HW 5.2-3

Monday, July 27, 2020 8:59 AM

Q: Apply the Composite Simpson's Rule with $m = 1, 2,$ and 4 panels to

to the integrals in exercise #1 and report the errors

a) $\int_0^1 x^2\,dx$

- $m=1 \Rightarrow (b-a)/2m = h \Rightarrow h = 1/2$

$$\Rightarrow \int_0^1 x^2\,dx \approx (1/2)/3\left[f(x_0)+f(x_2)+4\sum_{i=1}^{n} f(x_{2i-1}) + 2\sum_{i=1}^{m-1} f(x_{2i})\right]$$
$$\approx (1/6)[f(0)+f(1)+4f(x_1)]$$
$$\approx (1/6)[0^2+1^2+4(1/2)^2]$$
$$\approx (1/6)[1+1]$$
$$\approx 1/3$$

- then $\int_0^1 x^2\,dx = x^3/3\big|_0^1 = 1/3 - 0 = 1/3$ (actual)
- error $= 1/3 - 1/3 = 0$

$m = 2 \Rightarrow (b-a)/2m = h \Rightarrow h = 1/4$

$$\Rightarrow \int_0^1 x^2\,dx \approx (1/4)/3\,[f(x_0) + f(x_{2m}) + 4(f(x_1)+f(x_3)) + 2f(x_2)]$$
$$\approx 1/12[f(0)+f(1)+4(f(1/4)+f(3/4))+2f(1/2)]$$
$$\approx 1/12[0^2+1^2+4(1/16+9/16)+2(1/4)]$$
$$\approx 1/12[1+40/16+1/2] = 1/12[16/16+40/16+8/16]$$
$$\approx 1/12[64/16] = 1/12[4]$$
$$\approx 1/3$$

- thus again $1/3 - 1/3 = 0$, so the error is 0.

$m = 4 \Rightarrow (b-a)/2m = h \Rightarrow h = 1/8$

$$\Rightarrow \int_0^1 x^2\,dx \approx (1/8)/3[f(x_0)+f(x_{2m})+4[f(x_1)+f(x_3)+f(x_5)+f(x_7)] + 2[f(x_2)+f(x_4)+f(x_6)]]$$
$$=(1/24)[f(0)+f(1)+4[f(1/8)+f(3/8)+f(5/8)+f(7/8)] + 2[f(2/8)+f(4/8)+f(6/8)]]$$
$$\approx(1/24)[0^2+1^2+4(1/64+9/64+25/64+49/64)+2(4/64+16/64+36/64)]$$
$$\approx 1/3$$

- thus again $1/3 - 1/3 = 0$, so the error is 0.

b) $\int_0^{\pi/2} \cos x\,dx$

- $m=1 \Rightarrow (b-a)/2m = h \Rightarrow h = \pi/4$
- $$\int_0^{\pi/2} \cos x\,dx \approx (\pi/4)/3\left[f(x_0)+f(x_2)+4\sum_{i=1}^{m} f(x_{2i-1}) + 2\sum_{i=1}^{m-1} f(2i)\right]$$
$$\approx (\pi/12)[f(0)+f(\pi/2)+4(f(x_1))]$$
$$\approx (\pi/12)[\cos(0)+\cos(\pi/2)+4\cos(\pi/4)]$$
$$\approx 1.002280$$
- $\int_0^{\pi/2} \cos(x)\,dx = \sin(x)\big|_0^{\pi/2}$
$= 1$

$\Rightarrow$ the error $= |1.002280 - 1| = 0.002280$

$m = 2 \Rightarrow (b-a)/2m = h \Rightarrow h = \pi/8$

$$\int_0^{\pi/2} \cos(x)\,dx \approx (\pi/8)/3[f(x_0)+f(x_4)+4(f(x_1)+f(x_3)) + 2(f(x_2))]$$
$$\approx (\pi/24)[f(0)+f(\pi/2)+4(f(\pi/8)+f(3\pi/8))+2(f(2\pi/8))]$$
$$\approx (\pi/24)[\cos(0)+\cos(\pi/2)+4(\cos(\pi/8)+\cos(3\pi/8))+2\cos(\pi/4)$$
$$\approx 1.000135$$

$\Rightarrow$ the error $= |1 - 1.000135| = 0.000135$

$m = 4 \Rightarrow (b-a)/2m = h \Rightarrow h = \pi/16$

$$\int_0^{\pi/2} \cos(x)\,dx \approx (\pi/16)/3[f(x_0)+f(x_8)+4(f(x_1)+f(x_3)+f(x_5)+f(x_7))+2(f(x_2)+f(x_4)+f(x_6))]$$
$$\approx(\pi/48)[f(0)+f(\pi/2)+4(f(\pi/16)+f(3\pi/16)+f(5\pi/16)+f(7\pi/16))+2(f(2\pi/16)+f(4\pi/16)+f(6\pi/16))$$
$$\approx 1.000008$$

$\Rightarrow$ the error equals $|1.000008 - 1| = 0.000008$

c) $\int_0^1 e^x\,dx$

$m = 1 \Rightarrow (b-a)/2m \Rightarrow h = 1/2$

$$\int_0^1 e^x\,dx \approx (1/2)/3\,[f(x_0)+f(x_2)+4(f(x_1))]$$
$$\approx (1/6)[e^0 + e^1 + 4e^{1/2}]$$
$$\approx 1.718861$$

$\int_0^1 e^x = e^1 - e^0 = e - 1 = 1.718282$

$\Rightarrow$ the error $= |1.718861 - 1.718282|$

$= 0.000579$

$m = 2 \Rightarrow (b-a)/2m = h \Rightarrow h = 1/4$

$$\int_0^1 e^x\,dx \approx (1/4)/3\,[f(x_0)+f(x_4)+4(f(x_1)+f(x_3))+2f(x_2)]$$
$$\approx (1/12)[e^0+e^1+4(e^{1/4}+e^{3/4})+2e^{1/2}]$$
$$\approx 1.718319$$

$\Rightarrow$ the error $= |1.718319 - 1.7182821| = 0.0000369$

$m = 4 \Rightarrow (b-a)/2m = h \Rightarrow h = 1/8$

$$\int_0^1 e^x\,dx \approx (1/8)/3\,[f(x_0)+f(x_8)+4[f(x_1)+f(x_3)+f(x_5)+f(x_7)] + 2[f(x_2)+f(x_4)+f(x_6)]]$$
$$\approx 1/24[e^0+e^1+4(e^{1/8}+e^{3/8}+e^{5/8}+e^{7/8})+2(e^{2/8}+e^{4/8}+e^{6/8})]$$
$$\approx 1.718289$$

$\Rightarrow$ the error is $|1.718289 - 1.718282| = 0.000002$