

ENCYCLOPEDIA OF MATHEMATICAL SCIENCES

INCLUDING THEIR APPLICATIONS

VOLUME ONE:
ARITHMETIC AND ALGEBRA

FELIX KLEIN

Translated from the original German work
"Encyklopädie der Mathematischen Wissenschaften mit
Einschluss ihrer Anwendungen"

English Translation by
S. Vahid Ghayoomie, Claude A. Sonnet

Feb 2025

ENCYCLOPEDIA
OF
MATHEMATICAL
SCIENCES

INCLUDING THEIR APPLICATIONS

PUBLISHED

ON BEHALF OF THE ACADEMIES OF SCIENCES
OF GÖTTINGEN, LEIPZIG, MUNICH AND VIENNA,
WITH THE COLLABORATION OF NUMEROUS SPECIALISTS

VOLUME ONE IN TWO PARTS
ARITHMETIC AND ALGEBRA

EDITED BY
WILHELM FRANZ MEYER

IN KÖNIGSBERG I. PR.

PART ONE

LEIPZIG, PRINTED AND PUBLISHED BY B. G. TEUBNER

1898—1904

LIBRARY
ROYAL PRUSSIAN ACADEMY

ALL RIGHTS RESERVED, INCLUDING THE RIGHT OF TRANSLATION.

Translator's Note: This work was published over 100 years ago and has entered the public domain. The original copyright restrictions, including translation rights held by the Royal Prussian Academy, are no longer in effect. This translation is legally permitted under public domain laws.

Introductory Report on the Enterprise of Publishing the Encyclopedia of Mathematical Sciences

In September of 1894, *Felix Klein* and *Heinrich Weber* met with *Franz Meyer*, then professor at the Mining Academy in Clausthal, on a journey to the Harz Mountains. There, the first plan for the Encyclopedia of Mathematical Sciences was drafted. *Franz Meyer* developed his idea of composing a *dictionary of pure and applied mathematics*.

The ending century has, as in many areas of human knowledge, given rise to the desire for a comprehensive presentation of the scientific work accomplished during its course, which should also include the manifold applications to natural science and technology. Exhaustive, of course, in the sense of a complete presentation delving into all details of the widely branched structure, indicating all paths in both historical and methodological directions, such a work could not be planned, given the lack of comprehensive preliminary work, if one did not want to jeopardize its implementation. Thus, it was initially the intention to compile and characterize only the "most necessary", the fundamental "concepts" of our mathematical knowledge in the form of a *Lexicon*.

"It should" — as *Franz Meyer* explained in a first draft — "provide the explanation of the concept falling under a given keyword in the form in which it first appeared, along with indication of the literary source, as far as possible. While this was mainly intended for newer concepts, the old and even obsolete expressions should nevertheless be mentioned, to preserve them as in a museum. This would be followed by the historical development of the concept

follow up to the most recent times. Almost every concept differentiates and splits over time, takes on different nuances and applications, branches according to the uses made of it, deepens and generalizes. The respective technical term undergoes corresponding changes, additions, and compositions. The most important sections in this concept's career should again be provided with evidence." Thus, the developmental history of each individual concept should, in its part, provide a picture of progressive science.

The plan found full approval from *Klein* and *Weber*.

Fresh courage to execute it might strengthen during the wandering through mountain and forest. A great goal had been brought before their eyes, worth investing the effort and enduring the difficulties that the path would present. The enterprise exceeded the power of the individual; it was to become a collective effort of our German mathematicians, to which each would contribute according to their special field of work, and beyond that, where development brought it with it, researchers from abroad were to be recruited as well.

At that time, the *Cartel of German Academies* had just been formed, determined to implement and promote large scientific enterprises in collaborative work. The task set here appeared genuinely as a task for the Cartel. Through the academies, not only financial support should be offered, but also in scientific terms, the progress of work that would not be completed quickly — at that time, they thought of implementation in six to seven years — should be secured.

The *German Mathematical Association*, however, should primarily make the enterprise their own through the cooperation of their members. For them, the successfully begun plan of large detailed scientific reports on all current areas of mathematics, which were to be recorded in the annual reports, was complemented by this new comprehensive task, for which preliminary work could be drawn from those, at least in part.*)

*) Already at the first meeting of the German Mathematical Association in Halle, autumn 1891, *Felix Müller* during the discussion of "literary under-

Thus, the importance and need for comprehensive presentation of widely branched knowledge was naturally accompanied by the necessity of uniting their representatives for collaborative work.

* * *

At the *Natural Scientists' Meeting* in *Vienna* in September 1894, the *German Mathematical Association* decided to adopt the plan of composing a dictionary of pure and applied mathematics and commissioned *Franz Meyer* to seek scientific and financial support from the academies and learned societies of *Göttingen*, *Leipzig*, *Munich*, and *Vienna* united in the Cartel.

At the beginning of 1895, the first draft of the book, combined with a preliminary financing plan (which was established with the involvement of *B. G. Teubner* in *Leipzig*) was presented to the academies and received principal approval from *Göttingen*, *Munich*, and *Vienna*, while the *Society of Sciences* in *Leipzig*, due to lack of available funds, found itself compelled to abstain from participation in the enterprise for the time being.

The learned societies commissioned *F. Klein* (*Göttingen*), *W. v. Dyck* (*Munich*), *G. v. Escherich* (*Vienna*) to initiate discussions with the editorial board and with a publisher to be considered, and to draft a detailed plan of the enterprise regarding both its scientific and financial aspects. This academic commission subsequently stood as a permanent institution alongside the editorial board. It strengthened itself right at the beginning through *H. Weber* (*Strasbourg*) as representative of the *German Mathematical Association* and *L. Boltzmann* (*Vienna*) as advisor in scientific matters. Later, *H. v. Seeliger* (*Munich*) and more recently *O. Hölder* (*Leipzig*), who will be mentioned later, joined as well.

In detailed preliminary work, which concerned the organization of the material and its classification into larger comprehensive as well as smaller

takings suitable for facilitating the study of mathematics" (1st Annual Report of the *G.M.A.*, p. 59), in connection with the presentation of the draft of his (meanwhile published) mathematical vocabulary, pointed to such an alphabetically arranged mathematical encyclopedia.

individual articles, and then the anticipated scope of the entire work, occupied the summer of 1895. The decision about the feasibility of the enterprise, however, came from a conference of the *academy delegates* with *Franz Meyer* in September 1896 in Leipzig, in which *A. Wangerin*, in place of *H. Weber*, as well as publisher *Alfred Ackermann-Teubner* participated. Alongside a first draft of a keyword-based content arrangement, the manuscript of *Felix Müller's* previously mentioned lexicon of mathematical terminology was present — and it became apparent that for the intended purposes of an encyclopedia, an alphabetical arrangement could not be maintained. If one wanted to link the presentation of our current mathematical knowledge to individual concepts and technical terms and their transformation, as was the original plan mentioned at the beginning, the proper selection of keywords to be included, free from unnecessary ballast, around which the entire presentation would have to be grouped, would already present considerable difficulties. Nevertheless, such an arrangement would result in extensive fragmentation of the content, while on the other hand, particularly in presenting research results and methods, repetitions would hardly be avoidable. The lexicon would moreover acquire a completely inhomogeneous character, because alongside coherent developments about individual areas, very short sections, mere explanations, and countless cross-references would have to be inserted.

Thus in Leipzig, upon *Dyck's* proposal, the decision was made to abandon the idea of a proper *lexicon* and replace the *artificial* system of alphabetical ordering with the *natural* system of a purely subject-based arrangement and presentation of mathematical fields of knowledge. Even in such an arrangement, the manifold connections between individual disciplines are often enough severed, the mutual interweaving in subject matter or methodological terms can only partially be expressed, and the sequential presentation must completely replace the simultaneity of facts. But it is still possible to follow the main thread of guiding thoughts in the simply laid out presentation and incorporate into it the development of individual areas with their further elaboration.

Based on this new principle, the arrangement for the volumes dedicated to pure mathematics was first established. For its development, as well as for the preparation of two articles on "Surfaces of Third Order" and "Potential Theory," they succeeded in gaining *Heinrich Burkhardt*, then lecturer at the University of Göttingen, alongside *Franz Meyer*, and persuaded the former to join the editorial board, for it became apparent from the start that the editorial task could not be managed by a single person. Specifically, *Franz Meyer* later took on the editorship of Volume I (Arithmetic and Algebra) and Volume III (Geometry), while *Heinrich Burkhardt* took that of Volume II (Analysis).

It cannot be denied that with the change in the system of presentation, there was also a shift in content or at least a different emphasis of the same. Not the individual concept, but the structure of content in the results and methods of mathematical research forms the principle of grouping. Thus, the following was established as the task of the "*Encyclopedia of Mathematical Sciences*," as the work was called from then on:

"The task of the *Encyclopedia* shall be to provide, in a concise form suitable for quick orientation, but with the greatest possible completeness, a comprehensive presentation of the mathematical sciences according to their current content of established results, and at the same time to demonstrate through careful literature references the historical development of mathematical methods since the beginning of the 19th century. It shall not limit itself to so-called pure mathematics, but shall also consider applications to mechanics and physics, astronomy and geodesy, the various branches of technology and other fields, and thereby give an overall picture of the position that mathematics holds within today's culture."

A further difficulty now lay in measuring the scope of the entire work and in a proper distribution of space across the individual areas. Comparisons with earlier works of similar nature, with analogous ones from other disciplines, offered only slight

guidance. Here, an initial approach could only be established as a desirable limitation, not as a reliable norm, yet such an estimate had to form the basis for measuring the resources to be contributed by the academies as well as for negotiations with the publishing house.

It was agreed to set six large octavo volumes of forty sheets each as the starting point for space allocation. Three volumes were to serve pure mathematics, two applied mathematics, and another was to be dedicated to historical, philosophical, and didactic questions. Each volume was to be provided with its own index. The final volume should also contain a comprehensive overview and, to make the work usable as a reference work, a detailed alphabetically arranged *index*.

For the entire implementation of the enterprise, the editorial board was to work together with the commission appointed by the academies:

The *editorial board* was tasked with structuring the material in detail based on the work's arrangement established in joint consultations with the commission; to gain contributors, to reach understanding with them about the distribution of areas and to mediate the mutual reference of reviewers concerning neighboring areas; to ensure a unified character of the various articles; to oversee the printing; to compile the indexes; finally, through the commission, to provide regular reports to the participating academies about the work's progress.

The *academic commission* was to be responsible for maintaining the special interest of the academies in the work's prosperity and providing vigorous scientific support to the editorial board. In particular, this commission's approval should be required for any changes proving necessary in the work's plan or in the composition of the editorial board, as well as for the selection of contributors.

* * *

In spring 1896, the presented plans and proposals of the commission and editorial board received the approval of the aca-

demies of *Göttingen, Munich, and Vienna*, and the contract for publication was concluded with *B. G. Teubner* publishing house in Leipzig.

And now the work began — under favorable auspices, for right from the start, the editorial board succeeded in securing a large, significant circle of contributors, ready to put their work in service of the common cause, setting aside their special interests. "*General Principles*" had been issued, which were intended to ensure as much as possible a common basis for the structure of articles and uniform treatment of the material, without overly restricting the scientific freedom and individuality of the individual who bears full responsibility for their presentation.*)

Regarding the arrangement of individual volumes, as it gradually took shape based on these foundations, this will be reported in the special introductions by the editorial board. Here it should only be emphasized how the establishment and gradual completion of the comprehensive arrangement, the mutual alignment of the content of individual essays, and the clarification of their mutual relationships were particularly promoted in the frequent personal conferences between contributors, editors, and commission members. They represent a sacrifice by all participants that must be acknowledged with the utmost gratitude, but also a lasting

*) We believe we should reproduce them at this point with those modifications and additions which they later received, particularly when undertaking the volumes of applied mathematics.

General Principles for the Preparation of Articles.

1. Within each article, the mathematical *concepts* peculiar to the respective field, their most important *properties*, the most fundamental *theorems*, and the *investigation methods* that have proven fruitful are presented.

2. The execution of *proofs* of the communicated theorems must be omitted; only where principally important proof methods are concerned can a brief indication of them be given.

3. The parts of the work relating to *applications* should fulfill a dual purpose: they should, *on one hand*, orient the mathematician about what questions the applications pose to them, and *on the other hand*, inform the astronomer, physicist, engineer about what answer mathematics gives to these questions. Accordingly, they limit themselves to the mathematical side of applications; instrumentation, observation techniques, collection of constants, regulations fall outside the framework of the work.

benefit for the entire work and for all who have participated in it. The Natural Scientists' Meetings of the last decade, starting from the Vienna meeting of 1894, where the foundation stone of the work was laid, the national Mathematics Congress in Zurich (1897), as well as other conferences of the academic commission and editorial board, which were almost regularly combined with the annual meetings of the Cartel of German Academies, offered important opportunities for joint consultation about the work's progress and exchange of ideas about its detailed development.

The necessity of personal discussion became particularly apparent when, in 1897, after the most essential steps for the arrangement and implementation of the first three volumes of pure mathematics had been taken, it was time to approach the volumes dedicated to applied mathematics. From the outset, it had become clear that only an expansion of the editorial board could ensure the implementation of the enterprise and likewise, that — if one did not want to delay completion

If the first of these goals is to be achieved, it will be necessary to: briefly indicate the considerations that led to the mathematical formulation of the problem in question; explicitly establish this formulation; indicate the limits within which the occurring constants lie in practical cases; indicate the degree of accuracy up to which the formulation in question is to be considered correct.

If the second goal is also to be achieved, one must not limit oneself to mere references to those places in the first three volumes where the problem in question is treated; one must briefly state the result of the required mathematical operations (equation solving, geometric construction, integration). However, repetition of literature references is not necessary.

4. Strictly chronological arrangement of the material would necessitate many repetitions for which there is no space; but the *gradual development of concepts and methods* will be explained at appropriate points and documented through *precise literature references*.

5. The existing historical monographs and bibliographic resources will provide good initial orientation services to the contributors; however, the first principle of all historical criticism requires that the presentation ultimately be based on personal *study of the original works*.

6. While results from *older developmental periods* should be included, specific proof of their origin will have to be omitted; otherwise, following principle (5) would delay the completion of the work beyond measure, as the required orienting works are still lacking, especially for the 18th and partly also for the 17th century

of the whole into the distant future — it was necessary to tackle the work from all sides. The academic commission hoped to persuade *F. Klein* to join the editorial board and specifically to take charge of the volume relating to mechanics, and likewise to gain *A. Sommerfeld* (then private lecturer in Göttingen) for the editing of the mathematical-physical part. Initially, *Klein* undertook several major journeys (to England, France, Holland, Italy, and Austria), for which the academies had granted the necessary means in a liberal way, to make the necessary preliminary work for the arrangement, development, and collaboration on these volumes. While the participation of non-German authors had already become of essential importance for the character of the reports in the first volumes, here, with the volumes dedicated to applied mathematics, it is particularly important to be able to count on the collaboration of non-German authors according to the development of individual areas.

As much as we want to claim the entire enterprise as German in its foundation and execution, it is from

lacking. Accordingly, the historical presentation will generally begin with the start of the nineteenth century. Insofar as citations to earlier times are given at all, they are to be understood in the sense that no guarantee is provided whether an even earlier source could have been cited.

7. The individual mathematical subjects are not considered as isolated from each other; on the contrary, it is one of the main tasks of the work to bring general awareness to the manifold *interweaving and overlapping of the most diverse areas*.

8. One-sided emphasis of a particular school standpoint runs counter to the work's purpose. The most desirable would be if everywhere it were possible to integrate the results obtained by different paths into an *objective presentation*; where this appears unfeasible, at least each of the opposing views should be given a voice.

9. The Encyclopedia is not called upon to decide pending *disputes*, particularly those about priority.

10. If concepts or theorems belonging to another area are used in one field, reference is simply made to the section treating the *latter area* (using the signature used in the arrangement), even if it appears at a later point in the Encyclopedia. Moreover, things about which one can doubt whether they belong in an earlier or later section will generally be included at the earlier point.

11. As far as it can be done without compromising principles (7) and (10), the requirements for readers' prior knowledge will be kept such that

of utmost importance, if it is not to represent a one-sided viewpoint, that in the conception and presentation of individual areas, all voices that have contributed to the uniqueness of their development are heard. The permanent holdings of any science are an international good, gained from the collective work of scholars of all times and all countries. But in different directions, with varying emphasis and appreciation of individual areas, with characteristic differences in methods and form of presentation, different nations and different epochs have participated in this work. This must be expressed in the Encyclopedia in the presentation of content according to its historical development as well as in the recruitment of contributors. Indeed, the enterprise today counts, alongside its foundation of German authors, scholars from America, Belgium, England, France, Holland, Italy, from Norway, Austria, Russia, Sweden among its contributors.

In the years 1898 and 1899, particularly through the personal efforts and connections of *F. Klein*, the implementation

the work can also be useful to someone who seeks orientation about only a specific area.

12. Bibliographic completeness of *literature references* is neither possible nor even desirable, just as exhaustive enumeration of all main proposed theorems or suggested technical terms.

13. However, all important *technical terms* actually in use should appear and find explanation, so they can later be included in the index. Cases should be noted particularly where the same term or symbol is used by different authors with different meanings, especially those where the meaning of a term has imperceptibly expanded over time. Among obsolete terms, a sparing selection should be made.

14. Wherever necessary for understanding, *figures* will be included *in the text*.

15. The enterprise does not have the means for including extensive *collections of formulas*, or similar *tables of numerical values* of the functions treated — which should not be copied from other works without prior verification anyway. However, information about where such can be found is desirable, if necessary with a warning against uncritical use. — Very small tables can find space, which illustrate the behavior of a function through a few appropriately chosen numerical values; often a graphical representation will serve the same purpose even better.

16. *Citations* to frequently used journals will be given in uniform abbreviated

of the volumes dedicated to applied mathematics could be secured and a first arrangement of the same could be drafted. In doing so, it proved necessary to distribute the entire abundant material of applications across three volumes instead of two as planned, of which the fourth would encompass mechanics, the fifth mathematical physics, and the sixth geodesy, geophysics and astronomy, while a seventh volume was reserved for historical, philosophical and didactic questions.

In 1899, *Klein* definitively took over the editorship of the volume dedicated to mechanics, soon after *Sommerfeld* took the editorship of the fifth volume, mathematical physics.

An arrangement of the sixth volume could only be approached after multiple preliminary negotiations in 1900. It was undertaken by *E. Wiechert* in Göttingen for geodesy and geophysics, and by *R. Lehmann-Filhés* in Berlin for astronomy, both thereby joining the encyclopedia's editorial board. Unfortunately, the latter found himself compelled to step down from the editorial board in 1902, where he had conducted the initial negotiations with the selected contributors in a most commendable way. In his

form (according to a specially established scheme); books are cited, where they appear in an article for the first time, with family name and abbreviated first name of the author, main part of the title, place and year, in case of multiple occurrences the later times in shorter form. Where the matter doesn't seem important enough for more detailed information, mere enumerations of names usually have little use for the reader.

17. Generally meaningless *ornamental epithets*, such as groundbreaking, ingenious, magnificent, classical etc. should be avoided. Instead, it will be indicated in which direction progress lies in each case: whether in finding *new results* — or in *rigorous foundation* of previously only conjecturally proposed or insufficiently proven theorems — or in shortening cumbersome developments through the use of *new tools* — or finally in *systematic arrangement* of an entire theory.

Specifically for the preparation of volumes for applied mathematics, the following remarks apply:

1. Since the encyclopedia essentially addresses a mathematical audience, it must place emphasis on the *mathematical* side of theories. This will include, on one hand, the mathematical *formulation* of the tasks under consideration, and on the other hand, their mathematical *implementation*. The latter viewpoint, which often recedes in specifically physical and scientific books, will be essentially kept in mind here. On the other hand, however, in contrast to the presentation in the majority of mathematical works, the experimental foundation of individual

place, *K. Schwarzschild*, who had just been appointed to the Göttingen Observatory, joined in 1903.

At Easter 1904, *Conrad H. Müller*, who had been involved in the editorial work for some time, was appointed by the academic commission as co-editor of the fourth volume to support *F. Klein*; finally in July 1904, *Ph. Furtwängler* (in Potsdam) was called to edit the first part of Volume VI (Geodesy and Geophysics) in collaboration with *E. Wiechert*.

Meanwhile, on November 7, 1898, the first issue of the first volume was published, containing *H. Schubert's* report on the foundations of arithmetic, *E. Netto's* report on combinatorics, and *A. Pringsheim's* extensive work on irrational numbers and convergence of infinite processes. In August 1899, the publication of the second volume then began with *Pringsheim's* foundations of general function theory, followed by *A. Voss's* essay on differential and

areas should be described, namely to the extent that the reader gains a general judgment about the foundation and accuracy limit of the mathematical theory.

2. The general plan of the encyclopedia corresponds, as emphasized in the writing, to the *historical arrangement of material* and provision of the main moments of *historical development*. However, for the present volumes in this regard, it should be noted that the results of applied mathematics become outdated more quickly than those of pure mathematics, and therefore the historical development here does not have the same importance for understanding the current state of theory as there. Nevertheless, historical presentation will generally be desirable in the following volumes as well, insofar as it is compatible with systematics and clarity.

3. In the fields of applied mathematics, the literature is often very scattered and disconnected. The editorial board has therefore made it their concern to establish connections in advance in as many directions as possible, with mathematicians, physicists, technicians, ... astronomers, as well as in different countries; they will be gladly ready to communicate or at least point out otherwise difficult-to-obtain literature to the contributors based on these connections.

4. Finally, it does not seem necessary that each article be prepared by a single author. Rather, *smaller contributions* that cover only part of the material to be treated in the respective article are sometimes desirable. Such contributions can be printed as an appendix to the comprehensive article or, if the author agrees, be made available to the main reviewer of the area and incorporated by them. The authorship of such contributions will be appropriately expressed in the article's heading.

integral calculus, as well as that of the late *G. Brunel* on definite integrals. In October 1902, the first issue of the third volume appeared with essays by *H. v. Mangoldt* and *R. v. Lilienthal* on differential geometry. The publication of parts dedicated to applied mathematics began in June 1901 with the fourth volume with *M. Abraham's* presentation of geometric concepts for mechanics of deformable bodies and two treatises by *A.E.H. Love* on hydrodynamics. At Easter 1903, Volume V (Mathematical Physics) followed, introduced by *C. Runge's* essays on measurement and measuring and *J. Zenneck's* on gravitation, to which *G. H. Bryant's* general foundation of dynamics is joined. Still in the course of this year, the publication of the first issues of both parts of the sixth volume will begin. They will contain, on one hand, essays by *C. Reinhardt* and *P. Pizzetti* on geodesy, by *S. Finsterwalder* on photogrammetry, and on the other hand (in the astronomical part) treatises by *E. Anding* and *F. Cohn* on the theory of coordinates.

Not everyone has approved of this activity beginning on all sides of the work, fearing that the completion of individual volumes might be delayed too much. Also, the reader currently receives a not easily overlooked patchwork of individual issues, which libraries are also reluctant to release for use. However, it must be said that a delay in publication due to the broad scope of editorial activity does not occur, because it almost always involves different editors and contributors; on the contrary, the uniform progress of the whole is of essential importance for utilizing the mutual relationships between individual volumes and individual essays. On the other hand, the publishing house has recently accounted for the ease of use of individual issues through special equipment and binding of the issues.

Here is the place to emphasize with special thanks the extremely great accommodation of the publishing house *B. G. Teubner*. On one hand, the firm has most willingly fulfilled all extensive wishes and requirements of the editors and authors regarding printing, and on the other hand, through its own commitment to the honoraria to be expended, has made it possible

to meet the increasingly strong need arising in the course of development to carry out the work in a scope considerably expanded compared to the initial plan.

One may regret that the original approach of presenting a very concise overview of our current mathematical knowledge in six manageable volumes has been abandoned, and may not without concern see how from volume to volume the work extends beyond the boundaries drawn at the beginning. However, the striving for greatest possible completeness in individual sections and the desire to be clear and comprehensible, even at the expense of brevity — a desire that has repeatedly been expressed to us from reader circles — form the immediate reason for the growth in scope. But the essential reason lies perhaps deeper: The work is a *first* according to its task, so it cannot be a complete one in fulfilling it. Only when the vast field it encompasses lies before us in this first version as a whole, when the circle of problems it has to present has been measured once, will one be able to see how much remains to be done for deepening its content, for simplifying and making the presentation more concise, for aligning and interconnecting all individual parts.

* * *

Two circumstances significant for the recognition of the scientific work accomplished so far are still to be mentioned in our historical report:

The first is the publication of a French adaptation of the Encyclopedia, for which the publishers *B. G. Teubner* in Leipzig and *Gauthier-Villars et fils* in Paris received authorization from the academies in 1900. *J. Molk*, Professor at the Faculty of Sciences in Nancy, was entrusted with directing this edition initially for the volumes dedicated to pure mathematics, while for publishing the volumes of applied mathematics he collaborated with *P. Appell*, member of the Institut de France (mechanics), as well as with *A. Potier* (physics), *Ch. Lallemant* (geodesy and geophysics) and *H. Andoyer* (astronomy).

It is not merely a translation, but an adaptation envisioned, in which the leading French scholars have promised their participation

have promised. While fully preserving the character of the German original, this edition shall take into account the usage of French circles and, on the other hand, through joint collaboration of authors and editors, the individual articles shall experience manifold additions, especially regarding literature citations.*)

Thus the German work in its French edition will be made accessible to and appreciated by even wider circles.

We may see a further recognition of the work's implementation so far, which we welcome with particular joy, in the fact that recently the Royal Saxon Society of Sciences in *Leipzig* has also made it possible to participate in the publication of the Encyclopedia. They have delegated *O. Hölder* to the academic commission on their part.

Thus the publication now appears as a joint enterprise of the learned societies united in the Cartel of German Academies in Göttingen, Leipzig, Munich and Vienna, and it also demonstrates the significance of this union for the implementation of tasks that are only possible in united work; at the same time, however, the authority of the academies, which have made the enterprise their own, provides the guarantee that future development, completion of the whole, as well as later revisions are placed in the best hands and secured in their scientific foundation.

* * *

*) The prospectus of the French edition characterizes the nature of the adaptation in the following way:

In the French edition, efforts have been made to reproduce the essential features of the articles from the German edition; however, in the adopted mode of presentation, French customs and habits have been extensively taken into account.

This French edition will offer a very particular character through the collaboration of German and French mathematicians. The author of each article in the German edition has, in fact, indicated the modifications they deemed appropriate to introduce in their article and, on the other hand, the French editing of each article has given rise to an exchange of views in which all interested parties have participated; additions due particularly to French collaborators will be placed between two asterisks. The importance of such collaboration, of which the French edition of the Encyclopedia will offer the first example, will escape no one.

Thus may the Encyclopedia under this friendly aspect, under the protection of the united Academies, serve the sciences in its part:

Pure mathematical research, by preparing the old furrowed ground for new sowing and harvest, and opening up conquered land to fertilizing intellectual work;

The *applied branches of knowledge*, by bringing together the often separated paths of mathematical and scientific consideration, and preparing the foundation and method for their further development;

The *totality of all intellectual work*, by designating and circumscribing the position that belongs to the mathematical sciences in the realm of human knowledge.

Munich, July 30, 1904

Walther von Dyck,
as Chairman of the academic commission
for the publication of the Encyclopedia.