Assignment - 2

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***Abstract*—This document contains the solution to Exercise 2.32 of Oppenheim.**

Problem 1. Consider a system with input $x[n]$ and output $y[n]$. The input-output relation for the system is defined by following two properties:

1) $y[n] - ay[n-1] = x[n]$,
2) $y[0] = 1$.

Determine whether the system is linear.

Solution:

We can re-write the system function $H(e^{j\omega})$ as :

$$H(e^{j\omega})) = e^{j\pi/4} \cdot e^{-j\omega} \left(\frac{1 + e^{-j2\omega} + 4e^{-j4\omega}}{1 + \frac{1}{2}e^{-j2\omega}} \right)$$

$$= e^{j\pi/4} G(e^{j\omega})$$

Let $y_1[n] = x[n] * g[n]$, then

$$x[n] = \cos(\frac{n\pi}{2}) = \frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2}$$

$$y_1[n] = \frac{G(e^{j\pi/2})e^{j\pi n/2} + G(e^{-j\pi/2})e^{j\pi n/2}}{2}$$

Evaluating the frequency response at $\omega = \pm\pi/2$:

$$G(e^{j\pi/2}) = e^{-j\pi/2} \frac{1 + e^{-j\pi} + 4e^{-j2\pi}}{1 + \frac{1}{2}e^{-j\pi}} = 8e^{-j\pi/2}$$

$$G(e^{-j\pi/2}) = 8e^{j\pi/2}$$

Therefore,

$$y_1[n] = (8e^{j(\pi n/2 - \pi/2)} + 8e^{j(-\pi n/2 + \pi/2)})/2 = 8\cos(\frac{\pi}{2}n - \frac{\pi}{2})$$

and

$$y[n] = e^{j\pi/4} y_1[n] = 8e^{j\pi/4} \cos(\frac{\pi}{2}n - \frac{\pi}{2})$$