Lecture 1

Tuesday, January 7, 2020 2:04 PM

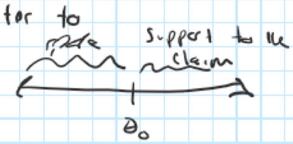
Example: Suppose θ in the population of defection in batch #2.

* the claim we want to test is: $\theta = \theta_0$??

What is θ ?

↳ θ is a parameter of distribution of defection in batch #2.

↳ In general, it decides the membership of a parameter to different subsets of the parameter space.



- Suppose Ω is the parameter space.
- The null hypothesis would correspond to the subset of Ω, notated as Ω_0, which pertains to the claim we are interested in.
- The alternative hypothesis is then notated $\Omega - \Omega_0$

Set of hypotheses
$$\begin{cases} H_0: \Omega = \Omega_0 & (=\theta_0) \\ H_A: \Omega - \Omega_0 & (\text{for instance, } \theta = \theta_1) \end{cases}$$

* If the set of hypotheses completely defines the distribution of our data, then the hypothesis is called a simple hypothesis

* A composite hypothesis is one that is not completely defined by set of hyp.

Simple Steps for HP problem

S1) Define what you are interested in.

S2) Come up with a decision rule (decides if $\theta \in$ null space or not)

Here, $\boxed{1-\beta = \text{power of a test}}$

* Given a desired α, the researcher attempts to maximizes $(1-\beta)$

ERROR TYPES w/ Decision Rule

TRUTH

Decision	H_0	H_A
H_0	✓ $1-\alpha$	Type II β
H_A	Type I α	✓ $1-\beta$

$\alpha =$ P(Type I ERROR) = P(H_0 is rejected | H_0 is true)

$\beta =$ P(Type II ERROR) = P(H_0 is not rejected | H_A is true)

Def: Critical Region (CR)

- A CR of a test is a subset of the sample space which corresponds to rejecting the null hypothesis
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$$C = \{ \underbrace{(x_1, \dots, x_n)}_{= \underline{x}} \mid \bar{x} \geq c \}$$

where c is appropriately chosen to maximize $(1-\beta)$.

Ex 12.1.1 (Yield of chemical reaction)

- $X \sim N(\mu, \sigma^2 = 16)$
- Past evidence suggest that $\mu = 10$, if a particular mineral is not present and that $\mu = 11$ if a certain element is present.
- Suppose we draw a random sample of 25 yields $(n = 25)$

$H_0 : \mu = 10$

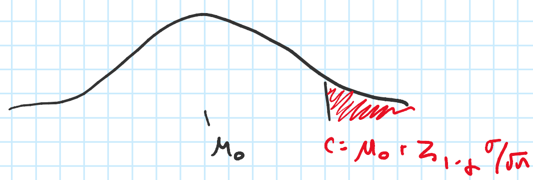
$H_A : \mu = 11$

- The researcher has decided $\alpha = 0.05$

- Suppose $T(X) = \bar{X}$ is our statistic making the decision (sufficient statistic)

$$C : \{ \underline{x} \mid \bar{x} \geq c \}$$

- Suppose $c = \mu_0 + z_{1-\alpha} \cdot \sigma/\sqrt{n}$, where $z_{1-\alpha}$ is the probability $P(Z \geq z_{1-\alpha}) = \alpha$



- If $\alpha = 0.05$, then $z_{1-\alpha} = 1.64$

$$\Rightarrow \quad c = 10 + (1.64)(4/\sqrt{25})$$

$$\approx 11.316$$

Then, $P(\bar{x} \geq c \mid \mu = \mu_0)$

$$= P\left(\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq \frac{11.316 - 10}{4/5}\right)$$

$$= \boxed{P(Z \geq 1.645)}$$

Type II Error:

$$P(\text{fail to reject null} \mid H_A \text{ is true}) = P(\bar{X} < 11.316 \mid \mu = 11)$$

$$= P\left(\frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} < \frac{11.316 - 11}{4/\sqrt{25}}\right)$$

$$= P(Z < 0.395)$$

$$\boxed{= 0.654 = \beta}$$

Now, suppose we had some different rejection region:

$$C = \{ \underline{x} \mid 10 < \bar{x} < 10.1006 \}$$

Then under the null hypothesis:

$$P\left(\frac{10 - 10}{\quad} < \quad < \frac{10.1006 - 10}{\quad}\right)$$

Then under the null hypothesis:

$$P\left(\frac{10-10}{0.8} < Z < \frac{10.1006-10}{0.8}\right)$$

$= P(0 < Z < 0.1257)$

$= \Phi(0.1257) - \Phi(0)$

$\boxed{= 0.05 = \alpha}$

$P(\text{Type II Error}) = 1 - P(10 \le \bar{x} < 10.1006 \mid \mu = 11)$

$= 1 - P\left(\frac{10-11}{0.8} < Z < \frac{10.10006-11}{0.8}\right)$

$= 1 - P(-1.25 < Z < -1.124)$

$\boxed{= 0.975}$

☆ We can have multiple tests with the same alpha level, with each corresponding to a different beta.

☆ The goal of hypothesis testing is to find the test that maximizes (1 - B) from amongst the class of tests that all correspond to type I error = α