#1) Show that the least squares estimators of B_0, B_1, σ^2 minimize the sum of squared errors.

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - E[Y_i])^2$$

$$= \sum_{i=1}^{n} (Y_i - (B_0 + B_1 x_i))^2$$

$$= \sum_{i=1}^{n} [(Y_i - (B_0 + B_1 x_i))(Y_i - (B_0 + B_1 x_i))]$$

$$= \sum_{i=1}^{n} [Y_i^2 - 2Y_i(B_0 + B_1 x_i) + (B_0 + B_1 x_i)^2]$$

$$= \sum_{i=1}^{n} [Y_i^2 - 2Y_i(B_0 + B_1 x_i) + [(B_0 + B_1 x_i)(B_0 + B_1 x_i)]]$$

$$= \sum [Y_i^2 - 2Y_i(B_0 + B_1 x_i) + B_0^2 + 2B_0 B_1 x_i + B_1^2 x_1^2]$$

$$= \sum [Y_i^2 - 2y_i B_1 x_i - 2y_i B_0 + B_1^2 x_i^2 + 2B_0 B_1 x_i + B_0^2]$$

$$= \sum [y_i^2 - 2B_1(y_i x_i) - 2B_0(y_i) + B_1^2(x_i^2) + 2B_0 B_1(x_i) + B_0^2]$$

$$= \sum(y_i^2) - 2B_1 \sum(y_i x_i) - 2B_0 \sum(y_i) + B_1^2 \sum(x_i^2) + 2B_0 B_1 \sum(x_i) + nB_0^2$$

$$SE = n\overline{y^2} - 2B_1 n\overline{xy} - 2B_0 n\bar{y} + B_1^2 n\overline{x^2} + 2B_0 B_1 n\bar{x} + nB_0^2$$

• Then we can take the partial derivatives with respect to B_1 and B_0 and set them to 0 to find the values of $\hat{B}_1$ & $\hat{B}_0$ that minimize the function.

$$\frac{\partial SE}{\partial \hat{B}_1} = -2n\overline{xy} + 2n\overline{x^2}\hat{B}_1 + 2\hat{B}_0 n\bar{x} = 0 \Rightarrow \overline{xy} = \overline{x^2}B_1 + B_0\bar{x}$$

$$\Rightarrow \frac{\overline{xy}}{\bar{x}} = \frac{\overline{x^2}}{\bar{x}} B_1 + B_0$$

$$\frac{\partial SE}{\partial B_0} = -2n\bar{y} + 2\hat{B}_1 n\bar{x} + 2n\hat{B}_0 = 0 \Rightarrow \bar{y} = \hat{B}_1\bar{x} + \hat{B}_0$$

Thus $\hat{B}_1\bar{x} + \hat{B}_0 - \bar{y} = \hat{B}_1\frac{\overline{x^2}}{\bar{x}} + \hat{B}_0 - \frac{\overline{xy}}{\bar{x}} \Rightarrow \hat{B}_1\bar{x} - \hat{B}_1\frac{\overline{x^2}}{\bar{x}} = \bar{y} - \frac{\overline{xy}}{\bar{x}}$

$\boxed{\therefore \hat{B}_1 = \dfrac{\bar{y} - \frac{\overline{xy}}{\bar{x}}}{\bar{x} - \frac{\overline{x^2}}{\bar{x}}}}$, which is exactly equivalent to the least squares estimator for B_1 &

$\boxed{B_0 = \bar{y} - B_1\bar{x}}$

#2) Let $d_i = \frac{1}{n} - \bar{x} b_i$ where $b_i = \frac{x_i - \bar{x}}{\sum (x_i - \bar{x})(x_i - \bar{x})}$

Q: show that $\sum d_i^2 = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2}$

$$\sum d_i^2 = \sum \left(\frac{1}{n} - \bar{x} \left[\frac{x_i - \bar{x}}{\sum (x_i - \bar{x})(x_i - \bar{x})} \right] \right)^2$$

$$= \sum \left(\frac{1}{n} - \frac{\sum x_i}{n} \left[\frac{\cancel{x_i - \bar{x}}}{\cancel{\sum (x_i - \bar{x})}(x_i - \bar{x})} \right] \right)^2$$

$$= \sum \left(\frac{1}{n} - \frac{\sum x_i}{n} \left[\frac{1}{\sum (x_i - \bar{x})} \right] \right)^2$$

$$= \frac{\sum x_i}{n \sum (x_i - \bar{x})^2}$$

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