Pattern Recognition – Homework 3

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Question 17

Dataset

We are given five movies with three binary features and their class labels:

great	fine	terrible	class
1	0	0	+
0	1	1	–
0	1	1	–
0	0	0	+
1	0	1	–

A new movie is observed with only the feature `terrible = 1`. We use the Naïve Bayes classifier to predict its class.

Naïve Bayes Decision Rule

We compute:

$$\text{Predicted class} = \arg\max_{c \in \{+,-\}} P(c) \cdot P(\text{terrible} = 1 \mid c)$$

Step 1: Prior Probabilities

There are 2 samples in class + and 3 in class −.

$$P(+) = \frac{2}{5}, \quad P(-) = \frac{3}{5}$$

Step 2: Class-Conditional Probabilities

For class + (2 samples):
- terrible = 1 appears in 0 of them.

$$P(\text{terrible} = 1 \mid +) = \frac{0}{2} = 0$$

For class − (3 samples):
- terrible = 1 appears in all 3.

$$P(\text{terrible} = 1 \mid -) = \frac{3}{3} = 1$$

Step 3: Posterior Scores (Unnormalized)

$$\text{Score}(+) = P(+) \cdot P(\text{terrible} = 1 \mid +) = \frac{2}{5} \cdot 0 = 0$$

$$\text{Score}(-) = P(-) \cdot P(\text{terrible} = 1 \mid -) = \frac{3}{5} \cdot 1 = \frac{3}{5}$$

Since $\frac{3}{5} > 0$, the Naïve Bayes classifier assigns the new movie to class $-$.