

# ENCYCLOPEDIA OF MATHEMATICAL SCIENCES

INCLUDING THEIR APPLICATIONS

## VOLUME ONE: ARITHMETIC AND ALGEBRA

FELIX KLEIN

Translated from the original German work  
"Encyklopädie der Mathematischen Wissenschaften mit  
Einschluss ihrer Anwendungen"

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ENCYCLOPEDIA  
OF  
MATHEMATICAL  
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VOLUME ONE IN TWO PARTS  
ARITHMETIC AND ALGEBRA

EDITED BY  
WILHELM FRANZ MEYER

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## Introductory Report on the Enterprise of Publishing the Encyclopedia of Mathematical Sciences

In September of 1894, Felix Klein and Heinrich Weber met with Franz Meyer, then professor at the Mining Academy in Clausthal, on a journey to the Harz Mountains. There, the first plan for the Encyclopedia of Mathematical Sciences was drafted. Franz Meyer developed his idea of composing a dictionary of pure and applied mathematics.

The ending century has, as in many areas of human knowledge, given rise to the desire for a comprehensive presentation of the scientific work accomplished during its course, which should also include the manifold applications to natural science and technology. Exhaustive, of course, in the sense of a complete presentation delving into all details of the widely branched structure, indicating all paths in both historical and methodological directions, such a work could not be planned, given the lack of comprehensive preliminary work, if one did not want to jeopardize its implementation. Thus, it was initially the intention to compile and characterize only the "most necessary", the fundamental "concepts" of our mathematical knowledge in the form of a lexicon.

"It should" — as Franz Meyer explained in a first draft — "provide the explanation of the concept falling under a given keyword in the form in which it first appeared, along with indication of the literary source, as far as possible. While this was mainly intended for newer concepts, the old and even obsolete expressions should nevertheless be mentioned, to preserve them as in a museum. This would be followed by the historical development of the concept"

follow up to the most recent times. Almost every concept differentiates and splits over time, takes on different nuances and applications, branches according to the uses made of it, deepens and generalizes. The respective technical term undergoes corresponding changes, additions, and compositions. The most important sections in this concept's career should again be provided with evidence." Thus, the developmental history of each individual concept should, in its part, provide a picture of progressive science.

The plan found full approval from Klein and Weber.

Fresh courage to execute it might strengthen during the wandering through mountain and forest. A great goal had been brought before their eyes, worth investing the effort and enduring the difficulties that the path would present. The enterprise exceeded the power of the individual; it was to become a collective effort of our German mathematicians, to which each would contribute according to their special field of work, and beyond that, where development brought it with it, researchers from abroad were to be recruited as well.

At that time, the Cartel of German Academies had just been formed, determined to implement and promote large scientific enterprises in collaborative work. The task set here appeared genuinely as a task for the Cartel. Through the academies, not only financial support should be offered, but also in scientific terms, the progress of work that would not be completed quickly — at that time, they thought of implementation in six to seven years — should be secured.

The German Mathematical Association, however, should primarily make the enterprise their own through the cooperation of their members. For them, the successfully begun plan of large detailed scientific reports on all current areas of mathematics, which were to be recorded in the annual reports, was complemented by this new comprehensive task, for which preliminary work could be drawn from those, at least in part.\*)

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\*) Already at the first meeting of the German Mathematical Association in Halle, autumn 1891, Felix Müller during the discussion of "literary under-

Thus, the importance and need for comprehensive presentation of widely branched knowledge was naturally accompanied by the necessity of uniting their representatives for collaborative work.

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At the Natural Scientists' Meeting in Vienna in September 1894, the German Mathematical Association decided to adopt the plan of composing a dictionary of pure and applied mathematics and commissioned Franz Meyer to seek scientific and financial support from the academies and learned societies of Göttingen, Leipzig, Munich, and Vienna united in the Cartel.

At the beginning of 1895, the first draft of the book, combined with a preliminary financing plan (which was established with the involvement of B. G. Teubner in Leipzig) was presented to the academies and received principal approval from Göttingen, Munich, and Vienna, while the Society of Sciences in Leipzig, due to lack of available funds, found itself compelled to abstain from participation in the enterprise for the time being.

The learned societies commissioned F. Klein (Göttingen), W. v. Dyck (Munich), G. v. Escherich (Vienna) to initiate discussions with the editorial board and with a publisher to be considered, and to draft a detailed plan of the enterprise regarding both its scientific and financial aspects. This academic commission subsequently stood as a permanent institution alongside the editorial board. It strengthened itself right at the beginning through H. Weber (Strasbourg) as representative of the German Mathematical Association and L. Boltzmann (Vienna) as advisor in scientific matters. Later, H. v. Seeliger (Munich) and more recently O. Hölder (Leipzig), who will be mentioned later, joined as well.

In detailed preliminary work, which concerned the organization of the material and its classification into larger comprehensive as well as smaller

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takings suitable for facilitating the study of mathematics" (1st Annual Report of the G.M.A., p. 59), in connection with the presentation of the draft of his (meanwhile published) mathematical vocabulary, pointed to such an alphabetically arranged mathematical encyclopedia.

individual articles, and then the anticipated scope of the entire work, occupied the summer of 1895. The decision about the feasibility of the enterprise, however, came from a conference of the academy delegates with Franz Meyer in September 1896 in Leipzig, in which A. Wangerin, in place of H. Weber, as well as publisher Alfred Ackermann-Teubner participated. Alongside a first draft of a keyword-based content arrangement, the manuscript of Felix Müller's previously mentioned lexicon of mathematical terminology was present — and it became apparent that for the intended purposes of an encyclopedia, an alphabetical arrangement could not be maintained. If one wanted to link the presentation of our current mathematical knowledge to individual concepts and technical terms and their transformation, as was the original plan mentioned at the beginning, the proper selection of keywords to be included, free from unnecessary ballast, around which the entire presentation would have to be grouped, would already present considerable difficulties. Nevertheless, such an arrangement would result in extensive fragmentation of the content, while on the other hand, particularly in presenting research results and methods, repetitions would hardly be avoidable. The lexicon would moreover acquire a completely inhomogeneous character, because alongside coherent developments about individual areas, very short sections, mere explanations, and countless cross-references would have to be inserted.

Thus in Leipzig, upon Dyck's proposal, the decision was made to abandon the idea of a proper lexicon and replace the artificial system of alphabetical ordering with the natural system of a purely subject-based arrangement and presentation of mathematical fields of knowledge. Even in such an arrangement, the manifold connections between individual disciplines are often enough severed, the mutual interweaving in subject matter or methodological terms can only partially be expressed, and the sequential presentation must completely replace the simultaneity of facts. But it is still possible to follow the main thread of guiding thoughts in the simply laid out presentation and incorporate into it the development of individual areas with their further elaboration.



Based on this new principle, the arrangement for the volumes dedicated to pure mathematics was first established. For its development, as well as for the preparation of two articles on "Surfaces of Third Order" and "Potential Theory," they succeeded in gaining Heinrich Burkhardt, then lecturer at the University of Göttingen, alongside Franz Meyer, and persuaded the former to join the editorial board, for it became apparent from the start that the editorial task could not be managed by a single person. Specifically, Franz Meyer later took on the editorship of Volume I (Arithmetic and Algebra) and Volume III (Geometry), while Heinrich Burkhardt took that of Volume II (Analysis).

It cannot be denied that with the change in the system of presentation, there was also a shift in content or at least a different emphasis of the same. Not the individual concept, but the structure of content in the results and methods of mathematical research forms the principle of grouping. Thus, the following was established as the task of the "Encyclopedia of Mathematical Sciences," as the work was called from then on:

"The task of the Encyclopedia shall be to provide, in a concise form suitable for quick orientation, but with the greatest possible completeness, a comprehensive presentation of the mathematical sciences according to their current content of established results, and at the same time to demonstrate through careful literature references the historical development of mathematical methods since the beginning of the 19th century. It shall not limit itself to so-called pure mathematics, but shall also consider applications to mechanics and physics, astronomy and geodesy, the various branches of technology and other fields, and thereby give an overall picture of the position that mathematics holds within today's culture."

A further difficulty now lay in measuring the scope of the entire work and in a proper distribution of space across the individual areas. Comparisons with earlier works of similar nature, with analogous ones from other disciplines, offered only slight

guidance. Here, an initial approach could only be established as a desirable limitation, not as a reliable norm, yet such an estimate had to form the basis for measuring the resources to be contributed by the academies as well as for negotiations with the publishing house.

It was agreed to set six large octavo volumes of forty sheets each as the starting point for space allocation. Three volumes were to serve pure mathematics, two applied mathematics, and another was to be dedicated to historical, philosophical, and didactic questions. Each volume was to be provided with its own index. The final volume should also contain a comprehensive overview and, to make the work usable as a reference work, a detailed alphabetically arranged index.

For the entire implementation of the enterprise, the editorial board was to work together with the commission appointed by the academies:

The editorial board was tasked with structuring the material in detail based on the work's arrangement established in joint consultations with the commission; to gain contributors, to reach understanding with them about the distribution of areas and to mediate the mutual reference of reviewers concerning neighboring areas; to ensure a unified character of the various articles; to oversee the printing; to compile the indexes; finally, through the commission, to provide regular reports to the participating academies about the work's progress.

The academic commission was to be responsible for maintaining the special interest of the academies in the work's prosperity and providing vigorous scientific support to the editorial board. In particular, this commission's approval should be required for any changes proving necessary in the work's plan or in the composition of the editorial board, as well as for the selection of contributors.

In spring 1896, the presented plans and proposals of the commission and editorial board received the approval of the aca-