HW 5.1-11

Q: Find a second-order formula for approximating $f'(x)$ by applying extrapolation to the two-point forward-difference formula.

- The two-point difference of $f'(x)$ is given by the following:

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2} f''(c) \text{ where } c \text{ is between } x \text{ and } x+h$$

- Letting $F(h) = \frac{f(x+h) - f(x)}{h}$ => $f'(x) = F(h) - \frac{h}{2} f''(c)$

- Substituting $h/2$ for h gives $f'(x) = F(h/2) - h/4 f''(c)$

=> $2f'(x) = 2F(h/2) - h/2 f''(c)$

- Then subtracting $f'(x)$ from $2f'(x)$ gives the following:

$$2f'(x) - f'(x) = \left[2F(h/2) - (h/2)f''(c) \right] - \left[F(h) - (h/2)f''(c) \right]$$

$$= 2F(h/2) - F(h)$$

$$= 2\left[\frac{f(x + h/2) - f(x)}{h/2} \right] - \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$= 4\left[\frac{f(x + h/2) - f(x)}{h} \right] - \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$= \frac{4f(x + h/2) - 4f(x) - f(x+h) + f(x)}{h}$$

$$= \frac{4f(x + h/2) - 3f(x) - f(x+h)}{h}$$

∴ Thus a second order formula for approximating $f'(x)$ is

$$f'(x) = \frac{4f(x + h/2) - 3f(x) - f(x+h)}{h}$$