https://storage.googleapis.com/met-cs-777-data/taxi-data-sorted-large.csv.bz2.

Simple Linear Regression

• Calculate the m slope and the b intercept for the small dataset

Slope (m): 2.698633750152394 Intercept (b): 3.970225230889111

• Calculate the m slope and the b intercept for the large dataset

Slope (m): 2.6656349732213727 Intercept (b): 4.532229802078271

https://storage.googleapis.com/met-cs-777-data/taxi-data-sorted-large.csv.bz2.

Additional Task using BatchGradientDescent and Desired Iterations and LR for Testing Aim

• Calculate the m slope and the b intercept for the small dataset

Final parameters for small dataset (iteration=2000, learning_rate = 0.01):

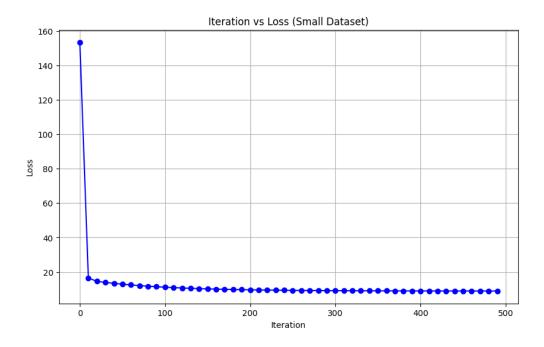
Slope (m): 2.6986391293785004 Intercept (b): 3.9701887413601833

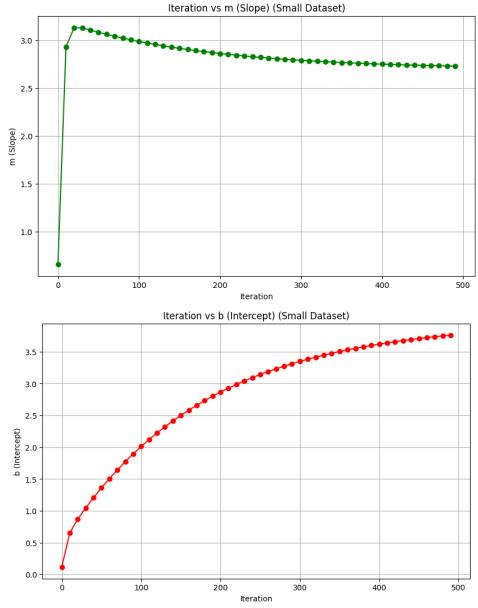
Approximately constant loss after 2000 iterations.

Final parameters for small dataset (iteration=1000, learning_rate = 0.0001):

Slope (m): 2.780701661507673 Intercept (b): 0.608503802919607

This needs to more time and bigger Alpha like 0.001 to decrease the Loss to get stable value





Above diagrams show the limitation of the range for answer to optimize the iteration and learning rate well approximately.

• Calculate the m slope and the b intercept for the large dataset

Final parameters for large dataset (iteration=35, learning_rate = 0.01):

Slope (m): 2.804932076974661

Intercept (b): 1.4434139285985357 → ~ 3.9

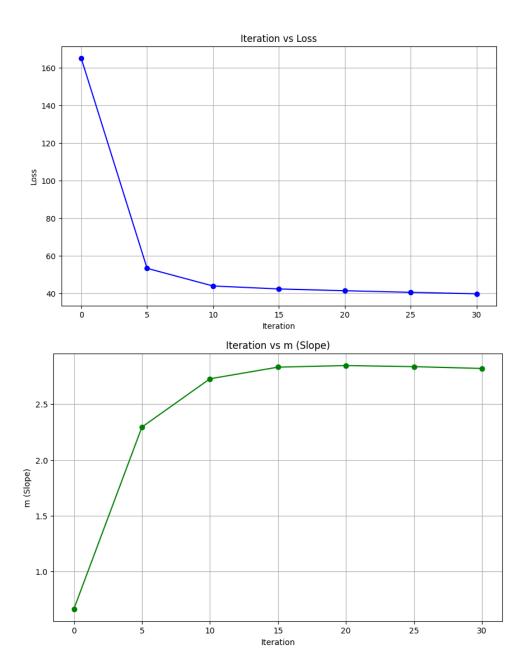
This model with mentioned hyperparameters needs more iterations (more time) to have stable and less Loss value

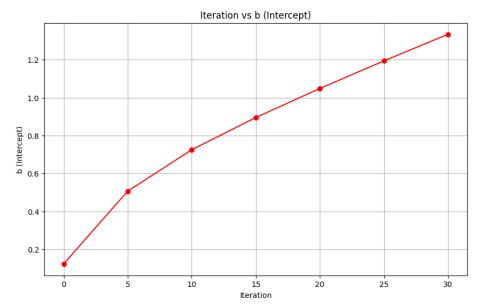
Final parameters for large dataset (iteration=100, learning_rate = 0.0001):

Slope (m): 0.5958643526175813

Intercept (b): 0.11339460453658075

This model with mentioned hyperparameters needs more iterations (more time) and a little more learning rate like 0.001





Above shows we need more iteration for large dataset, but there was some credit limitation for me to stop earlier. Diagrams show there is a limitation for m and loss however b can still increase well.

Note: All above models could estimate the slope m (parameter) with good accuracy however for getting intercept b (parameter) exists some issues and need to change the hyperparameters like LR and iterations.

Find the Parameters using Gradient Descent

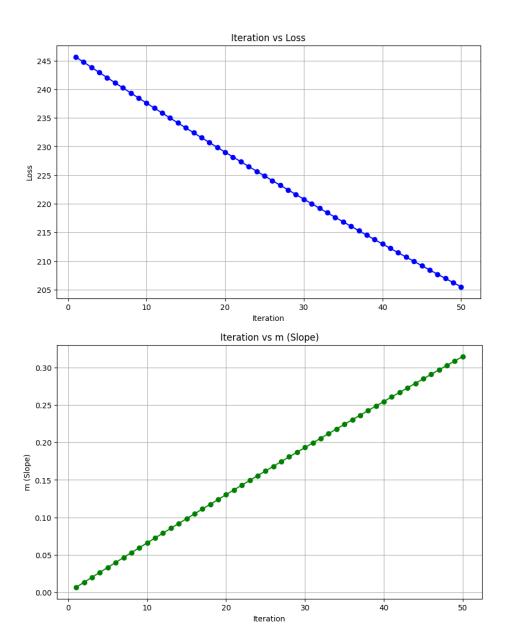
• Print out the costs and model parameters in each iteration. The maximum number of iterations is 50.

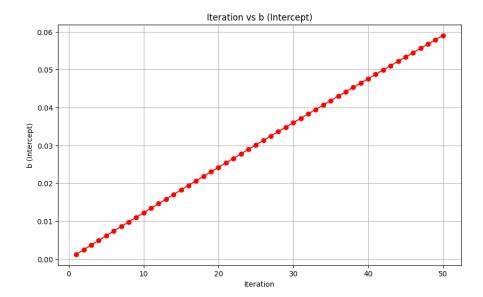
```
Iteration 1: Loss = 245.6573, m = 0.006661, b = 0.001229
Iteration 2: Loss = 244.7450, m = 0.013306, b = 0.002456
Iteration 3: Loss = 243.8368, m = 0.019937, b = 0.003680
Iteration 4: Loss = 242.9327, m = 0.026552, b = 0.004903
Iteration 5: Loss = 242.0327, m = 0.033152, b = 0.006124
Iteration 6: Loss = 241.1368, m = 0.039737, b = 0.007342
Iteration 7: Loss = 240.2450, m = 0.046307, b = 0.008559
Iteration 8: Loss = 239.3572, m = 0.052861, b = 0.009774
Iteration 9: Loss = 238.4735, m = 0.059401, b = 0.010986
Iteration 10: Loss = 237.5937, m = 0.065926, b = 0.012197
Iteration 11: Loss = 236.7180, m = 0.072436, b = 0.013405
Iteration 12: Loss = 235.8462, m = 0.078930, b = 0.014611
Iteration 13: Loss = 234.9783, m = 0.085410, b = 0.015816
Iteration 14: Loss = 234.1144, m = 0.091876, b = 0.017018
Iteration 15: Loss = 233.2545, m = 0.098326, b = 0.018219
Iteration 16: Loss = 232.3984, m = 0.104761, b = 0.019417
Iteration 17: Loss = 231.5462, m = 0.111182, b = 0.020614
Iteration 18: Loss = 230.6979, m = 0.117588, b = 0.021808
Iteration 19: Loss = 229.8534, m = 0.123980, b = 0.023001
Iteration 20: Loss = 229.0127, m = 0.130356, b = 0.024191
```

```
Iteration 21: Loss = 228.1759, m = 0.136719, b = 0.025380
Iteration 22: Loss = 227.3429, m = 0.143066, b = 0.026566
Iteration 23: Loss = 226.5136, m = 0.149399, b = 0.027751
Iteration 24: Loss = 225.6881, m = 0.155718, b = 0.028933
Iteration 25: Loss = 224.8664, m = 0.162022, b = 0.030114
Iteration 26: Loss = 224.0483, m = 0.168311, b = 0.031293
Iteration 27: Loss = 223.2340, m = 0.174586, b = 0.032469
Iteration 28: Loss = 222.4234, m = 0.180847, b = 0.033644
Iteration 29: Loss = 221.6164, m = 0.187094, b = 0.034817
Iteration 30: Loss = 220.8131, m = 0.193326, b = 0.035988
Iteration 31: Loss = 220.0135, m = 0.199543, b = 0.037157
Iteration 32: Loss = 219.2175, m = 0.205747, b = 0.038324
Iteration 33: Loss = 218.4251, m = 0.211936, b = 0.039489
Iteration 34: Loss = 217.6362, m = 0.218111, b = 0.040653
Iteration 35: Loss = 216.8510, m = 0.224272, b = 0.041814
Iteration 36: Loss = 216.0693, m = 0.230419, b = 0.042973
Iteration 37: Loss = 215.2912, m = 0.236552, b = 0.044131
Iteration 38: Loss = 214.5166, m = 0.242671, b = 0.045286
Iteration 39: Loss = 213.7455, m = 0.248776, b = 0.046440
Iteration 40: Loss = 212.9779, m = 0.254866, b = 0.047592
Iteration 41: Loss = 212.2138, m = 0.260943, b = 0.048742
Iteration 42: Loss = 211.4531, m = 0.267006, b = 0.049890
Iteration 43: Loss = 210.6959, m = 0.273055, b = 0.051036
Iteration 44: Loss = 209.9422, m = 0.279090, b = 0.052180
Iteration 45: Loss = 209.1918, m = 0.285111, b = 0.053323
Iteration 46: Loss = 208.4449, m = 0.291118, b = 0.054463
Iteration 47: Loss = 207.7013, m = 0.297112, b = 0.055602
Iteration 48: Loss = 206.9611, m = 0.303092, b = 0.056739
Iteration 49: Loss = 206.2243, m = 0.309058, b = 0.057874
Iteration 50: Loss = 205.4908, m = 0.315011, b = 0.059007
Final parameters:
Slope (m): 0.3150106118230866
Intercept (b): 0.059007033958606216
```

Calculate the m slope and the b intercept based on your calculations

Final parameters: Slope (m): 0.3150106118230866 Intercept (b): 0.059007033958606216





• Comment on how you can interpret the parameters of the model. What is the meaning of m and b in this case?

The model parameters consist of the slope m and the intercept b. As the model iterates, it minimizes the loss function (in this case, the Mean Squared Error) to find the best fit line for the data points representing the relationship between trip time and fare amount. A decrease in loss with each iteration indicates that the model is learning to better predict fare amounts based on trip times. Slope (m) quantifies the change in fare amount per unit increase in trip time. In our results, a slope of approximately 0.315 suggests that for each additional second of trip time, the fare amount increases by about 0.315 currency units. Intercept (b) represents the estimated fare amount when the trip time is zero. An intercept of approximately 0.059 indicates that if there were a trip with a duration of zero seconds (hypothetically), the starting fare would be around 0.059 currency units. So, these parameters help create a linear regression model that estimates fare amounts based on the trip time, enabling fare prediction for taxi services. Simple Linear Regression gives an exact result but can be computationally expensive for large datasets. Gradient Descent provides an approximate result, but it's faster and more scalable for large datasets, especially in distributed environments like PySpark on GCP. If tuned correctly, Gradient Descent should give results very close to Simple Linear Regression, but for very large datasets, the trade-off between accuracy and efficiency comes into play.

Fit Multiple Linear Regression using Gradient Descent

Print out the costs and model parameters in each iteration.

Iteration 1, Cost: 3945948.4832964763, m: [1.76455744e-01 5.58341313e-03 5.05884956e-03 1.30863370e-04], b: 0.00014766810137397475

Iteration 2, Cost: 30620522546237.824, m: [-4.91568090e+02 -3.33534778e+00 -1.82126249e-02 -7.16602758e-04], b: -0.0012832041487742204

Iteration 3, Cost: 5.9407654746179265e+19, m: [6.84703545e+05 1.68969899e+03 4.00796484e+01 1.37903008e+00], b: 2.2165874171162856

```
Iteration 4, Cost: 2.877334532430775e+25, m: [-4.76515564e+08 -8.18270020e+05 -2.78798311e+04
-9.59435614e+02], b: -1542.2917867199478
Iteration 5, Cost: 3.4739967184554596e+30, m: [1.65575739e+11 2.62777662e+08 9.68721520e+06
3.33373838e+05], b: 535899.5592234605
Iteration 6, Cost: 1.0425697423701348e+35, m: [-2.86836605e+13 -4.48788097e+10 -
1.67816599e+09 -5.77522193e+07], b: -92836930.88757718
Iteration 7, Cost: 7.732009906073271e+38, m: [2.47017783e+15 3.85541679e+12 1.44520094e+11
4.97350091e+09], b: 7994923915.752392
Iteration 8, Cost: 1.400472674113528e+42, m: [-1.05128226e+17 -1.64015511e+14 -6.15062574e+12
-2.11667071e+11], b: -340255724232.15234
Iteration 9, Cost: 6.047059187804543e+44, m: [2.18451046e+18 3.40793687e+15 1.27806836e+14
4.39833281e+12], b: 7070338849884.747
Iteration 10, Cost: 5.914447880207313e+46, m: [-2.16042465e+19 -3.37033098e+16 -
1.26397672e+15 -4.34983800e+13], b: -69923832302923.44
Iteration 11, Cost: 1.1685104604038749e+48, m: [9.60280966e+19 1.49806738e+17 5.61821396e+15
1.93344704e+14], b: 310802440471880.06
Iteration 12, Cost: 3.4667189488073156e+48, m: [-1.65402220e+20 -2.58032475e+17 -
9.67701222e+15 -3.33023818e+14], b: -535337214920475.7
Iteration 13, Cost: 4.52331699996137e+47, m: [5.97462333e+19 9.32059332e+16 3.49550948e+15
1.20294145e+14], b: 193373354179930.3
Iteration 14, Cost: 1.1188275563055752e+47, m: [-2.97141694e+19 -4.63550049e+16 -
1.73845538e+15 -5.98270454e+13], b: -96172231955605.62
Iteration 15, Cost: 4.684578319225702e+46, m: [1.92272543e+19 2.99950988e+16 1.12490857e+15
3.87125010e+13], b: 62230511415595.56
Iteration 16, Cost: 3.0870823484209363e+46, m: [-1.56083188e+19 -2.43494503e+16 -
9.13179345e+14 -3.14260708e+13], b: -50517543642953.7
Iteration 17, Cost: 3.0437633705785483e+46, m: [1.54984213e+19 2.41780065e+16 9.06749689e+14
3.12048013e+13], b: 50161851847885.0
Iteration 18, Cost: 4.326623988196347e+46, m: [-1.84780695e+19 -2.88263482e+16 -
1.08107680e+15 -3.72040787e+13], b: -59805716111811.95
Iteration 19, Cost: 3.998013143739844e+44, m: [1.77625015e+18 2.77100396e+15 1.03921183e+14
3.57633412e+12], b: 5748972437973.125
Iteration 20, Cost: 1.6923205374885324e+43, m: [-3.65446099e+17 -5.70107043e+14 -
2.13807673e+13 -7.35795778e+11], b: -1182794869048.3535
Iteration 21, Cost: 1.8019995916379536e+42, m: [1.19250268e+17 1.86033985e+14 6.97684881e+12
2.40100640e+11], b: 385962808486.2698
Iteration 22, Cost: 3.795572039794101e+41, m: [-5.47293938e+16 -8.53795457e+13 -
3.20199465e+12 -1.10193152e+11], b: -177135964598.563
Iteration 23, Cost: 1.3885406102596688e+41, m: [3.31025394e+16 5.16409400e+13 1.93669508e+12
6.66492482e+10], b: 107138952457.28644
Iteration 24, Cost: 8.132977016596443e+40, m: [-2.53341764e+16 -3.95220994e+13 -
1.48219985e+12 -5.10082907e+10], b: -81996046008.28568
Iteration 25, Cost: 7.214715230309111e+40, m: [2.38611634e+16 3.72241267e+13 1.39601970e+12
4.80424949e+10], b: 77228516745.63748
Iteration 26, Cost: 9.311255403326586e+40, m: [-2.71072923e+16 -4.22882040e+13 -
1.58593766e+12 -5.45783141e+10], b: -87734873915.50308
Iteration 27, Cost: 4.308206565767179e+38, m: [1.84386991e+15 2.87648727e+12 1.07877268e+11
3.71247858e+09], b: 5967825334.056229
```

```
Iteration 28, Cost: 1.3167251080243498e+37, m: [-3.22351412e+14 -5.02882460e+11 -
1.88595447e+10 -6.49030554e+08], b: -1043319297.9417276
Iteration 29, Cost: 1.1250438721083714e+36, m: [9.42251305e+13 1.46990577e+11 5.51265712e+09
1.89712180e+08], b: 304963242.9203994
Iteration 30, Cost: 1.999121463458819e+35, m: [-3.97193021e+13 -6.19659326e+10 -
2.32388767e+09 -7.99739595e+07], b: -128558382.68057543
Iteration 31, Cost: 6.352132398897435e+34, m: [2.23893657e+13 3.49261842e+10 1.30983827e+09
4.50767539e+07], b: 72461175.35443851
Iteration 32, Cost: 3.2934831683466013e+34, m: [-1.61216594e+13 -2.51516105e+10 -
9.43285410e+08 -3.24620394e+07], b: -52182767.394184604
Iteration 33, Cost: 2.6208859407853697e+34, m: [1.43815627e+13 2.24347798e+10 8.41334248e+08
2.89536990e+07], b: 46543295.334203824
Iteration 34, Cost: 3.064201606768887e+34, m: [-1.55503685e+13 -2.42596154e+10 -
9.09861491e+08 -3.13117919e+07], b: -50333739.65345676
Iteration 35, Cost: 5.05976026176973e+31, m: [6.31898012e+11 9.85313936e+08 3.68972101e+07
1.26987585e+06], b: 2041430.9389074743
Iteration 36, Cost: 1.05940429264371e+30, m: [-9.14350864e+10 -1.43014598e+08 -5.42207078e+06
-1.86496054e+05], b: -299693.1517055435
Iteration 37, Cost: 7.115869042255265e+28, m: [2.36971436e+10 3.66770631e+07 1.31384954e+06
4.53132725e+04], b: 72941.32783062349
Iteration 38, Cost: 1.055220608733595e+28, m: [-9.12543622e+09 -1.44568450e+07 -
6.06465324e+05 -2.07722882e+04], b: -33291.500770278915
Iteration 39, Cost: 2.892900304765641e+27, m: [4.77802893e+09 7.29164920e+06 2.06970830e+05
7.22121264e+03], b: 11708.165992209615
Iteration 40, Cost: 1.3218060367855156e+27, m: [-3.22972733e+09 -5.15329906e+06 -
2.61529708e+05 -8.90173666e+03], b: -14209.54934607969
Iteration 41, Cost: 9.405648707176155e+26, m: [2.72443478e+09 4.17227655e+06 8.68248000e+04
3.08649159e+03], b: 5061.574326429305
Iteration 42, Cost: 9.937888892560422e+26, m: [-2.80045807e+09 -4.41917640e+06 -
2.36413663e+05 -8.03741753e+03], b: -12820.161499050333
7.02840257e+04 -2.32025116e+03], b: -3629.7937925412825
Iteration 44, Cost: 2.573929465591671e+21, m: [-4.50684712e+06 -4.04396222e+04 -
7.28334385e+04 -2.40799021e+03], b: -3770.8365156652167
Iteration 45, Cost: 1.3250825911008849e+20, m: [1022581.02627742 -24668.12062785 -
72509.6916343 -2396.85256706], b: -3752.93457564294
Iteration 46, Cost: 1.6199481751689216e+19, m: [-357412.57419683 -20642.02083468 -
72590.18188679 -2399.62626118], b: -3757.395244946362
[CONTEXT ratelimit period="5 MINUTES [skipped: 11]"]
Iteration 47, Cost: 3.805904552407058e+18, m: [173220.90326966 -14616.41681331 -
72558.88261306 -2398.5528171 ], b: -3755.671705226879
Iteration 48, Cost: 1.525432535898835e+18, m: [-109607.11023038 -10819.69071639 -
72575.16715977 -2399.11692177], b: -3756.580630865344
Iteration 49, Cost: 9.668078649616465e+17, m: [87310.03685172 -7177.7890024 -72563.3726641
-2398.71475833], b: -3755.9363956638376
Iteration 50, Cost: 9.201627450789531e+17, m: [-85179.34391764 -4927.3247565 -72573.17639541
-2399.05596398], b: -3756.487273210688
[CONTEXT ratelimit_period="5 MINUTES [skipped: 11]"]
```

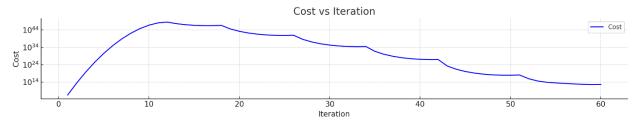
```
Iteration 51, Cost: 1.2666466520842488e+18, m: [ 99980.34951036 -2821.59408051 -
72562.03764286 -2398.67660339], b: -3755.880009207661
Iteration 52, Cost: 9660310018079194.0, m: [-8664.32029398 -2426.84273255 -72568.24502728 -
2398.89212252], b: -3756.227712844304
Iteration 53, Cost: 445849806920755.7, m: [ 1702.01961324 -1907.58818955 -72567.47600656 -
2398.86771714], b: -3756.1898541065125
Iteration 54, Cost: 85243361221846.03, m: [ -527.23676057 -1472.99562214 -72567.42896757 -
2398.86833547], b: -3756.1923502964314
Iteration 55, Cost: 35210825368322.54, m: [ 244.29948244 -1100.37455424 -72567.18987996 -
2398.8625411 l, b: -3756.1846805126147
Iteration 56, Cost: 17720952096710.367, m: [ -135.36945611 -795.42626237 -72566.99992693 -
2398.85865558], b: -3756.180236428806
[CONTEXT ratelimit period="5 MINUTES [skipped: 11]" ]
Iteration 57, Cost: 9383145192232.69, m: [ 110.80963556 -552.16882835 -72566.75320676 -
2398.85305861], b: -3756.173214834645
Iteration 58, Cost: 5403794061119.298, m: [ -92.94971855 -366.83743261 -72566.51051062 -
2398.84786937], b: -3756.1670404798792
Iteration 59, Cost: 3909671220976.619, m: [ 113.9229461 -230.83744453 -72566.21911202 -
2398.84130345], b: -3756.158865024209
Iteration 60, Cost: 4303293259074.533, m: [ -140.8427138 -137.08730976 -72565.92742287 -
2398.83506014], b: -3756.151442222664
Final parameters: m = [ -140.8427138 -137.08730976 -72565.92742287 -2398.83506014], b = -
3756.151442222664
```

• What are m and b values based on your calculations?

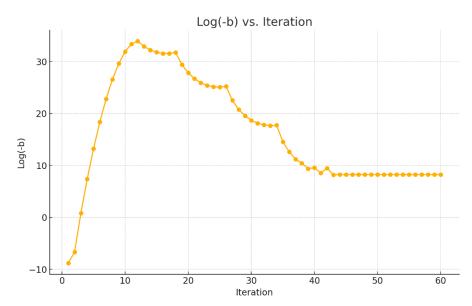
• Comment on how you can interpret the parameters of the model. What is the meaning of m_i and b in this case

About m values, these represent the coefficients for the independent variables (or features) in the regression model. Each element in the m vector corresponds to the weight (slope) assigned to each feature, which quantifies how much that feature contributes to the prediction of the dependent variable. A positive m means that the feature has a positive relationship with the dependent variable, while a negative m indicates a negative relationship. Regarding b value, this represents the intercept of the regression model. It is the value of the dependent variable when all independent variables are equal to zero. Essentially, it acts as the baseline value of the prediction when no other features contribute to the outcome. Based on the final results of the iterations: m=[-140.84, -137.08, -72565.92, -2398.83]: Each value corresponds to the slope for a specific feature. The larger negative values, like -72565.92, indicate a significant negative impact of the associated feature on the output. (m0: trip_time_in_secs, m1= trip_distance, m2= fare_amount, m3= tolls_amount). b=-3756.15: This is the intercept, indicating the baseline prediction of the dependent

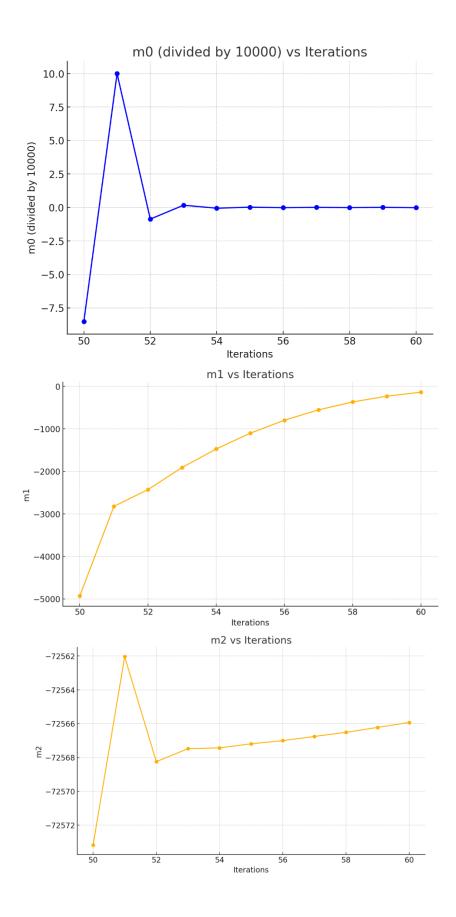
variable when all features are zero. The iterations indicate that the model adjusts these parameters over time to minimize the cost, eventually reaching the final set of values. But still we need more iteration for it, however due to credit I had to limit by 60 iterations.

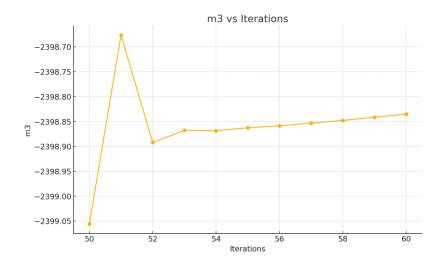


Above trend shows that if we have some more iterations like iteration=500, cost might decrease to near zero well (credit limitation)



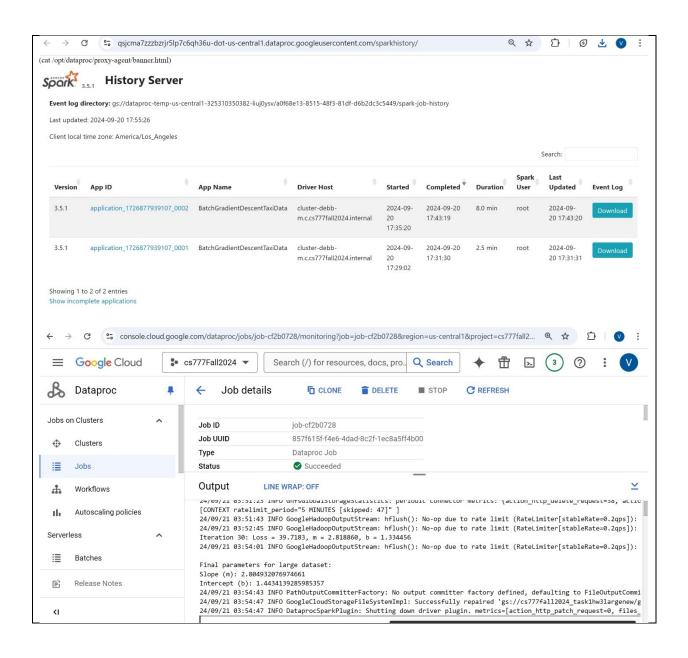
I want to show the convergence of method here to specific value. It needs more iterations.

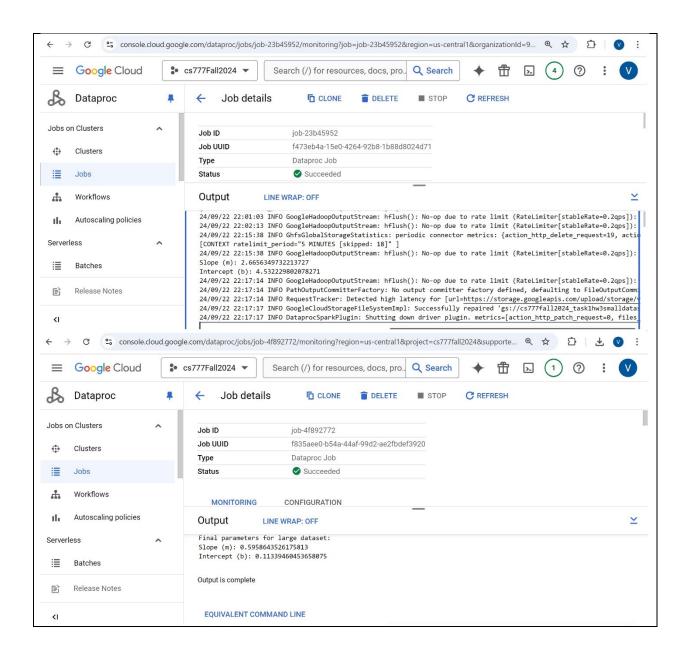


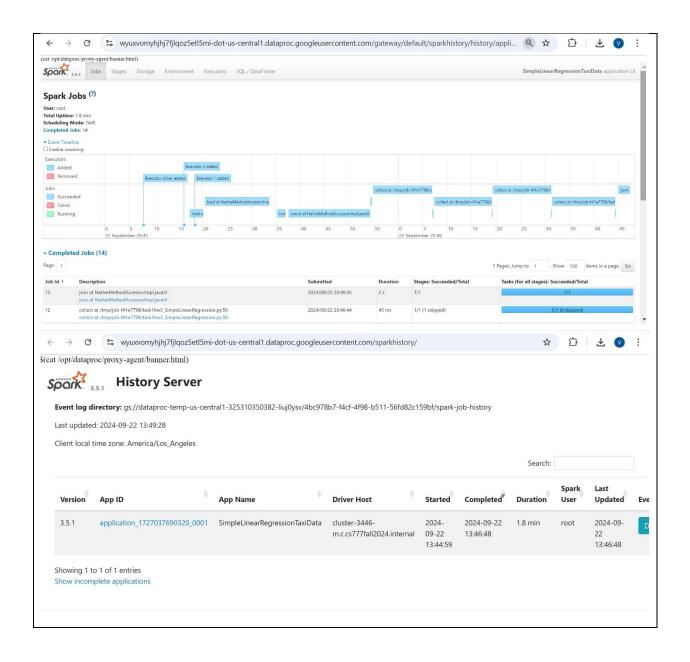


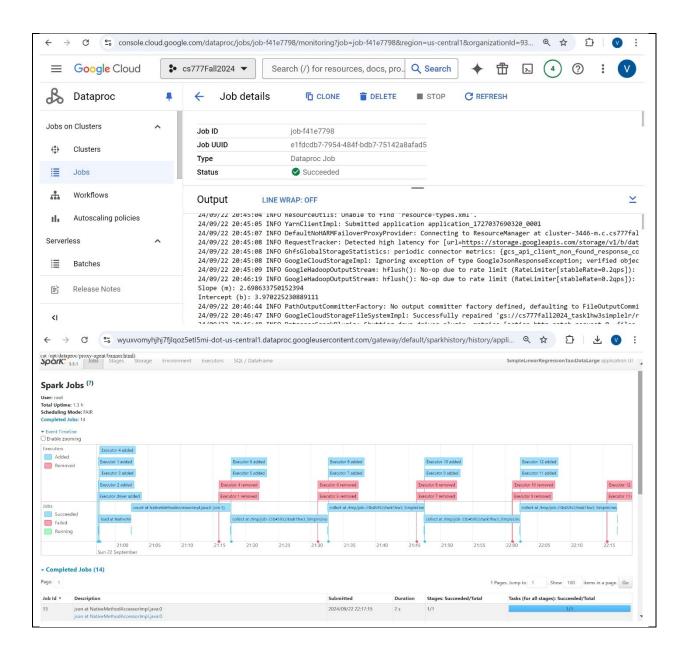
Spark History Output:

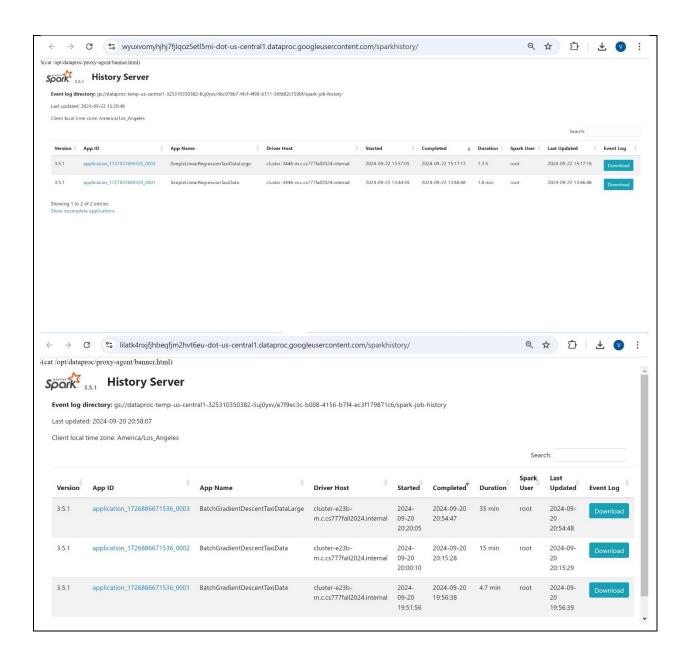
Simple Linear Regression

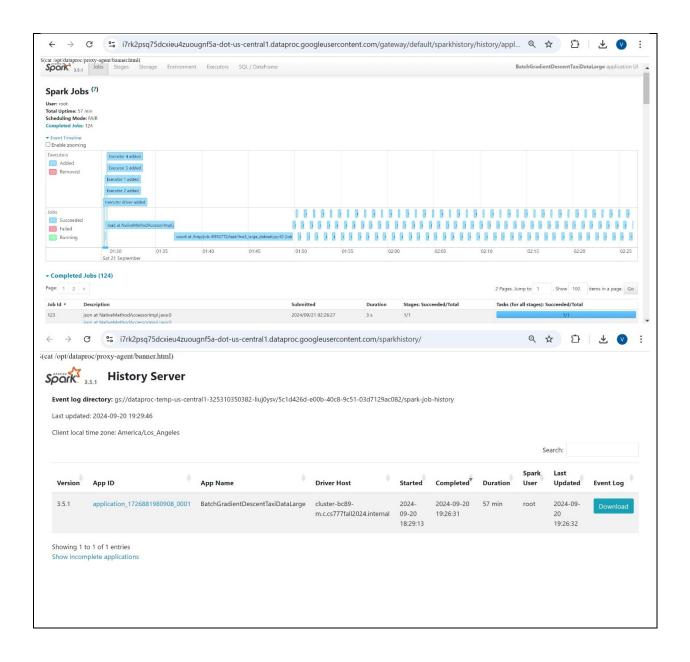


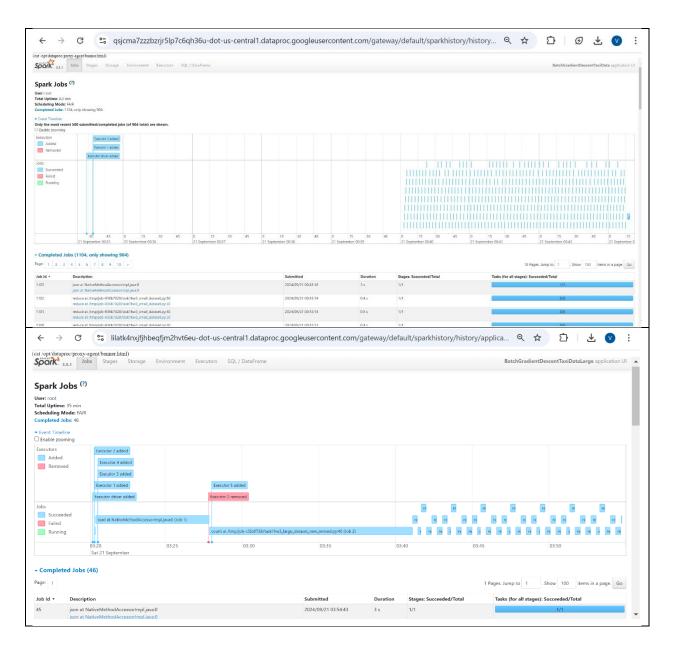




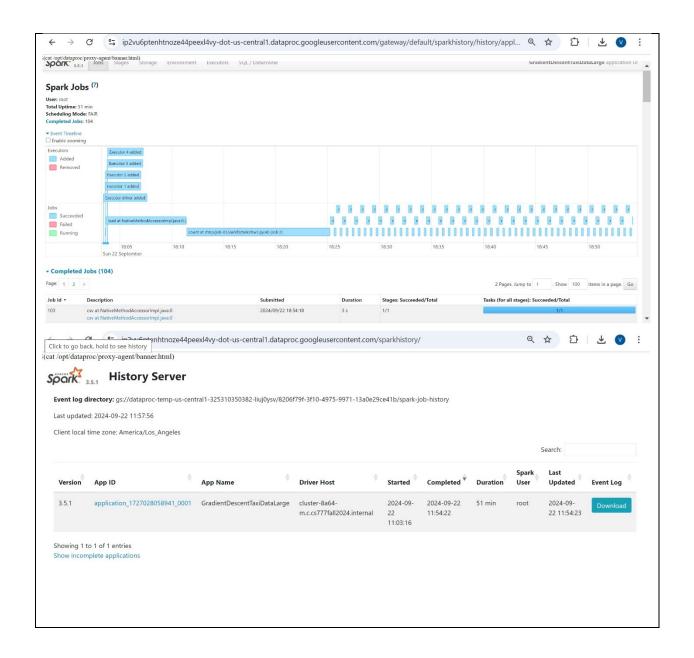








SGD





SLR/SGD

