

Binary adder Turing machine

1. Description

This machine adds two non-negative binary numbers of equal bit length.

If the inputs differ in length, the shorter number is padded with leading zeros to match the length. Inputs are assumed to be preprocessed this way.

* - delimiter for input numbers.

^ - empty cell

2. Algorithm

Algorithm is the following:

The result is constructed by adding two binary numbers, bit by bit from left to right.

It is being constructed 2 cells back from the first inputting cell.

The carry bit is being written 2 cells back from built result and with every iteration when the result is being updated the place of carry bit is also updated with its value (if it exists).

In every iteration the head of the machine reads all cells from left to right till reach the first empty cell. Then it moves left and makes the first non-empty cell to empty. Moving to left it makes the first non-empty cell before * to *, and moves to the carry bit cell to change the value if its exists and needed, and also its place shifting it 1 cell

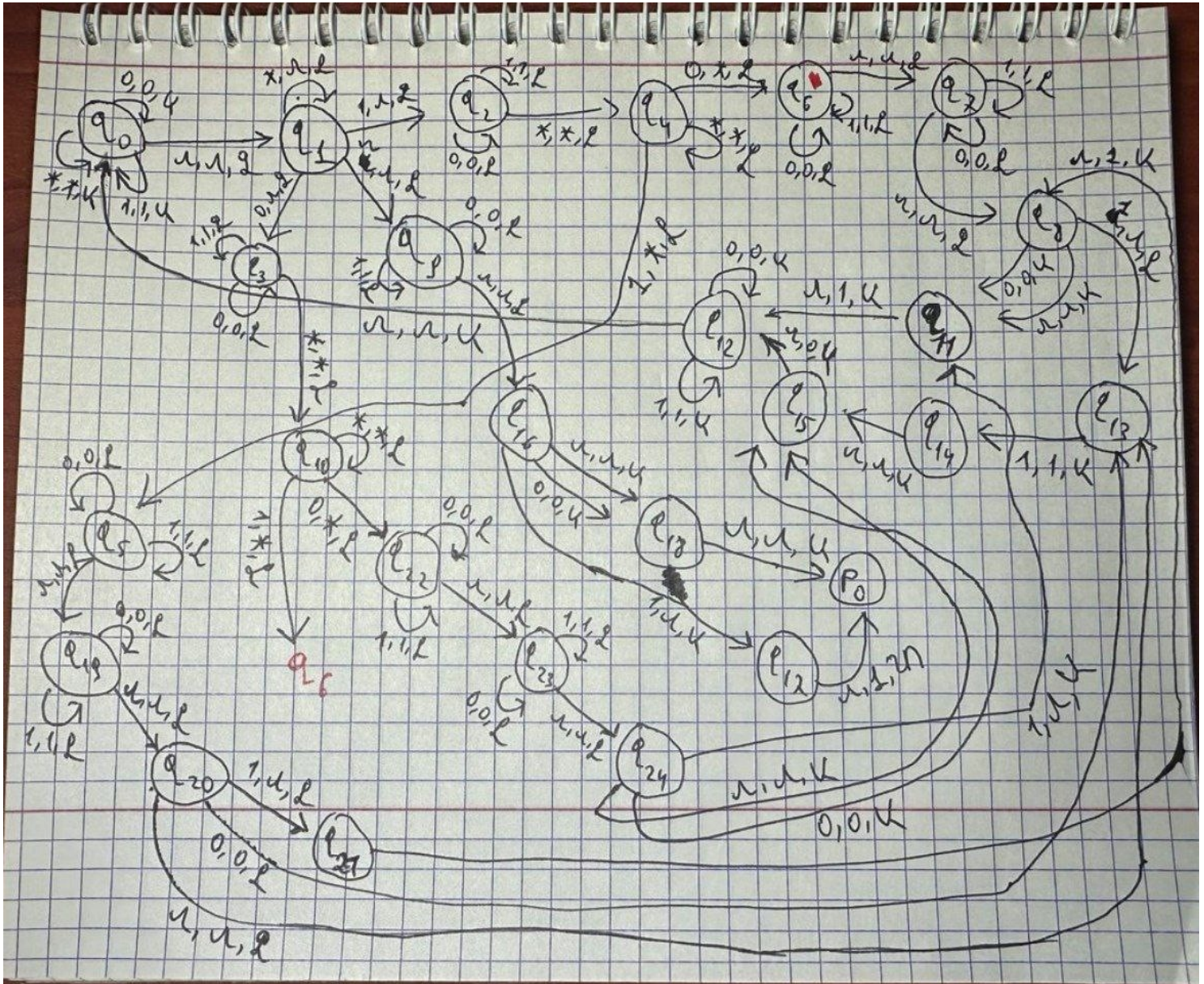
back. Then the sum of 2 left-most bits of the number is written its place.

The process of iteration ends , when the moment the head moves left first cell is *.

It starts to make all * to empty, then of course last time checks the carry bit , makes it empty if exists , and writes 1 in front of result if needed.

Having finished its job, the head stops on the first cell of the result.

2. Diagram



3. Table

	0	1	*	^
q_0	q_0, 0, U	q_0, 1, U	q_0, *, U	q_1, ^, a
q_1	q_3, ^, a	q_2, ^, a	q_1, ^, a	q_9, ^, a
q_2	q_2, 0, a	q_2, 1, a	q_4, *, a	
q_3	q_3, 0, a	q_3, 1, a	q_10, *, a	
q_4	q_6, *, a	q_5, *, a	q_4, *, a	
q_5	q_5, 0, a	q_5, 1, a		q_19, ^, a
q_6	q_6, 0, a	q_6, 1, a		q_7, ^, a
q_7	q_7, 0, a	q_7, 1, a		q_8, ^, a
q_8	q_11, 0, U	q_13, ^, a		q_11, ^, U
q_9	q_9, 0, a	q_9, 1, a		q_16, ^, a
q_10	q_22, *, a	q_6, *, a	q_10, *, a	
q_11				q_12, 1, U
q_12	q_12, 0, U	q_12, 1, U		q_0, ^, U
q_13		q_14, 1, U		
q_14				q_15, ^, U
q_15				q_12, 0, U
q_16	q_18, 0, U	q_17, ^, U		q_18, ^, U
q_17				p_0, 1, S
q_18				p_0, ^, U
q_19	q_20, 0, a	q_19, 1, a		q_20, ^, a
q_20	q_13, 0, a	q_21, ^, a		q_13, ^, a
q_21				q_8, 1, U
q_22	q_22, 0, a	q_22, 1, a		q_23, ^, a
q_23	q_23, 0, a	q_23, 1, a		q_24, ^, a
q_24	q_15, 0, U	q_11, ^, U		q_15, ^, U
p_0				