L14 Matching Pursuit & Compressed Sensing

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Ridge & Lasso Regression Review:

- Input $\tilde{X}, y$, $\tilde{X} \in \mathbb{R}^{n \times d}$, $y \in \mathbb{R}^n$, $x = [1; x]$

Goal:

- Ridge Regression

$$\alpha_s^o = \underset{\alpha \in \mathbb{R}^{d+1}}{\arg\min} \underbrace{\sum_{i=1}^{n} (y_i - \langle x_i, \alpha \rangle)^2}_{\text{OLS term}} + \underbrace{S\|\alpha\|_2^2}_{L_2 \text{ regularization term}}$$

- Lasso Regression

$$\alpha^\diamond_s = \underset{\alpha \in \mathbb{R}^{d+1}}{\arg\min} \underbrace{\sum_{i=1}^{n} (y_i - \langle x_i, \alpha \rangle)^2}_{\text{OLS term}} + \underbrace{S\|\alpha\|_1}_{L_1 \text{ regularization term}}$$

Similarly, these can be re-written as hard constraint solutions:

- Ridge Regression

$$\alpha_s^o = \underset{\alpha \in \mathbb{R}^{d+1}}{\arg\min} \sum_{i=1}^{n} (y_i - \langle x_i, \alpha \rangle)^2 \quad \text{s.t.} \quad \|\alpha\|_2^2 \leq t$$

- Lasso Regression

$$\alpha^\diamond_s = \underset{\alpha \in \mathbb{R}^{d+1}}{\arg\min} \sum_{i=1}^{n} (y_i - \langle x_i, \alpha \rangle)^2 \quad \text{s.t.} \quad \|\alpha\|_1 \leq t$$

$\rightarrow$ $\forall$ choices S, $\exists t$ such that:

$\alpha_s^o = \alpha_t^o$ or $\alpha^\diamond_s = \alpha^\diamond_t$

- let $t = \|\alpha^o_s\|_2^2$ or $t = \|\alpha^\diamond_s\|_2^2$ (for ridge vs lasso)
- thus as $S\downarrow$, the penalty of the norm gets smaller, thus $t\uparrow$

Matching Pursuit Algorithm:

- Goal:
  - Find $\alpha^* = \underset{\alpha \in \mathbb{R}^d}{\arg\min} \|\langle x, \alpha \rangle - y\|_2 + S\|\alpha\|_1$
- Algorithm

- set $r = y$, $\alpha = 0$
- for $i = 1$ to t do:
  - Set $X_j = \underset{X_j' \in X}{\arg\max} \, |\langle r, X_j' \rangle|$ • column with max dot product is the column most correlated with target
  - set $\alpha_j = \arg\min_{\alpha} \| r - X_j \alpha \| + S|\alpha|$ • get the coefficient for this column
  - set $r = r - X_j \alpha_j$ • get the new residual
- return α