Assignment - 1

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***Abstract*—This document contains the solution to Exercise 2.32 of Oppenheim.**

Problem 1. Consider an LTI system with frequency response:

$$H(e^{j\omega}) = e^{-j(\omega-\pi/4)}\left(\frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}}\right), \quad -\pi < \omega < \pi,$$

Determine output $y[n]$ for all n if the input for all n is :

$$x[n] = \cos(\frac{\pi n}{2})$$

Solution:

$$\begin{aligned} H(e^{j\omega}) &= e^{j\pi/4} \cdot e^{-j\omega}\left(\frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}}\right) \\ &= e^{j\pi/4}G(e^{j\omega}) \end{aligned}$$

Let $y_1[n] = x[n] * g[n]$, then

$$\begin{aligned} x[n] &= \cos\left(\frac{n\pi}{2}\right) = \frac{e^{j\pi n/2}+e^{-j\pi n/2}}{2} \\ y_1[n] &= \frac{G(e^{j\pi/2})e^{j\pi n/2}+G(e^{-j\pi/2})e^{j\pi n/2}}{2} \end{aligned}$$

Evaluating the frequency response at $\omega = \pm\pi/2$:

$$\begin{aligned} G(e^{j\pi/2}) &= e^{-j\pi/2}\frac{1+e^{-j\pi}+4e^{-j2\pi}}{1+\frac{1}{2}e^{-j\pi}} = 8e^{-j\pi/2} \\ G(e^{-j\pi/2}) &= 8e^{j\pi/2} \end{aligned}$$

Therefore,

$$y_1[n] = (8e^{j(\pi n/2-\pi/2)}+8e^{j(-\pi n/2+\pi/2)})/2 = 8\cos\left(\frac{\pi}{2}n-\frac{\pi}{2}\right)$$

and
$y[n] = e^{j\pi/4}y_1[n] = 8e^{j\pi/4}\cos(\frac{\pi}{2}n-\frac{\pi}{2})$