Question #3

Saturday, April 11, 2020 6:46 AM

- $X \perp Y$

- Let $n_x = |X| = 2$
- Let $n_y = |Y| = 3$
- Let $N = n_x + n_y$

- Wilcoxon $W_x = \sum \text{rank}(x_i)$, Mann-Whitney's $U_x = \sum I(\text{rank}(y_i) < \text{rank}(x_j))$ for any i, j

- There are $\binom{N}{n_x}$ arrangements of n_x amongst N choices

- Thus there are $\binom{5}{2} = 10$ ways to arrange the two x_i's amongst the 5 available rank indices

	1	2	3	4	5	W_x	W_y	U_x	U_y
1	X	X	Y	Y	Y	3	12	0	6
2	X	Y	X	Y	Y	4	11	1	5
3	X	Y	Y	X	Y	5	10	2	4
4	X	Y	Y	Y	X	6	9	3	3
5	Y	X	X	Y	Y	5	10	2	4
6	Y	X	Y	X	Y	6	9	3	3
7	Y	X	Y	Y	X	7	8	4	2
8	Y	Y	X	X	Y	7	8	4	2
9	Y	Y	X	Y	X	8	7	5	1
10	Y	Y	Y	X	X	9	6	6	0

- Now summing the count of each value w_x u_x

- w_x
  - $3 \to 1$
  - $4 \to 1$
  - $5 \to 2$
  - $6 \to 2$
  - $7 \to 2$
  - $8 \to 1$
  - $9 \to 1$

$\Rightarrow$

W_x	
3	1/10
4	1/10
5	2/10
6	2/10
7	2/10
8	1/10
9	1/10

- u_x
  - $0 \to 1$
  - $1 \to 1$
  - $2 \to 2$
  - $3 \to 2$
  - $4 \to 2$
  - $5 \to 1$
  - $6 \to 1$

U_x	
0	1/10
1	1/10
2	2/10
3	2/10
4	2/10
5	1/10
6	1/10

Part A ✓

- w_y
  - $6 \to 1$
  - $7 \to 1$
  - $8 \to 2$
  - $9 \to 2$
  - $10 \to 2$
  - $11 \to 1$
  - $9 \to 1$

$\boxed{\begin{array}{l} \bullet \text{ Thus } W_y = W_x + 3 \\ \therefore \text{ Thus } f_{w_y} = f_{w_x} \end{array}}$

- u_y
  - $0 \to 1$
  - $1 \to 1$
  - $2 \to 2$
  - $3 \to 2$
  - $4 \to 2$
  - $5 \to 1$
  - $6 \to 1$

$\boxed{\begin{array}{l} \bullet \text{ Thus } u_y = u_x \\ \therefore \text{ Thus } f_{u_y} = f_{u_x} \end{array}}$

Part B ✓

Part C:

$N = 10$, thus $(0.2)(10) = 2$

- Thus for each test statistic, the rejection statistic corresponds to being less than the second ranked stat.

$$\boxed{\begin{array}{ll} W_{x_{crit}} = 4 , & W_{y_{crit}} = 7 \\ U_{x_{crit}} = 1 , & U_{y_{crit}} = 1 \end{array}}$$ ✓

- we notice that the same shift applys to the critical values.