- 135 doutorados → $\bar{D} = 45$
- 146 investigação → $\bar{I} =$
- 114 doutorados e investigação

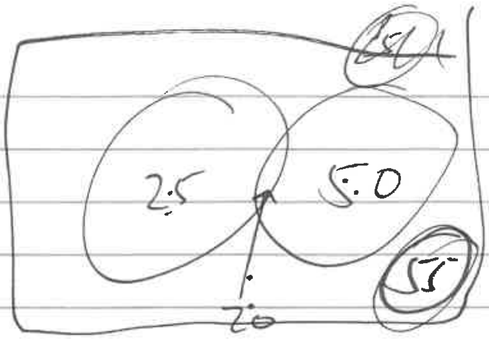
a) $P(D \cup I) = P(D) + P(I) - P(D \cap I) = \frac{135}{180} + \frac{146}{180} - \frac{114}{180}$

$= 0.92777$

b) $1 - P(D \cap I) = 1 - \frac{114}{180} = \frac{66}{180} = 0.366$ $\quad 1 - 0.927$

$= 0.073$

(4) 25% dep. investigação
50% lucros
10% dep. inv e lucros




a) 0.55

$P(D \cup L) = 0.25 + 0.5 - 0.20$
$= 0.55$

b) 0.75

c) $1 - P(D \cup L) = 0.45$

d) 0.58

e) $P(D \cap \bar{L}) =$

$0.25 + 0.55 = 0.8$

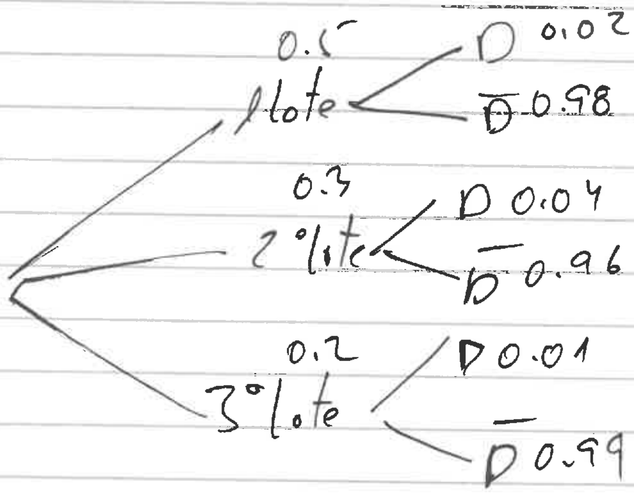
(5) 50% ar condicionado
49% direcção assistida
25% duas coisas

a) $0.5 - 0.25 = 0.25$

b) $1 - P(A \cup D) = 1 - (P(A) + P(D) - P(A \cap D))$
$= 1 - (0.5 + 0.49 - 0.25)$
$= 1 - 0.74$
$= 0.26$

e) 0.74

⑥




a) $P(D) = 0.02 \cdot 0.5 + 0.04 \cdot 0.3 + 0.01 \cdot 0.2$
$= 0.01 + 0.012 + 0.002 = 0.024$

b) $P(1 \cap 2 / D) = \dfrac{P((1 \cap 2) \cap D)}{P(D)} = \dfrac{0.022}{0.024} = 0.9166(6)$

$P(1 \cup 2) = 0.5 + 0.3 = 0.8$

$0.5 \cdot 0.02 + 0.3 \cdot 0.04 = 0.01 + 0.012 = 0.022$

⑦

| Região | kg | Qualidade A | B |
|---|---|---|---|
| I | 10000 | 2000 | 8000 |
| II | 20000 | 14000 | 6000 |
| III | 20000 | 10000 | 10000 |

A região mais favorável sabendo que é em A é a Região 2.

(8)

$C1$ — 8 brancas, 12 castanhas, 6 vermelhas

$C2$ — 4 brancas, 16 castanhas, 20 vermelhas

~~$\Omega = \{$branca, castanha, vermelha$\}$~~ 60 a)

bolas totais = 50

b) ~~$P(C1) = 0.5$~~

$P(C1) = 20/60$

c) $P(B) = 12/60$

d) $P(B \cap C1) = 8/60$

e) $P(B \cup C1) = P(B) + P(C1) - P(B \cap C1)$

$= 12/60 + 20/60 - 8/60$

$= 24/60$

f) $P(B) \cup P(C1) -$

$P(B \cup C1) - P(B \cap C1) = 24/60 - 8/60 = 0.266(6)$

g) $P(B|C1) = \frac{P(B \cap C1)}{P(C1)} = \frac{8/60}{20/60} = 0.4$

9) Caixa 1 = $3/5$ Caixa 2 = $2/6$ Caixa 3 = $1/7$

Caixa 1 tem maior probabilidade de sair branca

(10) H estar vivo daqui 25 anos é $3/5$

M " " " " " $2/3$

a) $P(\text{Ambos vivos}) = 2/3 \cdot 3/5 = 0.4 = 2/5$

b) $P(HV \cap \overline{MV}) = 3/5 \cdot 1/3 = 0.2 = 1/5$

c) $P(M) = 2/5 = \frac{4}{10}$

$P(HV \cap MV) = 2/3$

$2/5 \cdot 2/3 = 4/15$

$P(H \cap M) = 2/3$

d) $P(H \cup M) = 3/5 + \cancel{2/3} =$

$P(M) = ?$

$P(M) = 2/3$

$= 3/5 + 2/3 - 2/5 = \underset{(3)}{1/5} + \underset{(5)}{2/3} = 3/15 + 10/15$

$= 13/15$

(11) $P(E|NR) = \frac{P(E \cap NR)}{P(NR)} =$ ______

$P(E/NR) = \frac{P(E \cap NR)}{P(NR)}$ $\quad \frac{0.11 \times 0.91 \times 0.11}{0.11} = 0.091$

$\frac{0.1 \times 0.91 \times 0.11}{0.11} =$

12) Acreditar hoje = 0.25

$0.25^6 \cdot 0.65 = 0.0001586$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1ª | 2ª | 3ª | 4ª | 5ª | 6ª | 7ª | $= 0.04449$ |
| 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.25 | |

$0.75^6 \times 0.25$

13)

a) $P(A) = 0.5$

$P(A \cup B) = 0.5$

~~P(A∪~~ A e B independentes $A \cap B = 0$

$P(A \cup B) = P(A) + P(B) \Leftrightarrow 0.5 = 0.5 + P(B)$

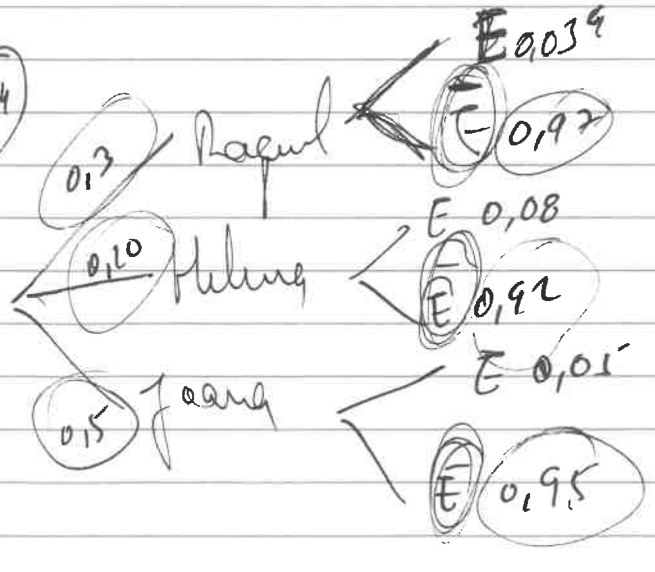
$\Leftrightarrow P(B) = 0$

b) $P(A \cap B) = 0$

$P(B) = 0$

$P(\bar{E})$

14)




$P(E) = 0{,}97 \cdot 0{,}3 + 0.92 \cdot 0{,}2 + 0.95 \cdot 0{,}5$

$= 0.291 + 0.184 + 0.475$

$= 0.945$

$$P(J \mid \bar{E}) = \frac{P(J \cap \bar{E})}{P(\bar{E})} = \frac{0.475}{0.945} = 0.5$$

(15) 30% recebem bolsa — L 0.85

70% não recebem — licenciatura 0,45

$P(L)\ 0.85 \times 0.30 + 0.45 \times 0.70 = 0.57$

$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{0.85 \times 0.3}{0.57} = 0.44737$$

(16)

0,4 1º ponto — MC 0,08 / BC 0,92

0,6 2º ponto — MC 0,02 / BC 0,98

$$P(1º|BC) = \frac{P(1º \cap BC)}{P(BC)} = \frac{0.92 \times 0.4}{0.98 \times 0.6 + 0.92 \times 0.4} =$$

2º lº [?] a fazer

$$\frac{0.368}{0.588 + 0.368} = \frac{0.368}{0.956} = 0.3535$$

$8/20 \quad 12/20$

(18) $\frac{8}{20} \cdot \frac{7}{19} = 0.4 \times = 0.14736$ $= 0.0000346$ ERRO

b) $\left(\frac{8}{20} \cdot \frac{7}{19} \cdot \frac{12}{18}\right)^3 = 0.09824^3$

1 2 3

E E E

E E E

E E E

E E E

$0.00094 \cdot \frac{8}{20} \cdot \frac{7}{19} \cdot \frac{6}{18}$

$= 0.04912$

19

a) T2 - 160

T3 - 160

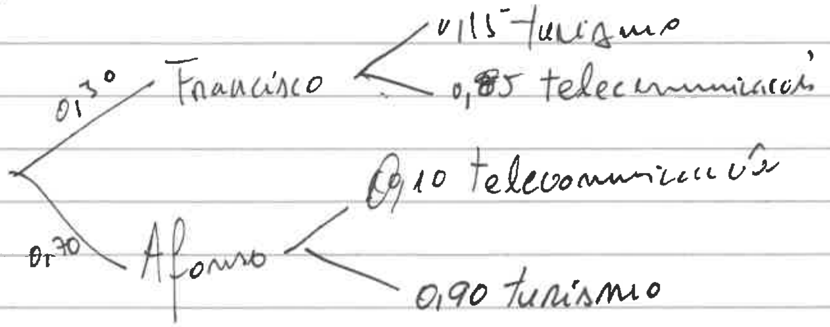
Tu = 80

$$\frac{0.4 \cdot 160 +}{0.4 \cdot 160 + 0.2 \times 160 + 0.5 \times 80} = \frac{64}{1344} =$$

$$\frac{0,4}{0,4+0,2+0.5} = 0.363636$$

b) P(6 meses 1ano

20

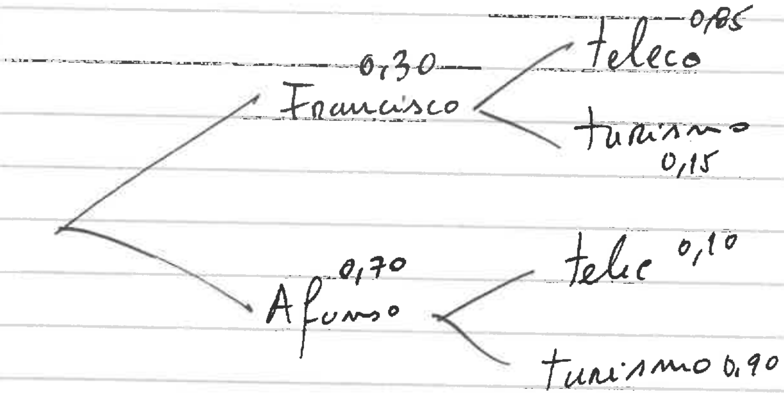



a) $P(F|T) = \frac{P(F \cap T)}{P(T)} = \frac{0.3 \times 0.85}{0.3 \times 0.85 + 0.7 \times 0.10} =$

0.784

b) $0.15 \times 0.3 + 0.9 \times 0.7 = 0.675$

20




a) $\dfrac{0.85 \times 0.30}{0.85 \times 0.3 + 0.10 \times 0.70} = \dfrac{0.255}{0.325} = 0.784$

$$P\left(F \middle| TE\right) = \frac{P(F \cap TE)}{P(TE)}$$

b) $0.15 \times 0.30 + 0.90 \times 0.70 = 0.675$

21

| | Nº empregados | Nº desempregados |
|---|---|---|
| Homens | 940 | 110 |
| Mulheres | 860 | 90 |

total = 2000

a) $\dfrac{950}{2000} = 0{,}475$

b) $P\left(H \middle| D\right) = \dfrac{P(H \cap D)}{P(D)} = \dfrac{110/2000}{200/2000} = \dfrac{0.055}{0.1} = 0.55$

22

a) 40% linux $\qquad\qquad$ $P(W) = 0.85$

30% linux e windows

95% linux ou windows

$P(L \cup W) = P(L) + P(W) - P(L \cap W)$

$0{,}95 = 0{,}4 + P(W) - 0{,}3$

$0.85 = P(W)$

$0{,}95 = 0.4 + 0.85 - 0.3$ ✓

b) $1 - P(L \cup W) =$

$P(\overline{L \cup W}) = 1 - P(L \cup W) = 0.05$

Cap II

1

$$f(x) \begin{cases} 0.2 & x \in \{2,4,6\} \\ a & x \in \{5,7\} \\ 0.3 & x = 8 \\ 0 & \text{o.v de } x \end{cases}$$

a) $\cancel{\sum x f(x) = 1}$

$\cancel{0.2 \times 2 + 0.2 \times 4 + 0.2 \times 6 + a \times 5 + a \times 7 + 0}$

$0.2 \times 3 + 2a + 0.3 = 1$

$0.6 + 2a + 0.3 = 1$

$2a = 0.1$

$a = 0.05$

b) função distribuição

$$F(x) = \begin{cases} 0 & \text{se } x < 2 \\ f(2) = 0.2 & \text{se } 2 \leq x < 4 \\ f(2)+f(4) = 0.4 & \text{se } 4 \leq x < 5 \\ f(2)+f(4)+f(5) = 0.45 & \text{se } 5 \leq x < 6 \\ f(2)+f(4)+f(5)+f(6) = 0.65 & 6 \leq x < 7 \\ f(2)+f(4)+f(5)+f(6)+f(7) = 0.70 & 7 \leq x < 8 \\ 1 & x \geq 8 \end{cases}$$

$P(X<5) = 0.4$ $\quad P(2<X<6) =$ 0.25

0.4 . 0.25

c) $P(X<5 | 2<X<6) = \dfrac{P(X<5 \cap 2<X<6)}{P(2<X<6)} = \dfrac{0.4}{0.45}$

0.45

$= 0.8$

$P(X<5 \cap 2<X\leq 6)$

$P(X<4.5) \Rightarrow F(4) = 0.4$

$P(X \geq 4) = \cancel{0.7 + 0.05 + 0.} \quad 1 - P(X<4) = 1 - F(2) = 0.8$

| | 2 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| $x$ | 0.2 | 0.2 | 0.05 | 0.2 | 0.05 | 0.3 |

0.25

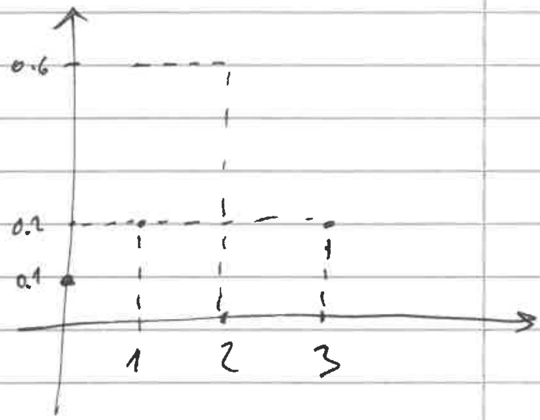
2

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | $1/10$ | $1/5$ | $K$ | $1/10$ |

a) $\frac{1}{10} + \underset{(2)}{\frac{1}{5}} + K + \frac{1}{10} = 1$

$\frac{1}{10} + \frac{2}{10} + K + \frac{1}{10} = 1$

$K = \frac{6}{10} = 0.6$




b)

$$F(x) = \begin{cases} 0 & \text{si } x < 0 \\ \frac{1}{10} & \text{si } 0 \leq x < 1 \\ \frac{3}{10} & \text{si } 1 \leq x < 2 \\ \frac{9}{10} & \text{si } 2 \leq x < 3 \\ 1 & \text{si } x \geq 3 \end{cases}$$

c) $P(X \leq a) \geq 0.5$

$a \in [2;3]$

$a = 2$

③

$$f(x) \begin{cases} \frac{x^2}{14} & \text{se } x \in \{1,2,3\} \\ 0 \text{ se } & x \in \mathbb{R} \setminus \{1,2,3\} \end{cases}$$

$$\int_{-\infty}^{+\infty} \frac{x^2}{14} = 1 \Leftrightarrow \frac{1}{14}\int_1^3 x^2 = 1 \Leftrightarrow \frac{1}{14}\left[\frac{x^3}{3}\right]_1^3 = 1$$

$$\frac{1}{14}\left[\frac{18}{3} - \frac{1}{3}\right] = \frac{1}{14}\left[\frac{17}{3}\right]$$

$x = 1 \Rightarrow f(x) = \frac{1}{14}$ $\qquad \frac{1}{14} + \frac{4}{14} + \frac{9}{14} = \frac{14}{14} = 1$ ✓

$x = 2 \Rightarrow f(x) = \frac{4}{14}$

$x = 3 \rightarrow f(x) = \frac{9}{14}$

c) $P(X=1 \mid X \leq 2)$

$$\frac{P(X=1 \cap X \leq 2)}{P(X \leq 2)} = \frac{\frac{1}{14}}{\frac{5}{14}} = \frac{14}{70} = 0{,}2$$

$$F(x) = \begin{cases} 0 & \text{se } x < 1 \\ \frac{1}{14} & \text{se } 1 \leq x < 2 \\ \frac{5}{14} & \text{se } 2 \leq x < 3 \\ 1 & \text{se } x \geq 3 \end{cases}$$

③ $F(x) =$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $F(x)$ | 0,1 | 0,4 | 0,9 | 1 |

$x = 1 \quad f(1) = 0{,}1$

$x = 2 \quad f(2) = 0{,}3$

$x = 3 \quad f(3) = 0{,}5$

$x = 4 \quad f(4) = 0{,}1$

b) $P(X \leq 2) = F(2) = 0.4$

$P(X > 1) = 1 - F(1) = 1 - 0.1 = 0.9$

(5) função densidade

$$f(x) = \begin{cases} ax & 0 \leq x < 1 \\ a & 1 \leq x < 2 \\ -ax + 3a & 2 \leq x < 3 \\ 0 & \text{o.v de } x \end{cases}$$

a)

$$\int_0^1 ax + \int_1^2 a + \int_2^3 -ax + 3a = 1$$

$$\cancel{ax = \frac{2x^2}{2}} \quad a\left[\frac{x^2}{2}\right]_0^1 + \int \left[ax\right]_1^2 + a\int_2^3 -x + 3$$

$$a\left[\frac{x^2}{2}\right]_0^1 + \left[ax\right]_1^2 + a\left[\frac{-x^2}{2} + 3x\right]_2^3 = 1$$

$$\Leftrightarrow a\left[\frac{1}{2} - \frac{0}{2}\right] + \left[2a - a\right] + a\left(\left(\frac{9}{2} + \frac{9}{(2)}\right) - \left(\frac{4}{2} + \frac{12}{(2)}\right)\right)$$

$\frac{9}{2} + \frac{18}{2}$ $\frac{27}{2} - \frac{28}{2} = -\frac{1}{2}$ $\frac{28}{2}$ $\frac{27}{2}$

$$\Leftrightarrow \cancel{\frac{a}{2}} + a \cancel{+} \cancel{\frac{a}{2}} = 1$$

$$\int_0^1 ax + \int_1^{1} a + \int_2^3 -ax+3a = 1$$

$$\Leftrightarrow a\int_0^1 x + \int_1^2 a + a\int_2^3 -x+3 = 1$$

$$\Leftrightarrow a\left[\frac{x^2}{2}\right]_0^1 + \left[ax\right]_1^2 + a\left[-\frac{x^2}{2}+3x\right]_2^3 = 1$$

$$\Leftrightarrow a\left[\tfrac{1}{2}-0\right] + \left[2a-a\right] + a\left[\left(-\tfrac{9}{2}+9\right)-\left(-\frac{2^2}{2}+3(2)\right)\right]$$

$$-4{,}5 - \quad 4$$

$$\Leftrightarrow \frac{a}{2} + a + \frac{a}{2} = 1$$

$$\Leftrightarrow 2a = 1 \Leftrightarrow a = \frac{1}{2}$$

b) Função distribuição

$$P(X \leq x) = \int_{-\infty}^{x} f(u)\,du$$

F(x)=

– se $0 \leq x < 1$

$$P(X \leq 0) = 0$$

$$P(X \leq x) = \int_0^1 \frac{1}{2}u = \frac{1}{2}\int_0^x u\,du = \frac{1}{2}\left[\frac{u^2}{2}\right]_0^x$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0$$

$$= \frac{x^2}{4}$$

si $x \leq 1 < 2$

$$\cancel{F(x) = \int_0^1 ax\,dx + \int_1^x a\,dx = a\left[x\right]_0^1 + \left[ax\right]_1^x}$$

$$\cancel{= a x}$$

$$F(x) = \int_0^1 \frac{1}{2} x\,dx + \int_1^x \frac{1}{2}\,du = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{1}{2}u\right]_0^x$$

$$= \frac{1}{4}x^2 + \frac{1}{2}x$$

si $2 \leq x < 3$

$$F(x) = \int_0^1 ax\,dx + \int_1^2 \frac{1}{2}\,dx + \int_2^x -\frac{1}{2}u + \frac{3}{2}\,du$$

$$F(x) = \frac{1}{4}x^2 + \frac{1}{2}x + \left[-\frac{u^2}{4} + \frac{3}{2}u\right]_2^x$$

si $x \geq 3$

$$F(x) = 1$$

(6)

$$f(x)=\begin{cases} 2x-1 & 1/2<x<k \\ 0 & \text{o.v. de } x \end{cases}$$

a) calcular $k$

$$\int_{1/2}^{k} 2x-1 = 1 \Leftrightarrow \left[\frac{\cancel{2}x^2}{\cancel{2}} - x\right]_{1/2}^{k} = 1$$

$$\left(k^2-k\right)-\left[\left(1/2\right)^2-1/2\right]=1$$

$$k^2-k-\left(\underset{(1)}{1/4}-\underset{(2)}{1/2}\right)=1$$

$$k^2-k \quad -1/4=1$$

$$k^2-k=1+1/4$$

$$k(k-1)=5/4$$

$$k=5/4 \lor k-1=5/4$$

$$k=5/4 \lor k=1/4$$

tem de

ser $k=5/4$

porque tem

de ser

superior a $1/2$

$$\int_{1/2}^{k} 2x-1=1$$

$$\Leftrightarrow \frac{\cancel{2}x^2}{\cancel{2}}-x$$

$$\left[x^2-x\right]_{1/2}^{k}$$

$$\left[\left(k^2-k\right)-\left(\left(1/2\right)^2-1/2\right)\right]$$

$$1/4-\underset{(2)}{1/2}$$

$$\left(\left(k^2-k\right)\right)-\left(1/4-2/4\right)$$

$$\left(k^2-k\right)-\left(-1/4\right)$$

$$k^2-k+1/4=\underset{(4)}{1}-1/4$$

$$k(k-1) \quad 3/4$$

$$k=3/4 \lor k-1=3/4+1$$

$$k=3/4 \lor \quad k=7/4$$

(8) $1-F(6) = 1-(0.05+0.06+0.06+0.08)$

$= 1-0.26 = 0.74$

(7) $f(x) = \begin{cases} \frac{1}{6}x+k & \text{se } 0 \le x < 3 \\ 0 & \text{o.v. de } x \end{cases}$

se $x<0$

a) $\int_0^3 \frac{1}{6}x+k = 1$

$F(x) = 0$

se $0 \le x < 3$

$\frac{1}{6}\left(\frac{x^2}{2}\right)+kx = 1$

$\left[\frac{x^2}{12}+kx\right]_0^3 = 1$

$F(x) = \int_0^x \frac{1}{6}u+\frac{1}{12}\,du$

$\frac{9}{12}+3k=1$

$= \left[\frac{1}{12}u^2+\frac{1}{12}u\right]_0^x$

$\frac{3}{4}+3k=1$

$= \frac{1}{12}x^2+\frac{1}{12}x$

$3k = 1-\frac{3}{4}$

$\frac{1}{12}(x^2+x)$

$\underset{(4)}{3k} = \frac{1}{4}$

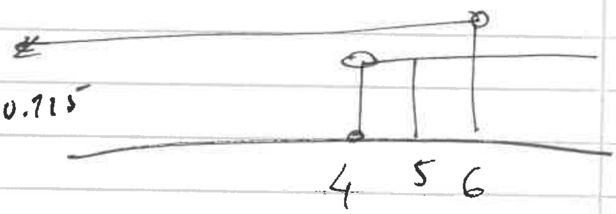
$12k=1$

$k=\frac{1}{12}$

⑧ 6 janeiros

$$P(X<6 \mid X>4) = \frac{P(X<6 \cap X>4)}{P(X>4)}$$

$$= \frac{f(5)}{f(4)} = \frac{0{,}25}{0.05+0.06+0.06+0.08+0.115}$$




$= 0.\cancel{5}$

⑨ 5 bolas amarelas, 2 bolas brancas, 7 bolas verdes } 14 bolas total

bola amarela ~~- um ponto~~
bola branca ~~dois pontos~~
bola verde ~~três pontos~~

X = soma dos produtos obtidos

$X = 2, 3, 4, 5, 6$

a) função de probabilidade X

$$\begin{cases} \frac{10}{91} & x=2 & \frac{5}{14}\cdot\frac{4}{13} = \\ \frac{5}{91}+\frac{5}{91}=\frac{10}{91} & x=3 & \frac{2}{14}\cdot\frac{5}{13}+\frac{5}{14}\cdot\frac{2}{13} \\ \left(\frac{2}{14}\cdot\frac{1}{13}\right)+\cancel{\left(\frac{2}{14}\cdot\frac{4}{13}\right)} + \frac{7}{14}\cdot\frac{5}{13}+\frac{5}{14}\cdot\frac{7}{13} & x=4 \\ \frac{7}{14}\cdot\frac{2}{13}+\frac{2}{14}\cdot\frac{7}{13} = & x=5 \\ \frac{7}{14}\cdot\frac{6}{13} = \end{cases}$$

10

$$f(x) = \begin{cases} (k-2)e^{-x} & \text{se } x \geq 0 \\ 0 & x < 0 \end{cases}$$

a) Calcular $k$

$$\int_0^{+\infty} (k-2)\cancel{e^{-x}}$$

$(k-2)e^{-x}$

$k=3$

$$\int_0^{+\infty} kx - 2x$$

$$\int_0^{+\infty} \frac{k-2}{e^x} \Rightarrow \left[\frac{kx-2x}{e^x}\right]_0^{+\infty} = 1$$

$e^{\infty}$

$$\left[(kx-2x) \cdot -\cancel{e^{-x}}\right]_0^{+\infty} = 1 \quad \lim_{x \to +\infty} \frac{kx-2x}{x} = 1$$

$kx - 2x = 1$

$x(k-2) = 1$

$x = 1 \vee k - 2 = 1$

$k = 3$

b)

$$P(X<3) = \int_0^3 (3-2)\, e^{-x} = \int_0^3 e^{-x} = \left[-e^{-x}\right]_0^3$$

$$= -e^{-3}$$

11

$$f(x) = \begin{cases} \frac{1}{10} & 0 \leq x < 5 \\ \frac{1}{25}(10-x) & 5 < x \leq 10 \\ 0 & \text{ou de } x \end{cases}$$

S = "Stock semanal da matéria prima A"

?

Resultado: $S \geq 8{,}42$

$f(x) \leq 0{,}05$

$\frac{1}{10} > 0{,}05$ !

$\frac{1}{25}(10-x) \leq 0{,}05$

$x \geq 8{,}75$

$\frac{1}{25}(10-x) \leq 0{,}05$

$\frac{10}{25} - \frac{x}{25} \leq 0{,}05$

$\cancel{\frac{x}{25}}\ 0{,}05 - \frac{10}{25}$

$-\frac{x}{25} \leq$

$x \geq 8{,}75$

12

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0,10 | 0,35 | 0,2 | 0,1 | 0,1 | 0,08 | 0,07 |

a) $E[X] = \sum x f(x) = 2 \times 0{,}10 + 3 \times 0{,}35 + 4 \times 0{,}2 + 5 \times 0{,}1 + 6 \times 0{,}1 + 7 \times 0{,}08 + 8 \times 0{,}07$

$= 4{,}27$

b) Variância

$4{,}25^2$

$E[X^2] - E[X]^2$

$2^2 \times 0.10 + 3^2 \times 0.35 + 4^2 \times 0.2 + 5^2 \times 0.1 + 6^2 \times 0.1 + 7^2 \times 0.08 + 8^2 \times 0.07$

$-4{,}27$

$= 17$

$=$

$(2-4{,}25)^2 \cdot 0.10 + (3-4{,}25)^2 \cdot 0.35 + (4-4{,}25)^2 \times 0.2$
$+ (5-4{,}25)^2 \times 0.10 + (6-4{,}25)^2 \times 0.1 + (7-4{,}25)^2 \times 0.08$
$+ (8-4{,}25)^2 \times 0.07 = 3.18$

13 ?

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | | | | | | |

14 $\int_0^2 x \cdot \frac{x}{2} = \left[\frac{x^2}{2} \cdot \frac{x^2}{4}\right]_0^2$ 1 2 2 3 6

$\int_0^2 x \cdot \frac{x}{2} = \int_0^2 \frac{x^2}{2} = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{1}{2}\left(\frac{2^3}{3} - 0\right) = \frac{1}{2} \cdot \frac{8}{3}$

$= \frac{8}{6} = \frac{4}{3}$

b) $\int_{-\infty}^{+\infty} \left(x - \frac{4}{3}\right)^2 \cdot \frac{x}{2}\, dx = \int \left(x - \frac{4}{3}\right)\left(x - \frac{4}{3}\right) \cdot \frac{x}{2}$

$= \int_0^2 \left(x^2 - \frac{4}{3}x - \frac{4}{3}x + \frac{16}{9}\right) \cdot \frac{x}{2} = \int_0^2 \left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) \cdot \frac{x}{2}$

$= \int_0^2 \left(\frac{x^3}{2} - \frac{8}{3}x + \frac{16}{9}\right) = \left[\frac{x^4}{4} - \frac{8}{3}x^2 + \frac{16}{9}x\right]_0^2$

13 66 55 44 33 22 11

$\left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6}\right)$

$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{6}$ $x = 0$

65 54 43 32 21 56 45 34 23 12

(15)

$E(X) = 6$

$E(X^2) = 62$

$Y = \frac{1}{2}X + 3$

a) $E(Y) =$

b) $Var(Y) =$

(16)

$$f(x) = \begin{cases} c(x^2+2x) & 0 < x < 1 \\ 0 & \text{ou de } x \end{cases}$$

a) $$\int_0^1 c(x^2+2x) = c\int_0^1 x^2+2x = 1 \Leftrightarrow c\int_0^1 \frac{x^3}{3} + x^2$$

$$\Leftrightarrow c\left[\frac{x^3}{3}+x^2\right]_0^1 = 1 \Leftrightarrow c\left[\frac{1}{3}+1\right] = 1$$

$$\Leftrightarrow \frac{c}{3} + \underset{(3)}{c} = \underset{(3)}{1} \Leftrightarrow c + 3c = 3 \Leftrightarrow 4c = 3 \Leftrightarrow c = \frac{3}{4}$$

b) $P(X > 1/3)$

$1 - P(X < 1/3)$

$= 1 - \int_0^{1/3} 3/4 (x^2 - x) = 1 - \left(3/4 \int_0^{1/3} x^2 - x\right)$

$= 1 - 3/4 \left[\frac{x^3}{3} - x^2\right]_0^{1/3} \Rightarrow 1 - \left(3/4 \left[\frac{(1/3)^3}{3} - (1/3)^2\right]\right)$

$= 1 - \left(3/4 \left(\frac{\cancel{1/9}}{\cancel{3}}\right) - 1/9\right)\Big] = 1 - 3/4 \left(1/27 - \underset{(3)}{1/9}\right)$

$= 1/1 - \left(3/4 \left(1/27 - 3/27\right)\right)$

$= 3/4 \left(-2/27\right)$

$1 - \left(\frac{-6}{108}\right)$

Função Distribuição

$\int_0^1 x \cdot 3/4 (x^2 + 2x)$

$= 3/4 \int_0^1 x^3 + 2x^2 \Leftrightarrow 3/4 \int_0^1$

$$P(X > 1/3) = \int_{1/3}^{1} 3/4 (x^2 + 2x) = 3/4 \int_{1/3}^{1} x^2 + 2x$$

$$= 3/4 \left[ \frac{x^3}{3} + x^2 \right]_{1/3}^{1} = \left[ \left( 1/3 + 1 \right) - \left( \frac{1/3}{3} + (1/3)^2 \right) \right]$$

$$= 3/4 \left[ 4/3 - \left( \frac{1/9}{3} + 1/9 \right) \right] = 3/4 \left[ 3/4 \right.$$

~~Fonç. distribuição~~

$P(X > 1/3)$

$$P(1/3 < X < 2/3) = \int_{1/3}^{2/3} 3/4 (x^2 + 2x) \quad 1/3 < X < 2/3$$

$$P(X > 1/3) \mid P(1/3 < X < 2/3) = \frac{P(X > 1/3 \cap 1/3 < X < 2/3)}{P(1/3 < X < 2/3)} = 1$$

e) $E[X] = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_{-\infty}^{+\infty} x \cdot 3/4 (x^2 + 2x)$

$$= \int_{0}^{1} x \cdot 3/4 (x^2 + 2x) = 3/4 \int_{0}^{1} x^3 + 2x^2 = 3/4 \left[ \frac{x^4}{4} + \frac{2x^3}{3} \right]_{0}^{1}$$

$$= 3/4 \left[ \frac{1^4}{4} + \frac{2(1)^3}{3} \right] = 3/4 \left[ 1/4 + \frac{2}{3} \right] = 3/4 \left[ 3/12 + 8/12 \right]$$

(3) (4)

$$= \frac{33}{48} = \frac{11}{16} \quad (\div 3)$$

$$= 11/12 \cdot 3/4$$

d) $Var(X) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx$

$= \int_0^1 x^2 f(x) - \mu^2 = \int_0^1 x^2 \cdot \frac{3}{4}(x^2+2x) - \mu^2$

$= \frac{3}{4}\int_0^1 x^4 + 2x^3 - \mu^2 = \frac{3}{4}\left[\frac{x^5}{5} + \frac{2x^4}{4}\right]_0^1$

$= \frac{3}{4}\left(\underset{(4)}{\frac{1}{5}} + \underset{(5)}{\frac{2}{4}}\right) - \mu^2 = \frac{3}{4}\left(\frac{4}{20} + \frac{10}{20}\right)$

$= \frac{14}{20} \cdot \frac{3}{4} = \frac{42}{80} - \left(\frac{11}{16}\right)^2$

$= 0.0523$

17 10000 billetes

18

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| f(x) | 0.1 | 0.15 | 0.2 | 0.2 | 0.2 | 0.1 | 0.025 | 0.025 |

19 a)

$(5x^4 - 4x^5)' = 20x^3 - 20x^4$

$= 20(x^3 - x^4)$ si $0<x<1$

b)

19

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 5x^4 - 4x^5 & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$

a) $P(X < 1/2) = F(1/2) = 5(1/2)^4 - 4(1/2)^5$

$P(X \geq 2/3) = 1 - F(2/3) = 1 - \left(5(2/3)^4 - 4(2/3)^5\right)$

$P(1/2 < X < 2/3) = F(2/3) - F(1/2) =$

$F(x) \rightleftarrows f(x)$

c) $E(X) = \int_{-\infty}^{+\infty} x f(x) = \int_{-\infty}^{+\infty} x \cdot 20(x^3 - x^4)\, dx$

$= 20 \int_0^1 x^4 - x^5 = 20 \left[ \frac{x^5}{5} - \frac{x^6}{6} \right]_0^1$

$= 20 \left( \frac{1}{5}_{(6)} - \frac{1}{6}_{(5)} \right) = 20 \left( \frac{6}{30} - \frac{5}{30} \right)$

$= 20 \left( \frac{1}{30} \right) = \frac{20}{30}$

$= \frac{2}{3}$

$Var(X) = \int_{-\infty}^{+\infty} (x - \mu)^2 f(x)\, dx$

ou

$\int_{-\infty}^{+\infty} x^2 f(x)\, dx - E[X]^2$

20

$$f(x) = \begin{cases} -0{,}04 + b & 10 \leq x \leq 15 \\ 0 & \text{o.v de } x \end{cases}$$

21 X = "nº da segunda bola extraída"

Y = ~~"nº da primeira bola extraída"~~

maior dos 2 números extraídos

a)

| X\|Y | 1 | 2 | |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |

1ª 2ª

1 2

1 1

(21) Extração sem reposição

X = "nº 1ª bola extraída"

Y = "menor nº das 2 bolas"

| 1ª | 2ª | X | Y | |
|---|---|---|---|---|
| 1 | 2 | 2 | 1 | $f_{XY}(2,1)$ |
| | 3 | 3 | 1 | $f_{XY}(3,1)$ |
| 2 | 1 | 1 | 1 | $f_{XY}(1,1)$ |
| | 3 | 3 | 2 | $f_{XY}(3,2)$ |
| 3 | ~~1~~ | 1 | 1 | $f_{XY}(1,1)$ |
| | 2 | 2 | 2 | $f_{XY}(2,2)$ |

| X \ Y | 1 | 2 |
|---|---|---|
| 1 | 2/6 | 0 |
| 2 | 1/6 | 1/6 |
| 3 | 1/6 | 1/6 |

$P(X=Y) = f(1,1) + f(2,2)$

$= 3/6 = \boxed{0,5}$

P(

(22)

| X\|Y | 1 | 2 | 3 | $f_X(x)$ |
|---|---|---|---|---|
| 1 | 1/12 | 3/12 | 1/12 | 1/3 |
| 2 | 2/12 | 1/4 | 1/12 | 6/12 |
| 3 | 1/12 | 1/12 | 0 | 1/6 |
| $f_Y(y)$ | 1/3 | 6/12 | 1/6 | 1 |

b) $E[Y] = 1 \cdot 1/3 + 2 \cdot 6/12 + 3 \cdot 1/6 = 1.83(3)$

$Var[Y] = 1^2 \cdot 1/3 + 2^2 \cdot 6/12 + 3^2 \cdot 1/6 - (1.83(3))^2$

c) $P(Y \text{ impar} \mid X \text{ impar}) = \dfrac{P(Y \text{ impar} \cap X \text{ impar})}{P(X \text{ impar})}$

$= \dfrac{3/12}{1/3 + 1/6} = \dfrac{3/12}{3/6}$

$= 0,5$

d) V.a independentes

$f_{XY}(x,y) = f_X(x) \cdot f_Y(y) \quad \forall (x,y) \in \mathbb{R}^2$

25

| X\|Y | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 2 | K | 0,24 | 0,18 | 0,06 | 0,6 |
| 4 | 0,08 | 0,16 | 0,12 | 0,04 | 0,4 |
| b) $f_Y(y)$ | 0,2 | 0,4 | 0,3 | 0,10 | 1 |

a) $K + 0{,}24 + 0{,}18 + 0{,}06 = 0{,}6$

$K = 0{,}6 - 0{,}24 - 0{,}18 - 0{,}06$

$K = 0{,}12$

b) $f_{XY}(x,y) = f_X(x) \cdot f_Y(y) \Rightarrow$ v.a. independentes

27

$$f_{XY}(x,y) \begin{cases} Kxy & 0 \le x \le 2,\ 0 \le y \le 1 \\ 0 & \text{o.v. de } x \text{ e } y \end{cases}$$

a) $K = ?$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} Kxy\ f_x f_y = 1 \qquad \int\int Kxy\, dx\,dy$$

$$\Leftrightarrow \int_{-\infty}^{+\infty} K \int_{0}^{2} xy\ dx\,dy = 1 \Leftrightarrow \qquad Ky\left[\frac{x^2}{2}\right]_0^2 = K\left(\frac{4}{2}y\right)$$

$$= \left[\frac{4}{2}y\right] dy \left[y^2\right]_0^2 = 2K = 1$$

$K = \frac{1}{2}$

$$f_{XY}(x,y)\begin{cases} kxy & 0\le x\le 2,\ 0\le y\le 1 \\ 0 & \text{o.v di } x \text{ e } y\end{cases}$$

$$\int_0^1\int_0^2 kxy\,dx\,dy = 1$$

$$\Leftrightarrow \int_0^1 ky\left[\frac{x^2}{2}\right]_0^2 dy = \int_0^1 \frac{1}{2}\cdot k\cdot y\,dy = k\int_0^1 \frac{1}{2}y\,dy$$

$$= \frac{1}{2}k\int_0^1 \left(\frac{y^2}{2}\right)_0^1 \quad \left(\frac{1}{2}\cdot\frac{1}{2}\right)k = 1 \qquad \frac{1}{4}k = 1$$

$$\frac{k}{4} = 1$$

$$k = 4$$

$$\int_0^1\int_0^2 kxy\,dx\,dy = \cancel{k}\int_0^1 ky\int_0^2 x\,dx\,dy$$

$$2x = \frac{2x^2}{x} \qquad = \int_0^1 ky\left[\frac{x^2}{2}\right]^2 = \cancel{kxy}\int_0^1 k\cdot 2y\,dy$$

$$= k\int_0^1 y^2 = 1 \quad \Leftrightarrow \quad k = 1$$

b) Distribuição conjunta do par aleatório $(X, Y)$

$0 \le x \le 2 \quad 0 \le y \le 1$

Para $x < 0$ e $y < 0$

$F_{XY}(x, y) = 0$

Para $0 \le x \le 2 \quad 0 \le y \le 1$

$$\int_0^x \int_0^y uv \, du \, dv = \int_0^x u \frac{y^2}{2} \, du = \frac{x^2}{2} \frac{y^2}{2} \quad \frac{x^2 y^2}{4}$$

Para $0 \le x \le 2 \quad y > 1$

$$\int_0^x \int_0^1 uv \, dv \, du \Rightarrow \cancel{F_{XY}(x,1)} = 1/4$$

$$\int_0^x u \left[\frac{1^2}{2}\right] = \int \frac{u^2}{2} = \frac{x^2}{4} \qquad F(x,1) = 1/4 \, x^2 \, 1^2 = 1/4 \, x^2$$

$F(x,y) = 1/4 \, 2^2 1^2 = 1$

Para $x > 2 \quad 0 \le y \le 1$

$$F_{XY}(x,y) = \int_0^2 \int_0^y uv \, dv \, du = F_{XY}(2, y)$$

$$= 1/4 \, 2^2 y^2 = y^2$$

c)

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1/4\, x^2 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases} \qquad F_Y(y) = \begin{cases} 0 & y \le 0 \\ y^2 & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

d) $$f_X(x) = \begin{cases} 0 & x < 0 \\ 1/2\, x & 0 \le x \le 2 \\ 0 & x > 2 \end{cases} \qquad f_Y(y) = \begin{cases} 0 & y < 0 \\ 2y & 0 \le y \le 1 \\ 0 & y > 1 \end{cases}$$

$$f_{XY}(x,y) = f_X(x) \cdot f_Y(y) = 1/2\, x \cdot 2y = xy = f_{XY}(x,y)$$

então X e Y são independentes

f) $E[X] = \int_{-\infty}^{+\infty} \int_0^y xv\,dx\,dv = \int_0^2 1/2\, x = 1$

$E[X] =$

(28)

$$f_{XY}(x,y) = \begin{cases} k e^{-x-y} & x>0 \ , \ y>0 \end{cases}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} k e^{-x-y} = 1 \Leftrightarrow$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} k e^{-x} \cdot e^{-y}\, dx\,dy \Leftrightarrow \int_{0}^{+\infty} k \int_{0}^{+\infty} e^{-x} e^{-y}\, dx\,d_y = 1$$

$$\int k e \int^{+\infty} \left[-e^{-x}\right]_{0}^{+\infty} = -e^{-\infty} = 0 \Rightarrow e$$

$$k\int^{+\infty} e^{-2y} = 1$$

$$\int_{0}^{+\infty}\int_{0}^{+\infty} e^{-x-y}\, du\,dy = \int_{0}^{+\infty} k\left[e^{-y}\right]_{0}^{+\infty} - k[$$

$$\int_{0}^{+\infty}\int_{0}^{+\infty} k\, e^{-x} \cdot e^{-y}\, du\,dy = \int_{0}^{+\infty} k \cdot e^{-y} \int_{0} e^{-u} = \int_{0}^{+\infty} k \cdot e^{-y} \left[-e^{-u}\right]_{0}^{+\infty}$$

$$\int_{0}^{+\infty} k \cdot e^{-y}\left[-e^{-\infty} - \cancel{e^{0}}\right] \quad \int_{0}^{+\infty} k \cdot e^{-y} = k \left[-e^{-y}\right]_{0}^{+\infty}$$

$$= k\left[0+1\right] = 1$$

$$k = 1$$

29

$$F_{XY}(x,y) = \begin{cases} (1-e^{-5x})(1-e^{-3y}) & x \geq 0 ,\ y \geq 0 \\ 0 & x < 0 ,\ y < 0 \end{cases}$$

a) verificar se as variáveis X e Y são independentes

$F_{XY}(x,y) = F_X(x) \cdot F_Y(y)$

b) $F_X(x) = \lim_{y \to +\infty} F_{XY}(x,y) = 1 - e^{-5x}$

$F_Y(y) = \lim_{x \to +\infty} F_{XY}(x,y) = 1 - e^{-3y}$

$F_{XY}(x,y) = (1-e^{-5x})(1-e^{-3y}) = F_{XY}(x,y)$ são v.a. independentes

$f_X(x) = (F_X(x))' = 1-e^{-5x} = 5e^{-5x}$

$f_Y(y) = (F_Y(y))' = 1-e^{-3y} = 3e^{-3y}$

(30)

| X \| Y | 12 | 17 | 18 | 20 | $f_X(x)$ | |
|---|---|---|---|---|---|---|
| 10 | $x$ | $x$ | $x$ | $x$ | $4x$ | 0,25 |
| 15 | 0,1 | $x$ | $x$ | $x$ | $0,1+3x$ | 0,2875 |
| 20 | 0,1 | 0,1 | $x$ | 0,2 | $0,4+x$ | 0,4625 |
| $f_Y(y)$ | 0 | 0 | 0 | 0 | 1 | 0,0 |
| | 0,2625 | 0,225 | 0,1875 | 0,325 | | |

$4x + 0,1 + 3x + 0,4 + x = 1$

$8x = 1 - 0,5$

$x = \frac{0,5}{8}$

$x = \frac{1}{16}$

$x = 0,0625$

$0,2 + x + 0,1 + 2x + 3x + 0,2 + 2x = 1$

$8x + 0,5 = 1$

$x = 1 - 0,5$

$x = 0,5/8$

| X \| Y | $Y < 12$ | $12 \le Y < 17$ | $17 \le Y < 18$ | $18 \le Y < 20$ | $Y \ge 20$ |
|---|---|---|---|---|---|
| $X < 10$ | 0 | 0 | 0 | 0 | 0 |
| $10 \le X < 15$ | 0 | 0,0625 | 0,125 | 0,1875 | 0,25 |
| $15 \le X < 20$ | 0 | 0,1625 | | | |
| $X \ge 20$ | 0 | | | | |

32

$$f_X(x_1) = \begin{cases} 2x_1 & 0 \leq x_1 \leq 1 \\ 0 & \text{o.v.d. } x_1 \end{cases} \qquad f_{X_2}(x_2) = \begin{cases} \frac{x_2^2}{9} & 0 < x_2 \leq 3 \\ 0 & \text{o.v.d. } x_2 \end{cases}$$

a) se $x_1$ e $x_2$ são v.a. independentes

$$f_{X_1X_2}(x_1,x_2) = f_{X_1}(x_1) \cdot f_{X_2}(x_2)$$

$$f_{X_1X_2}(x_1,x_2) = 2x_1 \cdot \frac{x_2^2}{9} \qquad 0 \leq x_1 \leq 1,\ 0 \leq x_2 < 3$$

$E[X_1] =$

P(

3.3)

a) $f_{X_2}(x_2) = \int_{-\infty}^{+\infty} f_{X_1X_2}(x_1,x_2)\,dx_1$

$= \int_0^1 4x_1x_2\,dx_1 = 4x_1\int_0^1 \frac{x^2}{2} = 4x_1\left[\frac{1}{2} - 0\right]$

$= 2x_2$

b)

d) $f(x_1,x_2) = f_{X_1}(x_1) \cdot f_{X_2}(x_2)$

$f_{X_1}(x_1) = \int_0^1 4x_1x_2\,dx_2 = 4x_1\left[\frac{x_2^2}{2}\right]_0^1 = 2x_1$

$f_{X_1X_2}(x_1,x_2) = 2x_1 \cdot 2x_2 = 4x_1 \cdot x_2$ são v.a independentes

35

| X \| Y | 0 | 1 | $f_X(x)$ |
|---|---|---|---|
| 0 | 0,5 | 0,1 | 0,6 |
| 1 | 0,03 | 0,05 | 0,08 |
| 2 | 0,02 | 0,3 | 0,32 |
| $f_Y(y)$ | 0,55 | 0,45 | 1 |

Cap III

1.

Binomial $p = 0{,}2$ $E(X) = 1$ $q \Rightarrow$ não sucesso

$E(X) = 0{,}2 \times n$ $V(X) = 0{,}2 \cdot 5 \cancel{\cdot\cdot} \cdot 0{,}8$

$1 = 0{,}2\, n$

$n = 5$

2. $p = 1/4$ $P(Y \geq 1) \geq 0{,}7$

Exame 2018

(2)
$$f(x) = \begin{cases} c(x^2+2x) & 0 < x < 1 \\ 0 & \text{o.v.} \end{cases}$$

a) O valor de c

$$\int_{-\infty}^{+\infty} c(x^2+2x)\,dx = c\int_0^1 x^2+2x\,dx = 1$$

$$\Leftrightarrow c\left[\frac{x^3}{3}+x^2\right]_0^1 \Leftrightarrow \frac{c}{3} + \underset{(3)}{c} = \underset{(3)}{1}$$

$$\Leftrightarrow 4c = 1 \Leftrightarrow c = 1/4$$

b) $P(X > 1/3) = \int_{1/3}^{1} 1/4\,(x^2+2x)\,dx = 1/4\left[\frac{x^3}{3}+x^2\right]_{1/3}^{1}$

$$= 1/4\left[\left(1/3+1\right) - \left(\frac{(1/3)^3}{3} + 1/9\right)\right] = 1/4\left[\left(1/3+\underset{(3)}{1}\right) - \left(1/27 + \underset{(3)}{1/9}\right)\right]$$

$$= 1/4\left[\underset{(9)}{4/3} - 4/27\right] = 1/4\left[36/27 - 4/27\right] = \frac{32}{27}$$

$\simeq 0{,}29$

$$P(1/3 < X < 2/3) = \int_{1/3}^{2/3} 1/4\,(x^2+2x)\,dx$$

c) $E(X) = \int_0^1 x f(x)\, dx$

$= \int_0^1 x \left(\frac{1}{4}(x^2 - 2x)\right) = \frac{1}{4}\int_0^1 x^3 - 2x^2$

$= \frac{1}{4}\left[\frac{x^4}{4} - \frac{2x^3}{3}\right]_0^1 = \frac{1}{4}\left(\frac{1}{\underset{(3)}{4}} - \frac{2}{\underset{(4)}{3}}\right)$

$= \frac{1}{4}\left(\frac{3}{12} - \frac{8}{12}\right) = -$

③

| X \ Y | 0 | 1 | 2 | $f_X(x)$ |
|---|---|---|---|---|
| 0 | 0,12 | 0,05 | 0,03 | 0,20 |
| 1 | 0,25 | 0,30 | 0,10 | 0,65 |
| 2 | 0,13 | 0,01 | 0,01 | 0,15 |
| $f_Y(y)$ | 0,5 | 0,36 | 0,14 | 1 |

a)

b) $P(X > Y) = 0{,}25 + 0{,}13 + 0{,}01 = 0{,}39$

c) $0{,}12 + 0{,}30 + 0{,}01 = 0{,}43 \times 100 = 43\%$

①

$\frac{1}{10000}$ Tem doença — P 0,95 / N 0,05

$\frac{9999}{10000}$ Não tem doença — P 0,05 / N 0,95

$P(T/P) = \frac{P(T \cap P)}{P(P)}$

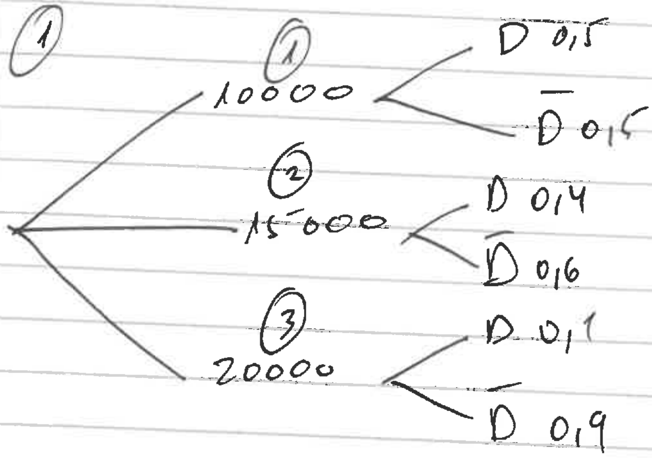
$= \frac{0{,}0001 \cdot 0{,}95}{0{,}0001 \cdot 0{,}95 + 0{,}05 \times 0{,}999}$

0,05007

$= 0{,}001877$

Exame Recurso 2018

1




$$P(D) = 0{,}1 \times \frac{20000}{45000} + 0{,}4 \times \frac{15000}{45000} + 0{,}5 \times \frac{10000}{45000}$$

$$P(1|D) = \frac{\frac{10000}{45000} \cdot 0{,}5}{P(D)}$$

$$P(2|D) = \frac{\frac{10000}{45000} \cdot 0{,}4}{P(D)}$$

$$P(3|D) = \frac{\frac{20000}{45000} \cdot 0{,}1}{P(D)}$$

| 1ª ext | 2ª ext | X |
|---|---|---|
| 4 | 1 | 1 |
| | 2 | 2 |
| | 3 | 3 |
| | 5 | 4 |
| 5 | 1 | 1 |
| | 2 | 2 |
| | 3 | 3 |
| | 4 | 4 |

2 a)

$$f(x) = \begin{cases} 8/20 & x=1 \\ 6/20 & x=2 \\ 4/20 & x=3 \\ 2/20 & x=4 \end{cases}$$

| 1ª ext | 2ª ext | X |
|---|---|---|
| 1 | 2 | 1 |
| | 3 | 1 |
| | 4 | 1 |
| | 5 | 1 |
| 2 | 1 | 1 |
| | 3 | 2 |
| | 4 | 2 |
| | 5 | 2 |
| 3 | 1 | 1 |
| | 2 | 2 |
| | 4 | 3 |
| | 5 | 3 |

b) $= f(1) + f(2) = 14/20$

c)