HW 2.5-1

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Q: Compute the first two steps of the Jacobi and the Gauss-Seidel Methods with the origin as a start.

a) $\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{matrix} 3u - v = 5 \\ -u + 2v = 4 \end{matrix} \Rightarrow \begin{matrix} u = \frac{5+v}{3} \\ v = \frac{4+u}{2} \end{matrix}$

Jacobi

$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \frac{5+v_0}{3} \\ \frac{4+u_0}{2} \end{bmatrix} = \begin{bmatrix} 5/3 \\ 2 \end{bmatrix}$

$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{5+v_1}{3} \\ \frac{4+u_1}{2} \end{bmatrix} = \begin{bmatrix} \frac{5+2}{3} \\ \frac{4+5/3}{2} \end{bmatrix} = \begin{bmatrix} 7/3 \\ 17/6 \end{bmatrix}$

Gauss-Seidel

$v_0 = 0$

$u_1 = \frac{5+v_0}{3} = 5/3$

$v_1 = \frac{4+u_1}{2} = \frac{4+5/3}{2} = 17/6 \Rightarrow \begin{bmatrix} 5/3 \\ 17/6 \end{bmatrix}$

$u_2 = \frac{5+v_1}{3} = \frac{5+17/6}{3} = 47/18$

$v_2 = \frac{4+u_2}{2} = \frac{4+47/18}{2} = 119/36 \Rightarrow \begin{bmatrix} 47/18 \\ 119/36 \end{bmatrix}$

Gauss-Seidel

b) $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{matrix} 2u - v = 0 \\ -u + 2v - w = 2 \\ -v + 2w = 0 \end{matrix} \Rightarrow \begin{matrix} u = v/2 \\ v = (u+w+2)/2 \\ w = v/2 \end{matrix}$

$\begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0/2 \\ 1 \\ 0 \end{bmatrix}$

Jacobi:

$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \end{bmatrix}$

$v_0 = 0 \Rightarrow u_1 = 0/2 \Rightarrow v_1 = (0+0+2)/2 = 1 \Rightarrow w_1 = 1/2$

$\Rightarrow \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1/2 \end{bmatrix} \Rightarrow \begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ 3/4 \end{bmatrix}$ Gauss-Seidel

c) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$

$\begin{matrix} 3u + v + w = 6 \\ u + 3v + w = 3 \\ u + v + 3w = 5 \end{matrix} \Rightarrow \begin{matrix} u = (6 - v - w)/3 \\ v = (3 - u - w)/3 \\ w = (5 - u - v)/3 \end{matrix}$

$\begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5/3 \end{bmatrix} \Rightarrow$

Jacob:

$u_2 = (6 - 1 - 5/3)/3$

$v_2 = (3 - 2 - 5/3)/3$

$w_2 = (5 - 2 - 1)/3 = \begin{bmatrix} 10/9 \\ -2/9 \\ 2/3 \end{bmatrix}$

$\begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{matrix} (6-0-0)/3 = 2 \\ (3-2-0)/3 = 1/3 \\ (5-2-1)/3 = 8/9 \end{matrix}$

$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{matrix} (6 - 4 - 8/9)/3 = (43/27) \\ (3 - 43/27 - 8/9)/3 = (3 - 67/27)/3 = (14/27)/3 = (19/81) \\ (5 - 43/27 - 19/81)/3 = (405/81 - 129/81 - 19/81)/3 = (262/243) \end{matrix} = \begin{bmatrix} 43/27 \\ 19/81 \\ 262/243 \end{bmatrix}$

Gauss-Seidal