Assignment 7

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***Abstract*—This document performs the QR decomposition on a matrix.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_7

1 Problem

Find the QR decomposition of $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

2 Solution

Let α and β be the column vectors of the given matrix.

$$\alpha = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{2.0.2}$$

The column vectors can be represented as,

$$\alpha = k_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\beta = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

where,

$$k_1 = \|\alpha\| \tag{2.0.5}$$

$$\mathbf{u_1} = \frac{\alpha}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u_2} = \frac{\beta - r_1 \mathbf{u_1}}{\|\beta - r_1 \mathbf{u_1}\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u_2}^T \beta \tag{2.0.9}$$

From (2.0.3) and (2.0.4),

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Where $\mathbf{R}$ is an upper triangular matrix and

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{3^2 + 1^2} = \sqrt{10} \tag{2.0.13}$$

$$\mathbf{u_1} = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix} \tag{2.0.14}$$

$$r_1 = \begin{pmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \sqrt{10} \tag{2.0.15}$$

$$\mathbf{u_2} = \begin{pmatrix} \frac{-1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \begin{pmatrix} \frac{-1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \sqrt{10} \tag{2.0.17}$$

Now putting the values from (2.0.13) to (2.0.17), we obtain the QR decomposition of given matrix,

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \sqrt{10} & \sqrt{10} \\ 0 & \sqrt{10} \end{pmatrix} \tag{2.0.18}$$