HW 1.2-8

Friday, June 26, 2020 9:25 AM

- Use Theorem 1.6 to determine whether fixed point iteration of $g(x)$ is locally convergent to the given fixed point r.

a) $g(x) = (2x-1)/x^2$, $r=1$

- Recall Theorem 1.6: Assume that g is continuously differentiable, that $g(r)=r$ and that $S = |g'(r)| < 1$. Then fixed point iteration converges linearly with rate S to the fixed point r for initial guesses sufficiently close to r.
- $g(1) = \frac{(2(1)-1)}{1^2} = 1$
- now $g'(x) = \frac{\partial (2x-1)x^{-2}}{\partial x} = \frac{x^2(2) - (2x-1)(2x)}{x^4} = \frac{2x^2 - (4x^2-2x)}{x^4} = \frac{2x-2x^2}{x^4}$
- now evaluate $g'(1)$: $\frac{2(1)-2(1)^2}{1^4} = 0$
  - $|0| < 1$
- Thus, the fixed point iteration converges locally to the fixed point at $r=1$

b) $g(x) = \cos x + \pi + 1$, $r = \pi$

- $g(x) = \cos\pi + \pi + 1$
  $= -1 + \pi + 1$
  $= \pi$
- $g'(x) = \frac{\partial \cos x + \pi + 1}{\partial x} = -\sin x$
- $g'(\pi) = -\sin\pi$
  $= 0$
- $|0| < 1$, thus the fixed point iteration converges locally to the fixed point at $r=\pi$

c) $g(x) = e^{2x}-1$, $r=0$

- $g(0) = e^0 - 1$
  $= 0$
- $g'(x) = \frac{\partial e^{2x}-1}{\partial x} = 2e^{2x}$
- $g'(0) = 2e^{20}$
  $= 2$
- $|2| > 1$, thus the fixed point iteration is not convergent to the fixed point at $r=0$