HW 8.1-4

Monday, August 3, 2020 8:49 AM

Q: Is the Backward Difference Method unconditionally stable for the heat equation if $c < 0$? Explain.

- The heat equation is given by: $u_t = cu_{xx}$
- Then the Backward Difference Method gives the following for the heat equation:

  $\Rightarrow w_t = A^{-1}w_{j-1} + b$
- Thus the eigenvalues are $1 + 2\sigma(1 - \cos(\pi j/m+1))$ where $\sigma = ck/h^2$ and where h, k are the stepsizes for x, t respectively.
- Then since $c < 0$ and $\sigma = ck/h^2$, $c < 0 \Rightarrow \sigma < 0$.
- Additionally, $-1 \leq \cos x \leq 1$ for all $x \in \mathbb{R}$. Thus $(1 - \cos x) \in [0, 2]$ for all x.
- Then since $2\sigma < 0$ and $(1 - \cos x) > 0$, $2\sigma(1 - \cos x) < 0$.
  - $2\sigma(1 - \cos x) < 0 \Rightarrow |1 + 2\sigma(1 - \cos x)| < 1$.
  - Thus, the Backward Difference Method is <u>not</u> unconditionally stable for the heat equation when $c < 0$. □