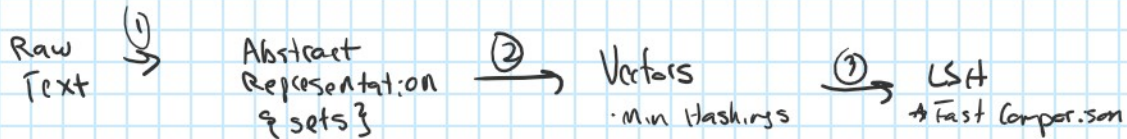


L3 Similarity

Monday, January 13, 2020 3:02 PM

• Goal



Distance:

Euclidean Distance

$a = (a_1, a_2, \dots, a_n) \in \mathbb{R}^n$
 $b = (b_1, b_2, \dots, b_n) \in \mathbb{R}^n$

$$d(a, b) = \|a - b\| = \sqrt{\sum_{j=1}^n (a_j - b_j)^2}$$

• The inverse of distance is known as similarity

Distance

$d(a, b)$

- small if a, b are close
- if large $\rightarrow a, b$ far.
- 0, if the same.
- $d(a, b) \in [0, \infty)$

$$\hookrightarrow d(a, b) = 1 - s(a, b)$$

$$\hookrightarrow d(a, b) = \sqrt{s(a, a) + s(b, b) - 2s(a, b)}$$

Similarity

$s(a, b)$

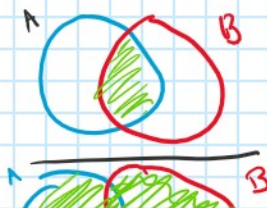
- large if a, b close
- if small $\rightarrow a, b$ far.
- 1, if the same.
- $s(a, b) \in [0, 1]$

Jaccard Similarity:

$JS(A, B)$ where $A = \{0, 1, 2, 5, 6\}$
 $B = \{0, 2, 3, 5, 7, 9\}$

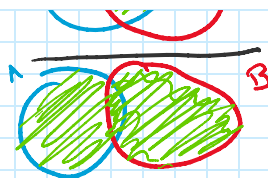
$$= \frac{|A \cap B|}{|A \cup B|} = \frac{|\{0, 2, 5\}|}{|\{0, 1, 2, 3, 5, 6, 7, 9\}|} = \frac{3}{8}$$

$$= 0.375$$



$$d_J(A, B) = 1 - JS(A, B)$$

"Jaccard Distance"



"Jaccard Distance"

Similarities Between sets:

$$S_{x,y,z,z'}(A,B) = \frac{x|A \cap B| + y|\overline{A \cap B}| + z|A \Delta B|}{x|A \cap B| + y|\overline{A \cap B}| + z'|A \Delta B|}$$

for $x, y, z, z' \geq 0, z' > z$

Ex.

$$\bullet JS(A,B) = S_{1,0,0,1}(A,B) = \frac{|A \cap B|}{|A \cap B| + |A \Delta B|}$$

$$\bullet Ham(A,B) = S_{1,1,0,1}(A,B) = 1 - \frac{|A \Delta B|}{|C \cap J|}$$

$$\bullet Andb(A,B) = S_{1,0,0,2}(A,B) = \frac{|A \cap B|}{|A \cup B| + |A \Delta B|}$$

$$\bullet RT(A,B) = S_{1,1,0,2}(A,B) = \frac{|C \cap J| - |A \Delta B|}{|C \cap J| + |A \Delta B|}$$

$$\bullet Dice(A,B) = S_{2,0,0,1}(A,B) = \frac{2|A \cap B|}{|A| + |B|}$$

Modeling Text:

I am Sam.

Sam I am.

I do not like green eggs and ham.

I do not like them, Sam I am.

Text \rightarrow vector $\in \mathbb{R}^d$
($d=11$)

Bag-of-words:

(am, and, do, eggs, green, ham, I, like, not, Sam, them, zebra)

Bag-of-words is a count of each word at i th coordinate

$$v_1 = (\textcircled{1}, 0, 0, 0, 0, 0, \textcircled{1}, 0, 0, 1, 0, 0)$$

$$v_2 = (\textcircled{1}, 0, 0, 0, 0, 0, \textcircled{1}, 0, 0, 1, 0, 0)$$

$$v_3 = (0, 1, 1, 1, 1, 1, \textcircled{1}, 1, 1, 0, 0, 0)$$

$$v_4 = (\textcircled{1}, 0, 1, 0, 0, 0, \textcircled{2}, 1, 1, 1, 1, 0)$$

K-grams with Words:

Words $K=1$:

{[I], [am], [Sam], ... [them]}

Words $K=2$ (I am Sam Sam I am...)

{[I am], [am Sam], [Sam Sam]}

words $k=2$ (I am Sam Sam I am ...)
 $\{ \underline{[I\ am]}, \underline{[am\ Sam]}, \underline{[Sam\ Sam]} \}$

characters $k=3$
 $\{ [iam], [ams], [msa], [sam] \dots \}$

↳ Jaccard needs sets (even though $[iam]$ occurs twice)

Modeling Choices:

- words vs characters
↳ more interpretable

More complex representation
(larger k , words vs characters, punctuation, etc)



more data.

- new lines

- value of k

- capitalization

- punctuation

↳ highlight '#'

K-grams and Jaccard Example:

$O_1: \underline{[I\ am]}, [am\ Sam]$

$O_2: [sam\ I], \underline{[I\ am]}$

$$JS(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{1}{3} = 0.333$$

Continuous bag of Words:

- each word $\xrightarrow{\text{maps}}$ vector $V_{\text{word}} \in \mathbb{R}^d$

↳ bow $(0, 0, 0, 1, 0, 0, \dots, 0)$