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Thursday, February 13, 2020 11:00 AM

Q: let $X_1, \dots, X_N$ be a random sample from a distribution with pdf $f(x;\theta) = \frac{1}{\theta}$ if $0 \le x \le \theta$ and zero otherwise.

Derive the GLR test of $H_0: \theta = \theta_0$ versus $H_A: \theta \neq \theta_0$

S1. The PDF...

$$f(x;\theta) = \frac{1}{\theta} \cdot I\{0 \le x \le \theta\}$$

S2. The JDF

$$f_{\underline{x}}(\underline{x};\theta) = \prod_{i=1}^{n} \frac{1}{\theta} \cdot \prod_{i=1}^{n} I\{0 \le x_i \le \theta\} = \left(\frac{1}{\theta}\right)^n \cdot I\{x_{N,N} \le \theta\}$$

- Thus $X_{N,N}$ is the MLE

Since the identity function depends on the parameter of interest, we know the MLE is an ordered statistic of X.

$$I\{0 \le X_i \le \theta\} = 1$$

$\Rightarrow$ $X_{N,N} \le \theta$ where $X_{N,N}$ is the largest ordered statistic.

S3. Then the GLR gives...

$$\lambda(x) = \frac{f_{\underline{x}}(\underline{x};\theta_0)}{f_{\underline{x}}(\underline{x};\hat{\theta})} = \frac{\left[\frac{1}{\theta_0}\right]^N}{\left[\frac{1}{X_{N,N}}\right]^N} = \left[\frac{X_{N,N}}{\theta_0}\right]^N = \frac{X_{N,N}^{\,N}}{\theta_0^{\,N}}$$

- Now, the test only depends on one observation of X (ie, $X_{N,N}$ only).

- we know we would reject the null hypothesis if $X_{N,N} \le \alpha$, so using the same logic and some algebra we get.

$$\left[\frac{X_{N,N}}{\theta_0}\right]^N \le \alpha$$

$$\Rightarrow X_{N,N}^{\,N} \le (\theta_0^{\,N})(\alpha)$$

$$\Rightarrow X_{N,N} \le (\theta_0)(\alpha)^{1/n}$$

$\therefore$ Thus we reject H_0 if $X_{N,N} \le (\theta_0)(\alpha)^{1/n}$

$X_{N,N} = (\theta_0)(\alpha)$

$\therefore$ Thus we reject H_0 if $X_{N,N} \le (\theta_0)(\alpha)^{1/n}$

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