Assignment 18

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***Abstract*—This document solves a problem based on polynomial vector spaces.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_18

1 Problem

Consider the vector space V of real polynomials of degree less than or equal to n. Fix distinct real numbers $a_0, a_1, \cdots, a_k$. For $p \in V$

$$max\left\{\left|p(a_j)\right| : 0 \leq j \leq k\right\}$$

defines a norm on V

1) only if $k < n$
2) only if $k \geq n$
3) if $k + 1 \leq n$
4) if $k \geq n + 1$

2 Solution

Options 2 and 4 are correct as verified in the table 2

Properties	Norm $\forall x \in V$
Positivity	$\|\|x\|\| \geq 0, \|\|x\|\| = 0 \iff x = 0$
Scalar Multiplication	$\|\|\alpha x\|\| = \|\alpha\| \, \|\|x\|\|, \alpha \in F$
Triangle Inequality	$\|\|x + y\|\| \leq \|\|x\|\| + \|\|y\|\|$

TABLE 1: Properties of Norm

For $p \in V$ then the norm, $max\left\{\left\|p(a_j)\right\| : 0 \leq j \leq k\right\} = 0 \iff \left\|p(a_j)\right\|_{0 \leq j \leq k} = 0$	
Conditions	**Explanation**
only if $k < n$	A polynomial doesn't necessarily have $k + 1$ distinct real roots, i.e., for some j, $\left\|p(a_j)\right\| \neq 0$ thus p is not identically zero. Thus condition fails.
only if $k \geq n$	p is a polynomial of degree ≤n, it can't have more than n roots and is only possible when, $p(x) = 0 \implies \left\|p(a_j)\right\|_{0 \leq j \leq k} = 0$ hence p is identically zero. Thus condition satisfies.
if $k + 1 \leq n$	Not a norm for $k < n$. Hence incorrect.
if $k \geq n + 1$	Norm for $k \geq n$. Hence correct.

TABLE 2: Verifying Positivity Property of Norm