Lecture 16

Thursday, March 5, 2020 2:02 PM

Agenda

- Chapter 14 Non-parametric tests
- 14.2 Tests for median
- 14.2 Tests for variation

Advantages

- Fewer assumptions
- Rank of data

Section 14.2

- Sign tests
  - Suppose $X_1, \dots, X_n \overset{iid}{\sim} F_X(x)$
  - Suppose m is the median of the distribution (a measure of location)
    - The median satisfies: $P[X_i \le m] = P[X_i \ge m] = \frac{1}{2}$
    - Suppose we had the following hypotheses:

$H_0: m = m_0$

$H_A: m > m_0$

Consider the sgn(.) function...

$$\text{sgn}(X_i - m_0) = \begin{cases} + & X_i \ge m_0 \\ - & x \le m_0 \end{cases}$$

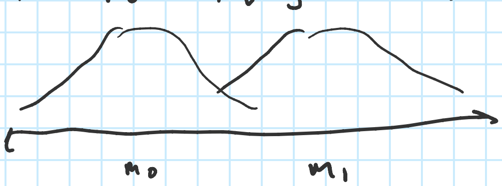
Under $H_0: m = m_0$, Thus...

$$P[X \le m_0] = P[X - m_0 \le 0]$$
$$= P[\text{sgn}(X - m_0) = \text{"}-\text{"}]$$

"-" = negative

- Let $p = P[X \le m_0]$

then $p_0 = P[\text{sgn}(X - m_0) = \text{"}-\text{"} \mid m_0]$



Thus $P[\operatorname{sgn}(X - m_0) = \text{"-"} \mid m_1] \leq p_0$

Thus

$$\begin{cases} H_0: m = m_0 \\ H_A: m > m_0 \end{cases} \Rightarrow \begin{cases} H_0: p = p_0 \\ H_A: p < p_0 \end{cases}$$ } starting to look like a binomial distribution...

- Now, let T be the RV representing the count of observations $\leq m_0$
- Then under the null H_0, $T \sim Bin(n, p_0)$

Thm 14.2.1 Let $X \sim F_X(x)$ and let $F_X(m) = \frac{1}{2}$

(a) A size α test of $\begin{cases} H_0: m = m_0 \\ H_A: m > m_0 \end{cases} \Rightarrow \begin{cases} H_0: p = p_0 \\ H_A: p \leq p_0 \end{cases}$

rejects H_0 if:

$B(t, n, p_0) \leq \alpha$ where B is the binomial CDF.

(b) Alternatively $H_0: p = p_0$, $H_A: p > p_0$ ...

we reject if:

$1 - B(t-1, n, p_0) \leq \alpha$

(c) Lastly for two-tail... ($H_A: p \neq p_0$)

... we reject if:

$B(t; n, p_0) \leq \alpha/2$

OR

$1 - B(t-1, n, p_0) \leq \alpha/2$