HW 1.5-2

Monday, June 29, 2020 9:47 AM

- Apply two steps of the Method of False Position with initial bracket $[1,2]$ to the equations of exercise #1.
  - Recall: If a and b are the initial guesses of the function f, such that $f(a)f(b) < 0$, the False Position Method to find the further approximation is given by: $c = \frac{bf(a) - af(b)}{f(a) - f(b)}$

a) $x^3 = 2x + 2$

$\Rightarrow x^3 - 2x - 2 = 0$

- $f_1(1) = (1)^3 - 2(1) - 2$
  $= -3$
  $-3 < 0$
- $f_1(2) = (2)^3 - 2(2) - 2$
  $= 8 - 4 - 2$
  $= 2$
  $2 > 0$

$\Rightarrow$ $c_1 = \frac{bf(a) - af(b)}{f(a) - f(b)} = \frac{2f_1(1) - 1f_1(2)}{f_1(1) - f_1(2)} = \frac{(2)(-3) - (1)(2)}{(-3) - (2)} = \frac{-6 - 2}{-5} = \frac{8}{5}$

Then $f_1(c) = \left(\frac{8}{5}\right)^3 - 2(8/5) - 2$
$= -1.104$
$-1.104 < 0$

- The new interval becomes $[8/5, 2]$

$c_2 = \frac{2f_1(8/5) - (8/5)f_1(2)}{f_1(8/5) - f_1(2)} = \frac{(2)(-1.104) - (8/5)(2)}{-1.104 - 2}$

$\boxed{c_2 = 1.742}$

b) $e^x + x - 7 = 0$

$f(1) = e^1 + 1 - 7$
$= e - 6$
$= -3.28$
$-3.28 < 0$

$f(2) = e^2 + 2 - 7$
$= e^2 - 5$
$= 2.39$
$2.39 > 0$

$c_1 = \frac{2(-3.28) - 1(2.39)}{-3.28 - 2.39}$
$c_1 = 1.578$

$f(c_1) = e^{1.578} + 1.578 - 7$
$= -0.573$
$-0.573 < 0$, thus the updated interval becomes $[1.578, 2]$

$c_2 = \frac{2(-0.573) - (1.578)(2.39)}{(-0.573) - 2.39} = \boxed{1.66}$

c) $e^x + \sin(x) - 4 = 0$

$f(1) = e^1 + \sin(1) - 4$
$= -0.44$
$-0.44 < 0$

$f(2) = e^2 + \sin(2) - 4$
$= 4.29$
$4.29 > 0$

$c_1 = \frac{2(-0.44) - 1(4.29)}{(-0.44) - (4.29)}$
$= 1.09$

$f(1.09) = e^{1.09} + \sin(1.09) - 4$
$= -0.129$
$-0.129 < 0$

- the updated interval is $[1.09, 2]$

$c_2 = \frac{2(-0.129) - (1.09)(4.29)}{(-0.129) - (4.29)} = \boxed{1.12}$