

# Assignment 18

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**Abstract**—This document solves a problem based on polynomial vector spaces.

Download latex-tikz codes from

[https://github.com/Vaibhav11002/EE5609/tree/master/Assignment\\_18](https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_18)

## 1 PROBLEM

Consider the vector space  $V$  of real polynomials of degree less than or equal to  $n$ . Fix distinct real numbers  $a_0, a_1, \dots, a_k$ . For  $p \in V$

$$\max \{|p(a_j)| : 0 \leq j \leq k\}$$

defines a norm on  $V$

- 1) only if  $k < n$
- 2) only if  $k \geq n$
- 3) if  $k + 1 \leq n$
- 4) if  $k \geq n + 1$

## 2 SOLUTION

Options 2 and 4 are correct as verified in the table

Properties	Norm $\forall x \in V$
Positivity	$\ x\  \geq 0, \ x\  = 0 \iff x = 0$
Scalar Multiplication	$\ \alpha x\  =  \alpha  \ x\ , \alpha \in F$
Triangle Inequality	$\ x + y\  \leq \ x\  + \ y\ $

TABLE 1: Properties of Norm

For $p \in V$ then the norm, $\max \{ p(a_j)  : 0 \leq j \leq k\} = 0 \iff  p(a_j) _{0 \leq j \leq k} = 0$	
Conditions	Explanation
only if $k < n$  Example:	<p>A polynomial doesn't necessarily have <math>k</math> distinct real roots, i.e., it may have repeated, complex roots.</p> <p>let <math>p</math> be polynomial of degree <math>n = 2</math> and <math>k = 1</math> given by:-</p> $p(x) = x^2 + 4x + 4 \quad (2.0.1)$ $ p(a_j) _{0 \leq j \leq 1} = 0 \implies a_0 = -2, a_1 = -2 \quad (2.0.2)$ <p>but <math>a_0, a_1, \dots, a_k</math> should be distinct real numbers.</p> <p>This contradicts the property of Norm. Thus condition fails.</p>
only if $k \geq n$	<p><math>p</math> is a polynomial of degree <math>\leq n</math>, it can't have more than <math>n</math> roots and is only possible when,</p> $p(x) = 0 \implies  p(a_j) _{0 \leq j \leq k} = 0$ <p>hence <math>p</math> is identically zero. Thus condition satisfies.</p>
if $k + 1 \leq n$	Not a norm for $k < n$ . Hence incorrect.
if $k \geq n + 1$	Norm for $k \geq n$ . Hence correct.

TABLE 2: Verifying Positivity Property of Norm