HW 8.2-4

Monday, August 3, 2020 8:49 AM

Q: Prove that if $S(x)$ is twice differentiable, then $u(x,t) = S(\alpha x + c\alpha t)$ is a solution of the wave equation (8.28)

- The wave equation is $u_{tt} = c^2 u_{xx}$
- Differentiating with respect to t gives:

$$u(x,t)_t = \frac{\partial}{\partial t}(S(\alpha x + c\alpha t))$$
$$= S(\alpha x + c\alpha t)_t \cdot \frac{\partial}{\partial t}(c\alpha t)$$
$$= S(\alpha x + c\alpha t)_t \cdot (c\alpha)$$

- Thus $u(x,t)_{tt} = \frac{\partial}{\partial t}(c\alpha S(\alpha x + c\alpha t)_t)$

$$= c^2 \alpha^2 S^2(\alpha x + c\alpha t)_{tt}$$

- Similarily $u(x,t)_x = \frac{\partial}{\partial x}(S(\alpha x + c\alpha t))$

$$= S(\alpha x + c\alpha t)_x \cdot \frac{\partial}{\partial x}(\alpha x)$$
$$= \alpha S(\alpha x + c\alpha t)_x$$

- Thus again, $u(x,t)_{xx} = \frac{\partial}{\partial x}(\alpha S(\alpha x + c\alpha t)_x)$

$$= \alpha^2 S(\alpha x + c\alpha t)_{xx}$$

- Comparing the two equations gives:

$u(x,t)_{tt} = c^2\alpha^2 S(\alpha x + c\alpha t)_{tt}$ and $u(x,t)_{xx} = \alpha^2 S(\alpha x + c\alpha t)_{xx}$

- Thus $u(x,t)_{tt} = c^2 \cdot u(x,t)_{xx}$ $\square$