Lecture 14

Thursday, February 27, 2020 2:00 PM

Agenda

- Section 13.7 (Goodness of fit)
- Section 13.8 (Other GOF tests not based on χ^2)

If $X_i \sim Bin(n, p)$... (assuming n is known)

$H_0: p = p_0$

- we know $E(X_i) = n \cdot p_0$ under null

$Var(X_i) = n p_0 (1 - p_0)$

- If we come up with approximate test, then under H_0

$\hat{p} \sim N\left(p, SE(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}\right)$

$\hat{p} = X_i / n$

This is a good approximation if:

$np_0 \geq 10$ AND $n(1-p_0) \geq 10$

(rule of thumb)

Going back to example 13.7.1 from Lecture 13...

- Let X represent repair time of some component in days $(n = 40)$

(A) Fully specified H_0:

$H_0: X \sim Poisson(3)$

Repair Time	0	1	2	3	4	5	6	7
observed count	1	3	7	6	10	7	6	0
p_{j0}								
Exp Count	2	5.96	8.96	8	~	4.04	2	1.36

$= 7.96$

$= 7.4$

- Reject H_0 at $\alpha = 0.1$

(B) Unspecified Case:

$H_0: X \sim Poisson(\theta)$ with θ unknown

- we estimate θ with $\hat{\theta} = MLE_\theta = \bar{X}$

$$\frac{\sum X_i}{n} = \frac{[0(1) + 1(3) + 2(7) + \ldots + 6(6)]}{n = 40}$$

$\Rightarrow \hat{\theta} = \bar{X} = 3.65$

Then $P(X = x) = \dfrac{e^{3.65} - 3.65^x}{x!}$

Thus the expected count for any x is:

$E[x] = n \cdot P(X = x)$

Repair Time	0	1	2	3	4	5	6	7
observed count$_j$	1	3	7	6	10	7	6	0
P_{j0} from Poisson	0.025	0.095	0.173	0.211	0.142	0.303		
Exp Count with poisson probabilities	4.83		6.92	8.44	7.68	12.2		

$\chi^2 = \chi^2(2)$

13.8 Other GOF Tests:

- These tests use the empirical distribution function (EDF)

Def: The EDF of random sample $X_1, \ldots, X_N$ is given by:

$$\hat{F}_{\underline{X}}(t) = \sum_{i=1}^{n} \frac{I(X_i \le t)}{n}$$

$$F_{\underline{X}}(t) = \begin{cases} 0 & t \le x_1 \\ i/n & x_i \le t \le x_{i+1} \\ 1 & t > x_n \end{cases}$$

(A) Completely Specified Hypothesis H_0:

H_0: $X \sim F_X(x)$ (all parameters assumed to be known)

Goal: Measure "distance" between $F_X(x)$ and $\hat{F}_{\underline{X}}(x)$ for X in sample.

- It turns out that this can be viewed as a test for uniformity because

  $F(X) = U \sim \text{Unif}(0, 1)$

- A measure of distance could be:

  $F_X(x_i) - \hat{F}_{\underline{X}}(x_i)$ evaluated at all x_i, $i = 1, \ldots, x$

- Then we could calculate the totals

$$\sum_{i=1}^{n} \left(F_X(x_i) - \hat{F}_{\underline{X}}(x_i) \right)^2$$

$$\Rightarrow \sum_{i=1}^{n} \left(F_X(x_i) - \frac{i}{n} \right)^2$$

or use half point adjustment to get

$$\sum_{i=1}^{n} \left(F_X(x_i) - \frac{i-0.5}{n} \right)^2$$

- The Cramer Von Mises test is one where

$$CM = \frac{1}{12(n)} + \sum_{i=1}^{n} \left(F_X(x_i) - \left(\frac{i - 0.5}{n} \right) \right)^2$$

- Then an appropriate size α test of $H_0: X \sim F_X(x)$ is to reject if $CM_{OB} \geq CM_{1-\alpha}$ where $CM_{1-\alpha}$ is given on pg 613 (Table 9, A C)