1. Two cards are drawn successively from a pack without replacing the first. If the first card is spade, then what is the probability that the second card is also a spade?

(A) $\frac{13}{51}$

(B) $\frac{12}{13}$

(C) $\frac{4}{17}$

~~(D)~~ $\frac{3}{17}$

Total ways $= {}^{52}C_2 = \frac{52 \times 51}{2}$

Favourable outcomes

$= {}^{13}C_2 = \frac{13 \times 12}{2}$

Prob. $= \dfrac{\frac{\cancel{13} \times \cancel{12}\, 3}{2}}{\frac{\cancel{52}\, 4 \times 51}{\cancel{2}}} = \frac{3}{17}$

2. Four of the seven balls in a box have odd numbers on them. What is the probability that all three balls picked at random, one after the other, without replacement, have odd numbers?

(A) $\frac{24}{343}$

~~(B)~~ $\frac{4}{35}$

(C) $\frac{2}{7}$

(D) $\frac{64}{343}$

Total ways $= {}^{7}C_3 = \frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5}{6}$

$= 35$

Favourable ways $= {}^{4}C_3$

Prob $= \frac{{}^{4}C_3}{{}^{7}C_3} = \dfrac{\frac{4!}{3!\,1!}}{\frac{7!}{\cancel{3!}\,4!}} = \frac{\cancel{24}\, 4}{7 \times \cancel{6} \times 5}$

$= \frac{4}{35}$

3. Two urns contain respectively 2 white and 1 black balls, and 1 white and 5 black balls. One ball is transferred from the first to the second urn, and then a ball is drawn from the second urn. What is the probability that the ball drawn is white?

(A) $\frac{1}{21}$

(B) $\frac{1}{3}$

(C) $\frac{4}{21}$

(D) $\frac{5}{21}$

Fav. case $= \frac{2}{3} \times \frac{2}{7} + \frac{1}{3} \times \frac{1}{7}$

$= \frac{5}{21}$

4. There are 2 boxes: Box-1 contains 3 silver spoons and 3 copper spoons and Box-2 contains 5 copper and 3 silver spoons. Assume that Box-1 is likely to be chosen with a probability of $\frac{2}{3}$ and Box-2 is likely to be chosen with a probability of $\frac{1}{3}$. A spoon is chosen at random from a box that has been randomly selected. What is the probability that it is a silver spoon?

(A) $\frac{11}{24}$

(B) $\frac{1}{3}$

(C) $\frac{1}{8}$

(D) $\frac{5}{24}$

Fav. case

$= \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{6} + \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{8}$

$= \frac{1}{3} + \frac{1}{8} = \frac{3+8}{24} = \frac{11}{24}$

5. The chance that a certain disease is diagnosed correctly is 60%. The chance that the patient will die under the treatment, after correct diagnosis is 40%; and the chance of death by wrong diagnosis is 70%. A patient who had the disease died. What is the probability that his / her disease was diagnosed correctly?

(A) $\frac{24}{25}$

(B) $\frac{6}{13}$

(C) $\frac{3}{14}$

(D) $\frac{1}{6}$

$P(A/B) = P(B|A) \times \frac{P(A)}{P(B)}$

$P\left(\frac{\text{Diag}}{\text{Died}}\right) = P\left(\frac{\text{Died}}{\text{Diag}}\right) \times \frac{P(\text{Diag})}{P(\text{Died})}$

$= \frac{0.4 \times 0.6}{0.7 \times 0.4 + 0.4 \times 0.6} = \frac{\cancel{24}\ 6}{\cancel{52}\ 13}$

$\begin{array}{r} 28 \\ 24 \\ \hline 52 \end{array}$

6. Let X be a discrete random variable with the p.m.f. as

X=x	-2	-1	0	1	2
p(x)	1/9	2/9	3/9	2/9	1/9

Find $P(|X| > 1)$.

(A) $\frac{1}{3}$

(B) $\frac{7}{9}$

~~(C)~~ $\frac{2}{9}$

(D) $\frac{8}{9}$

Solution:

$= P(X>1) + P(X<-1)$

$P(|X|>1) = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

7. It is known that 2.5% of mobile phone chargers fail during the warranty period provided they are kept dry. The failure percentage is 5.6, if they are ever wet during the warranty period. If 91% of the chargers are kept dry and 9% are wet during warranty period, what is the probability that a phone charger fails during the warranty period?

(A) 0.3321

(B) 0.4392

(C) 0.0391

~~(D)~~ 0.0278

Given $P\left(\frac{\text{fail}}{\text{dry}}\right) = 0.025$

$P\left(\frac{\text{fail}}{\text{wet}}\right) = 0.056$

$P(\text{dry}) = 0.91$

$P(\text{wet}) = 0.09$

$P(\text{fail}) = P\left(\frac{\text{fail}}{\text{dry}}\right) \times P(\text{dry}) + P\left(\frac{\text{fail}}{\text{wet}}\right) \times P(\text{wet})$

$= 0.025 \times 0.91 + 0.056 \times 0.09$

$= \frac{25 \times 91}{100000} + \frac{56 \times 9}{100000} = \frac{2275 + 504}{100000}$

$= 0.02779 = 0.278$

Q8 A random variables X has the following probability mass functions

X	0	1	2	3	4	5
$P(X=x)$	0	$2k$	$3k$	$2k^2$	$4k$	$9k^2+k$

Find $P(X<3)$.

(A) 0.622
(B) 0.378
~~(C)~~ 0.455
(D) 0.471

$P(X<3)$
$= P(0)+P(1)+P(2)$
$= 5k = \frac{5}{11} = 0.455$

$9k^2+k+4k+2k^2+3k+2k+0 = 1$
$11k^2+10k=1$
$11k^2+11k-k-1=0$
$11k(k+1)-1(k+1)=0$
$k=-1, \quad k=\frac{1}{11}$

9\. Suppose the probabilities of n mutually independent events are $p_1, p_2, \ldots, p_n$ respectively. Then what is the probability that at least one of the events will occur?

(A) $1 - p_1 \ldots p_n$
~~(B)~~ $1-(1-p_1)\ldots(1-p_n)$
(C) $(1-p_1)\ldots(1-p_n)$
(D) $p_1 \ldots p_n$

At least one
= Total - none
$= 1-(1-P_1).(1-P_2)\ldots-.(1-P_n)$

10. Some elements in a chemical laboratory are highly contaminated. A high amount of contamination in an element is likely to exist with a 0.1 probability. The levels of contamination are evaluated on four randomly chosen elements. How likely is it that at least one of them has a high level of contamination?

(A) 0.2048
(B) 0.4096
(C) 0.6561
(D) 0.7408

At least one

= Total − none

$= 1 - (0.9)^4$

$= 1 - 0.9 \times 0.9 \times 0.9 \times 0.9$

$= 1 - 0.81 \times 0.81$

$= 1 - 0.64 = 0.36$

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