Question #5

Sunday, April 12, 2020 2:14 PM

Test the claim that the 29th percentile is 10 versus the alternative that it is less than 10 via the sign test. Constructs a 70% confidence interval for the 25th percentile.

$X = 0\ 7\ 7\ 8\ 10\ 20\ 22\ 25\ 27\ 33\ 40\ 41\ 50\ 55\ 75\ 80$, $n = 16$

- If we hypothesize that 10 is the first quartile, then 1/4 of observations should be smaller than 10

  ie, $P(x_i - 10 < 0) = 1/4$

  $\sim B.n(16, p_0)$ where $p_0 = 1/4$

- Thus we can count the number of successes $(x_i \leq 10)$...

  $X = 0\ 7\ 7\ 8\ 10\ 20\ 22\ 25\ 27\ 33\ 40\ 41\ 50\ 55\ 75\ 80$ (10 counts as it)

Success (-)	Failure (+)
IIII	IIII IIII I

- Now let t = the number of successes...

  $t = 4$

- Then the p value for this t is given by:

$1 - B(3, 16, 1/4) = 1 - 0.405 = 0.595$

∴ Thus we do not have strong evidence at any reasonable α, so we fail to reject the null ✓

- Now we wish to construct confidence intervals.
- we can use the following: $P[X_{i:n} \leq x_p \leq X_{j:n}] = B(j-1; n, p) - B(i-1; n, p)$, where we can choose j, i to get the desired confidence coefficient.
- let $j = 7$, $i = 3$
- Then $B(j-1; n, p) = B(6, 16, 1/4) = 0.92$
- Then $B(i-1; n, p) = B(2, 16, 1/4) = 0.197$

$0.92 - 0.197 = 0.723$

∴ $P[X_{3:16} \leq x_{0.25} \leq X_{7:16}] \simeq 0.70$ ✓

∴ $[7, 22]$ corresponds to the 70% CI.