Assignment 14

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Abstract—This document solves a problem involving linear transformations.

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/
  master/Assignment_14

1 Problem

Let V be the set of all complex numbers regarded as a vector space over the field of real numbers(usual operations). Find a function from V into V which is a linear transformation on the above vector space, but which is not a linear transformation on $\mathbb{C}$ i.e., which is not complex linear.

2 Solution

Let

$$T : V \to V \tag{2.0.1}$$

be a function such that,

$$T(x + iy) = Re(x + iy) = x \tag{2.0.2}$$
$$\implies T : x + iy \to x \tag{2.0.3}$$

where $x, y \in \mathbb{R}$.

Let, $\alpha = a + ib$, $\beta = c + id$.

$$\implies T(k\alpha + \beta) = T(ka + ikb + c + id) \tag{2.0.4}$$
$$= T(ka + c + i(kb + d)) \tag{2.0.5}$$
$$= ka + c \tag{2.0.6}$$
$$= kT(\alpha) + T(\beta) \tag{2.0.7}$$

Now, let $z \in V$ such that,

$$z = i \tag{2.0.8}$$
$$\implies T(z) = T(i) = 0 \tag{2.0.9}$$

We can also write,

$$T(i) = T(i(1)) = iT(1) = i \neq 0 \tag{2.0.10}$$

Thus from (2.0.7), T is real linear transformation and from (2.0.10), T is not complex linear.