HW3

Parts A, C and D)

Answer:

(a) $p(\omega_i|\mathbf{x}) = \frac{p(\mathbf{x}|\omega_i)P(\omega_i)}{p(\mathbf{x})}$. Since $P(\omega_1) = P(\omega_2) = P(\omega_3) = 1/3$, we just need to compare the likelihood $p(\mathbf{x}|\omega_i)$.

$$p(\mathbf{x}|\omega_1) = \frac{1}{2\pi}\exp\left[-\frac{1}{2}[0.3,0.3][0.3,0.3]^t\right] = 0.1455$$

$$p(\mathbf{x}|\omega_2) = \frac{1}{2\pi}\exp\left[-\frac{1}{2}[-0.7,-0.7][-0.7,-0.7]^t\right] = 0.0975$$

$$p(\mathbf{x}|\omega_3) = \frac{1}{4\pi}\exp\left[-\frac{1}{2}[-0.2,-0.2][-0.2,-0.2]^t\right]$$
$$+\frac{1}{4\pi}\exp\left[-\frac{1}{2}[0.8,-0.2][0.8,-0.2]^t\right]$$
$$= 0.1331$$

Since $p(\mathbf{x}|\omega_1)$ is the largest likelihood, $\mathbf{x}$ is classified to be from class 1.

(b) Assume that $\mathbf{x} = [x_1, x_2]^t$,

$$p(\omega_i|x_2) = \frac{\int p(x_1, x_2|\omega_i)P(\omega_i)dx_1}{p(x_2)}$$

Still since $P(\omega_1) = P(\omega_2) = P(\omega_3) = 1/3$, we just need to compare $\int p(x_1, x_2|\omega_i)dx_1$.

$$\int p(x_1, x_2|\omega_1)dx_1 = \frac{1}{2\pi}\int \exp\left[-\frac{1}{2}(x_1^2 + 0.09)\right]dx_1$$
$$= \frac{1}{\sqrt{2\pi}}\exp(-\frac{0.09}{2}) = 0.3813$$

$$\int p(x_1, x_2|\omega_2)dx_1 = \frac{1}{2\pi}\int \exp\left[-\frac{1}{2}\left((x_1-1)^2 + 0.49\right)\right]dx_1$$
$$= \frac{1}{\sqrt{2\pi}}\exp(-\frac{0.49}{2}) = 0.3122$$

$$\int p(x_1, x_2|\omega_3)dx_1 = \frac{1}{4\pi}\int \exp\left[-\frac{1}{2}\left((x_1-0.5)^2 + 0.04\right)\right]dx_1$$
$$+\frac{1}{4\pi}\int \exp\left[-\frac{1}{2}\left((x_1+0.5)^2 + 0.04\right)\right]dx_1$$
$$= \frac{1}{2\sqrt{2\pi}}\exp(-\frac{0.04}{2}) + \frac{1}{2\sqrt{2\pi}}\exp(-\frac{0.04}{2})$$
$$= 0.3910$$

Thus $\mathbf{x}$ is classified to be from class 3.

(c) Still $\mathbf{x} = [x_1, x_2]^t$,

$$p(\omega_i|x_1) = \frac{\int p(x_1, x_2|\omega_i)P(\omega_i)dx_2}{p(x_1)}$$

We need to compare $\int p(x_1, x_2|\omega_i)dx_2$.

$$
\begin{aligned}
\int p(x_1, x_2|\omega_1)dx_2 &= \frac{1}{2\pi}\int \exp\left[-\frac{1}{2}(x_2^2 + 0.09)\right] dx_2 \\
&= \frac{1}{\sqrt{2\pi}}\exp(-\frac{0.09}{2}) = 0.3813 \\
\int p(x_1, x_2|\omega_2)dx_2 &= \frac{1}{2\pi}\int \exp\left[-\frac{1}{2}\left((x_2 - 1)^2 + 0.49\right)\right] dx_2 \\
&= \frac{1}{\sqrt{2\pi}}\exp(-\frac{0.49}{2}) = 0.3122 \\
\int p(x_1, x_2|\omega_3)dx_2 &= \frac{1}{4\pi}\int \exp\left[-\frac{1}{2}\left((x_2 - 0.5)^2 + 0.04\right)\right] dx_2 \\
&\quad + \frac{1}{4\pi}\int \exp\left[-\frac{1}{2}\left((x_2 - 0.5)^2 + 0.64\right)\right] dx_2 \\
&= \frac{1}{2\sqrt{2\pi}}\exp(-\frac{0.04}{2}) + \frac{1}{2\sqrt{2\pi}}\exp(-\frac{0.64}{2}) \\
&= 0.3403
\end{aligned}
$$

x belongs to class 1.

Part B) Bayes Error $= \frac{1}{3}(0.097) + \frac{1}{3}(0.133) = 0.077$

Q4:

Part A.

In general: $(M_2 - M_1)^T \Sigma^{-1}X + \frac{1}{2}(M_1^T\Sigma^{-1}M_1 - M_2^T\Sigma^{-1}M_2) \gtrless_{w_2}^{w_1} \ln\frac{P_2(\lambda_{21} - \lambda_{22})}{P_1(\lambda_{12} - \lambda_{11})}$ if $P_2 = P_1 = \frac{1}{2}$:
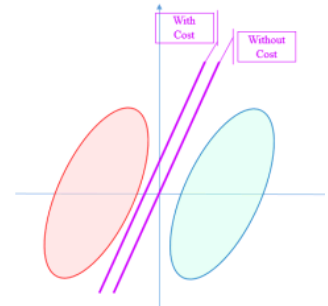
$[-2 \ 0]^T \frac{4}{3}\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2}\left([1 \ 0]\frac{4}{3}\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - [-1 \ 0]\frac{4}{3}\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = 0$

$\frac{4}{3}(-2x_1 + x_2) + \frac{1}{2}\left(\frac{4}{3} - \frac{4}{3}\right) = 0 \quad \rightarrow \quad 2x_1 = x_2$,

Part B. considering cost: $\frac{4}{3}(-2x_1 + x_2) = \ln\frac{1}{2} = -0.7 \rightarrow \quad 2x_1 - 0.5 = x_2$

The borders are given in this image:

Blue is class w_1 and red is class w_2.

Part C.

Bayes Error $= \frac{1}{2}(2\lambda_{21})\int\int N(M_1, \Sigma)dX + \frac{1}{2}(\lambda_{21})\int\int N(M_2, \Sigma)dX$

Q5:

Part A)

$$p(x|w_1) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{1}{2}x^t\Sigma_1^{-1}x\right) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{1}{2\sigma_n^4}[x_1\; x_2]\begin{bmatrix}\sigma_n^2 & 0 \\ 0 & \sigma_n^2\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}\right) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{1}{2\sigma_n^4}(x_1^2\sigma_n^2 + x_2^2\sigma_n^2)\right) =$$

$$\frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{1}{2\sigma_n^2}(x_1^2 + x_2^2)\right)$$

$$p(x|w_2) = \frac{1}{\sqrt{2\pi\sigma_n^2(\sigma_n^2+\sigma_s^2)}} \exp\left(-\frac{1}{2}x^t\Sigma_2^{-1}x\right) = \frac{1}{\sqrt{2\pi\sigma_n^2(\sigma_n^2+\sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2+\sigma_s^2)}[x_1\; x_2]\begin{bmatrix}\sigma_n^2 & 0 \\ 0 & \sigma_n^2 + \sigma_s^2\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}\right) =$$

$$\frac{1}{\sqrt{2\pi\sigma_n^2(\sigma_n^2+\sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2+\sigma_s^2)}(x_1^2\sigma_n^2 + x_2^2(\sigma_n^2 + \sigma_s^2))\right)$$

$$p(x|w_3) = \frac{1}{\sqrt{2\pi\sigma_n^2(\sigma_n^2+\sigma_s^2)}} \exp\left(-\frac{1}{2}x^t\Sigma_3^{-1}x\right) = \frac{1}{\sqrt{2\pi\sigma_n^2(\sigma_n^2+\sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2+\sigma_s^2)}[x_1\; x_2]\begin{bmatrix}\sigma_n^2 + \sigma_s^2 & 0 \\ 0 & \sigma_n^2\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}\right) =$$

$$\frac{1}{\sqrt{2\pi\sigma_n^2(\sigma_n^2+\sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2+\sigma_s^2)}(x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)\right)$$

$$e_1 = \lambda_{11}p(w_1)p(x|w_1) + \lambda_{12}p(w_2)p(x|w_2) + \lambda_{13}p(w_3)p(x|w_3) = pp(x|w_2) + (1 - 2p)p(x|w_3)$$

$$e_2 = \lambda_{21}p(w_1)p(x|w_1) + \lambda_{22}p(w_2)p(x|w_2) + \lambda_{23}p(w_3)p(x|w_3) = pp(x|w_1) + a(1 - 2p)p(x|w_3)$$

$$e_2 = \lambda_{31}p(w_1)p(x|w_1) + \lambda_{32}p(w_2)p(x|w_2) + \lambda_{33}p(w_3)p(x|w_3) = pp(x|w_1) + app(x|w_2)$$

w_1 vs. w_2, w_3:

$$e_1 < e_2: \; pp(x|w_2) + (1 - 2p)p(x|w_3) < pp(x|w_1) + a(1 - 2p)p(x|w_3):$$

$$\frac{p}{\sqrt{2\pi\sigma_n^2(\sigma_n^2 + \sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2 + \sigma_s^2)}(x_1^2\sigma_n^2 + x_2^2(\sigma_n^2 + \sigma_s^2))\right)$$

$$+ \frac{1 - 2p}{\sqrt{2\pi\sigma_n^2(\sigma_n^2 + \sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2 + \sigma_s^2)}(x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)\right)$$

$$< \frac{p}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{1}{2\sigma_n^2}(x_1^2 + x_2^2)\right) + \frac{a(1 - 2p)}{\sqrt{2\pi\sigma_n^2(\sigma_n^2 + \sigma_s^2)}} \exp\left(-\frac{1}{2\sigma_n^2(\sigma_n^2 + \sigma_s^2)}(x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)\right)$$

Use natural logarithm:

$$\ln a - \frac{1}{2\sigma_n^2(\sigma_n^2 + \sigma_s^2)}\left(x_1^2\sigma_n^2 + x_2^2(\sigma_n^2 + \sigma_s^2)\right) + \ln b - \frac{1}{2\sigma_n^2(\sigma_n^2 + \sigma_s^2)}(x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)$$

$$< \ln c - \frac{1}{2\sigma_n^2}(x_1^2 + x_2^2) + \ln d - \frac{1}{2\sigma_n^2(\sigma_n^2 + \sigma_s^2)}(x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)$$

$$2\sigma_n^2(\sigma_n^2 + \sigma_s^2)\ln a - \left(x_1^2\sigma_n^2 + x_2^2(\sigma_n^2 + \sigma_s^2)\right) + 2\sigma_n^2(\sigma_n^2 + \sigma_s^2)\ln b - (x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)$$

$$< 2\sigma_n^2(\sigma_n^2 + \sigma_s^2)\ln c - (\sigma_n^2 + \sigma_s^2)(x_1^2 + x_2^2) + 2\sigma_n^2(\sigma_n^2 + \sigma_s^2)\ln d - (x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)$$

$$\acute{a} - \left(x_1^2\sigma_n^2 + x_2^2(\sigma_n^2 + \sigma_s^2)\right) + \acute{b} - (x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2) < \acute{c} - (\sigma_n^2 + \sigma_s^2)(x_1^2 + x_2^2) + \acute{d} - (x_1^2(\sigma_n^2 + \sigma_s^2) + x_2^2\sigma_n^2)$$
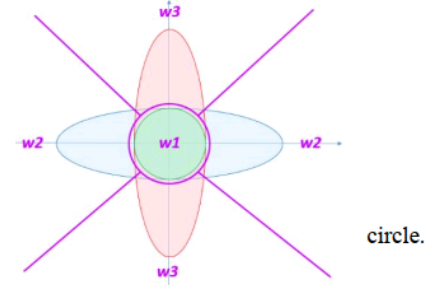
$$\acute{a} - x_1^2\sigma_n^2 - x_2^2\sigma_n^2 - x_2^2\sigma_s^2 + \acute{b} - x_2^2\sigma_n^2 - x_2^2\sigma_s^2 - x_2^2\sigma_n^2$$
$$< \acute{c} - x_1^2\sigma_n^2 - x_1^2\sigma_s^2 - x_2^2\sigma_n^2 - x_2^2\sigma_s^2 + \acute{d} - x_2^2\sigma_n^2 - x_2^2\sigma_s^2 - x_2^2\sigma_n^2$$

$$\acute{a} + \acute{b} < \acute{c} - x_1^2\sigma_s^2 + \acute{d} \qquad \rightarrow \qquad x_1^2 < \acute{f} \qquad \rightarrow \qquad Ax_1^2 < C$$

circle.

$$\rightarrow \qquad \left(\frac{\sigma_s^2}{\sigma_n^2(\sigma_s^2+\sigma_n^2)}\right)x_1^2 < \ln(a^2\,\frac{\sigma_s^2+\sigma_n^2}{\sigma_n^2})$$

$e_1 < e_3$: $pp(x|w_2) + (1 - 2p)p(x|w_3) < pp(x|w_1) +$
$app(x|w_2) \qquad \rightarrow \qquad \left(\frac{\sigma_s^2}{\sigma_n^2(\sigma_s^2+\sigma_n^2)}\right)x_2^2 < \ln(\frac{a^2}{(1-2p)^2} \times \frac{\sigma_s^2+\sigma_n^2}{\sigma_n^2})$

$Ax_1^2 < C$ and $Ax_2^2 < D$: $Ax_1^2 + Ax_2^2 < C + D \quad \rightarrow \quad x_1^2 + x_2^2 < B : A$

If we calculate like above for w_2 and w_3 we will get: $x_2 = \pm x_1$

Final borders are plotted here:

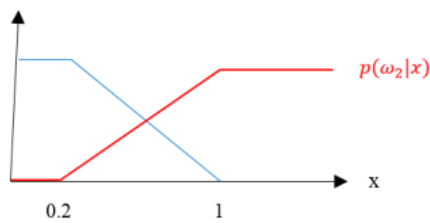Green area belongs to class w_1. Blue and red areas belong to

Classes w_2 and w_3, respectively.

Part B)

$$\text{Error} = \int_{R_1}[pp(x|w_2) + (1 - 2p)p(x|w_3)]dx + \int_{R_2}[pp(x|w_1) + a(1 - 2p)p(x|w_3)]dx +$$

$$\int_{R_3}[pp(x|w_1) + app(x|w_2)]dx$$

Q6:

Part A) Considering the shape, posterior is defined just for positive xs.

$$p(\omega_1|x) = \begin{cases} 1 & 0 \leq x \leq 0.2 \\ \frac{-5}{4}x + \frac{5}{4} & 0.2 < x \leq 1 \end{cases}$$

$$p(\omega_2|x) = \begin{cases} 1 & 1 \leq x \\ \frac{5}{4}x - \frac{1}{4} & 0.2 < x < 1 \end{cases}$$

Part B) $p(\omega_1|x) \underset{\omega_1}{\overset{\omega_2}{\lessgtr}} p(\omega_2|x)$ Therefore, $R_2 = \{x|x > 0.6\}$ and $R_1 = \{x|x < 0.6\}$ where 0.6 is the Bayes boundary:

$$\frac{-5}{4}x + \frac{5}{4} = \frac{5}{4}x - \frac{1}{4} \qquad \longrightarrow \qquad x = 0.6$$

Part C) $R(t) = \{x | r(x) \ge t\}$

$$r(x) = \min\{p(\omega_1|x), p(\omega_2|x)\} = \begin{cases} 0 & x \le 0.2 \ \cup \ 1 < x \\ \frac{5}{4}x - \frac{1}{4} & 0.2 < x \le 0.6 \\ \frac{-5}{4}x + \frac{5}{4} & 0.6 < x \le 1 \end{cases}$$

in $0.2 < x \le 0.6$: $\quad \frac{5}{4}x - \frac{1}{4} \ge t \quad \longrightarrow \quad \frac{5}{4}x - \frac{1}{4} \ge \frac{1}{4} \quad \longrightarrow \quad x > 0.4$

in $0.6 < x \le 1$: $\quad \frac{-5}{4}x + \frac{5}{4} \ge t \quad \longrightarrow \quad \frac{-5}{4}x + \frac{5}{4} \ge \frac{1}{4} \quad \longrightarrow \quad x < 0.8$

So, $R\left(\frac{1}{4}\right) = \{0.4 < x < 0.8\}$

Part D) We classify x to class i if $c_i(x)$ is smaller than λ. Considering the costs:

in $0.2 < x \le 0.6$, *we have* $c_1(x) = \frac{5}{4}x - \frac{1}{4}$ $\qquad \frac{5}{4}x - \frac{1}{4} \le \lambda \qquad x \le \frac{4}{5}\lambda + \frac{1}{5}$

in $0.6 < x \le 1$, *we have* $c_2(x) = \frac{-5}{4}x + \frac{5}{4}$ $\qquad \frac{-5}{4}x + \frac{5}{4} \le \lambda \qquad x \ge \frac{-4}{5}\lambda + 1$

$$d(x) = \begin{cases} 1 & x \le \frac{4}{5}\lambda + \frac{1}{5} \\ 2 & x \ge \frac{-4}{5}\lambda + 1 \\ 0 & \frac{4}{5}\lambda + \frac{1}{5} < x < \frac{-4}{5}\lambda + 1 \end{cases}$$
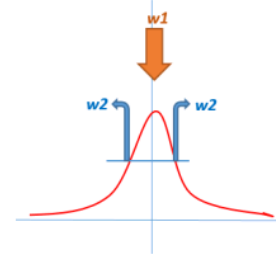
Part E) considering $d(x)$ above and $\lambda = \frac{1}{4}$: $\qquad \frac{4}{5}\left(\frac{1}{4}\right) + \frac{1}{5} < x < \frac{-4}{5}\left(\frac{1}{4}\right) + 1 \quad \longrightarrow \quad 0.4 < x < 0.8$

We get the same reject region from part C and part D.

Q7:

Part A) $p(\omega_1|x) \underset{\omega_1}{\overset{\omega_2}{\lessgtr}} p(\omega_2|x)$ $\qquad \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) = \frac{1}{4} \quad \rightarrow \quad x = \pm 1.36$

The borders are specified in the image:

Part B) let $p(\omega_1) = p$

$pp(x|\omega_1) > (1-p)p(x|\omega_1) \quad \rightarrow \quad \frac{p}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) > \frac{1-p}{4}$

$p\left(\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{4}\right) > \frac{1}{4} \quad \rightarrow \quad p > \frac{\frac{1}{4}}{\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{4}}$ $\qquad$ This inequality should be correct for all x. Therefore I put

$x = \pm 2$ in it. Then, $p > \frac{\frac{1}{4}}{\frac{1}{\sqrt{2\pi}}\exp(-2) + \frac{1}{4}} \quad \rightarrow \quad p > 0.82$

Part C) It is impossible! The domain of $p(x|\omega_2)$ is in range [-2 2]. Changing its prior does not extend the domain so there is always some areas out of this range that $p(x|\omega_1)$ is greater than $p(x|\omega_2) = 0$ for them.

Q10:

$$p(\omega_2|x) = \frac{1}{2}x \qquad\qquad p(\omega_1|x) = -\frac{1}{2}x + 1$$

Part A)

$$\varepsilon = \int_0^1 P(\omega_2|x)p(x)dx + \int_1^2 P(\omega_1|x)p(x)dx = \int_0^1 0.5xp(x)dx + \int_1^2 0.5(1-0.5x)p(x)dx$$

Part B)

Let's re-arrange the table:

تصمیم ＼ تشخیص	عدم وجود خرابی	وجود خرابی
ادامه کار با ظرفیت کامل	0	2ϵ
ادامه کار با ظرفیت محدود	2ϵ	ϵ
توقف کامل	4ϵ	0

$$D_1 \; vs. \, D_2 : 0 \times p(\omega_2|x) + 2\epsilon \times p(\omega_1|x) \underset{D_1}{\overset{D_2}{\gtrless}} 2\epsilon \times p(\omega_2|x) + \epsilon \times p(\omega_1|x)$$

$$2\left(-\frac{1}{2}x + 1\right) \underset{D_1}{\overset{D_2}{\gtrless}} 2\left(\frac{1}{2}x\right) + \left(-\frac{1}{2}x + 1\right) \quad\rightarrow\quad \frac{2}{3} \underset{D_1}{\overset{D_2}{\gtrless}} x$$

$$D_1 \; vs. \, D_3 : 0 \times p(\omega_2|x) + 2\epsilon \times p(\omega_1|x) \underset{D_1}{\overset{D_3}{\gtrless}} 4\epsilon \times p(\omega_2|x) + 0 \times p(\omega_1|x)$$

$$2\left(-\frac{1}{2}x + 1\right) \underset{D_1}{\overset{D_3}{\gtrless}} 4\left(\frac{1}{2}x\right) \quad\rightarrow\quad 1 \underset{D_1}{\overset{D_3}{\gtrless}} x$$

$$D_2 \text{ vs. } D_3: 2\epsilon \times p(\omega_2|x) + \epsilon \times p(\omega_1|x) \gtrless_{D_2}^{D_3} 4\epsilon \times p(\omega_2|x) + 0 \times p(\omega_1|x)$$

$$2\left(\tfrac{1}{2}x\right) + \left(-\tfrac{1}{2}x + 1\right) \gtrless_{D_2}^{D_3} 4\left(\tfrac{1}{2}x\right) \rightarrow \tfrac{2}{3} \gtrless_{D_2}^{D_3} x$$

$$\text{Therefore: } \begin{cases} D_1 & \text{if } x > 1 & \rightarrow & \text{full capacity} \\ D_2 & \text{if } \tfrac{2}{3} < x < 1 & \rightarrow & \text{Limited capacity} \\ D_3 & \text{if } \tfrac{2}{3} > x & \rightarrow & \text{stop working} \end{cases}$$

Q11:

$$P(C_1) = P(C_2) = 0.5$$

$X_1 =$ \ Class	C_1	C_2
-1	0.2	0.3
0	0.4	0.6
1	0.4	0.1

$X_2 =$ \ Class	C_1	C_2
-1	0.4	0.1
0	0.5	0.3
1	0.1	0.6

Part A)

$$P(C_1|X_1 = -1, X_2 = -1) = \frac{P(X_1=-1, X_2=-1|C_1)P(C_1)}{P(X_1=-1, X_2=-1)} = \frac{P(X_1=-1|C_1)P(X_2=-1|C_1)P(C_1)}{P(X_1=-1, X_2=-1)} = \frac{0.2 \times 0.4 \times 0.5}{(0.2 \times 0.4 \times 0.5)+(0.3 \times 0.1 \times 0.5)} = \frac{0.08}{0.11} = \frac{8}{11}$$

Part B)

$$\text{Error: } P(C_2|X_1 = -1, X_2 = -1) = 1 - P(C_1|X_1 = -1, X_2 = -1) = \frac{3}{11}$$

A) This is wrong for several reasons. First, to get the Bayes risk, you would need to scale each curve by its prior. Second, some of the shaded area has to be counted twice; e.g., the area under A where B and C are both above A.

B and C) The only way for something like this to occur is if the prior for a class is significant enough that its joint distribution raises above the other classes' joint distribution for all *x*. Class C can't do that, as it does not span all of the *x*-space. Class B can, as it covers the whole number line and its variance is greater than Class A's.

Q14-

$$p(w1) = p(w2) = \frac{1}{2} \quad \rightarrow \quad Ln(p(x|w1)) - Ln(p(x|w2)) = 0 \rightarrow$$

$$D = Ln\left(\frac{1}{2\times\pi^{\frac{1}{2}}\times|\Sigma|^{\frac{1}{2}}} \times e^{-\frac{1}{2}(x-\mu_1)^T\times\Sigma^{-1}\times(x-\mu_1)} \right) - Ln\left(\frac{1}{2\times\pi^{\frac{1}{2}}\times|\Sigma|^{\frac{1}{2}}} \times e^{-\frac{1}{2}(x-\mu_2)^T\times\Sigma^{-1}\times(x-\mu_2)} \right)$$

$$D = (\mu_1 - \mu_2)^T \Sigma^{-1} x - \frac{1}{2}\mu_1\Sigma^{-1}\mu_1 + \frac{1}{2}\mu_2^T\Sigma^{-1}\mu_2$$

یک متغیر تصادفی گوسی تحت هر تبدیل خطی به یک متغیر تصادفی گوسی دیگر با پارامتر های متفاوت تبدیل می شود.

$$\mu_1' = E\{D|w_1\} = E\{(\mu_1-\mu_2)^T\Sigma^{-1}x - \frac{1}{2}\mu_1\Sigma^{-1}\mu_1 + \frac{1}{2}\mu_2^T\Sigma^{-1}\mu_1\} = \frac{1}{2}(\mu_1-\mu_2)^T\Sigma^{-1}(\mu_1-\mu_2) = \frac{1}{2}dm^2$$

$$\mu_2' = E\{D|w_2\} = -\frac{1}{2}(\mu_1-\mu_2)^T\Sigma^{-1}(\mu_1-\mu_2) = -\frac{1}{2}dm^2$$

$$\sigma_1^2 = \sigma_2^2 = (\mu_1-\mu_2)^T\Sigma^{-1}(\mu_1-\mu_2) = dm^2$$

$$p_e = \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\sqrt{2\Pi}dm} e^{-\frac{\left(D-\frac{1}{2}dm^2\right)^2}{2dm^2}} dD + \frac{1}{2}\int_{0}^{\infty} \frac{1}{\sqrt{2\Pi}dm} e^{-\frac{\left(D+\frac{1}{2}dm^2\right)^2}{2dm^2}} dD$$

$$p_e = \frac{1}{\sqrt{2\Pi}dm}\int_{-\infty}^{0} e^{-\frac{1}{2}\left(\frac{D-\frac{1}{2}dm^2}{dm}\right)^2} dD$$

حالا کافی است تغییر متغیر بدهیم تا به عبارت مورد نظر برسیم.

$$z = \frac{D-\frac{1}{2}dm^2}{dm} \rightarrow dz = \frac{dD}{dm}$$

$$p_e = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{-\frac{1}{2}d_m} e^{-\frac{z^2}{2}} dz = \frac{1}{\sqrt{2\Pi}} \int_{\frac{1}{2}dm}^{0} e^{-\frac{z^2}{2}} dz$$