Pattern Recognition – Homework 3

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Question 16

Problem Setup

This is a three-class problem in four dimensions with equal priors $P(\omega_i) = 1/3$ for $i = 1, 2, 3$. The class-conditional densities are multivariate Gaussians with common covariance

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 30 \end{pmatrix}.$$

The means are

$$\boldsymbol{\mu}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \quad \boldsymbol{\mu}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}, \quad \boldsymbol{\mu}_3 = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 2 \end{pmatrix}.$$

We classify the point

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}.$$

Since priors are equal and Σ is shared, the discriminant is

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}_i)$$

. Classify to the class with maximum g_i.

First, $\Sigma^{-1} = \text{diag}(1, 1/10, 1/20, 1/30)$.

Calculations

For ω_1:

$$\mathbf{x} - \boldsymbol{\mu}_1 = \begin{pmatrix} 0 \\ -1 \\ -2 \\ -2 \end{pmatrix},$$

$$(\mathbf{x} - \boldsymbol{\mu}_1)^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}_1) = 0^2 \cdot 1 + (-1)^2 \cdot \frac{1}{10} + (-2)^2 \cdot \frac{1}{20} + (-2)^2 \cdot \frac{1}{30} = \frac{1}{10} + \frac{4}{20} + \frac{4}{30} = \frac{1}{10} + \frac{1}{5} + \frac{2}{15}.$$

Common denominator 30: $\frac{3}{30} + \frac{6}{30} + \frac{4}{30} = \frac{13}{30}$.

$$g_1(\mathbf{x}) = -\frac{1}{2} \cdot \frac{13}{30} = -\frac{13}{60}.$$

For ω_2:

$$\mathbf{x} - \boldsymbol{\mu}_2 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ -1 \end{pmatrix},$$

$$(-1)^2 \cdot 1 + 0 + 0 + (-1)^2 \cdot \frac{1}{30} = 1 + \frac{1}{30} = \frac{31}{30},$$

$$g_2(\mathbf{x}) = -\frac{1}{2} \cdot \frac{31}{30} = -\frac{31}{60}.$$

For ω_3:

$$\mathbf{x} - \boldsymbol{\mu}_3 = \begin{pmatrix} -2 \\ -3 \\ 0 \\ 0 \end{pmatrix},$$

$$(-2)^2 \cdot 1 + (-3)^2 \cdot \frac{1}{10} + 0 + 0 = 4 + \frac{9}{10} = \frac{49}{10},$$

$$g_3(\mathbf{x}) = -\frac{1}{2} \cdot \frac{49}{10} = -\frac{49}{20}.$$

Decision

Compare: $g_1 = -13/60 \approx -0.217$, $g_2 = -31/60 \approx -0.517$, $g_3 = -49/20 = -2.45$. The maximum is g_1, so classify $\mathbf{x}$ to ω_1.