Assignment 16

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Abstract—**This document solves a problem involving ordered basis of linear transformation.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_16

1 Problem

Let V be an n-dimensional vector space over the field F, and let $\mathcal{B} = \{\alpha_1, \ldots, \alpha_n\}$ be an ordered basis for V then there is a unique linear operator T on V such that

$$T\alpha_j = \alpha_{j+1}, j = 1, \ldots, n-1$$
$$T\alpha_n = 0.$$

What is the matrix A of T in the ordered basis $\mathcal{B}$?

2 Coordinates of a Vector

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be the ordered basis of an n-dimensional vector space V over field F and let $\mathbf{v} \in V$. If

$$\mathbf{v} = \beta_1\mathbf{v}_1 + \beta_2\mathbf{v}_2 + \ldots + \beta_n\mathbf{v}_n \tag{2.0.1}$$

then the tuple $(\beta_1, \beta_2, \ldots, \beta_n)$ is called the coordinate of the vector $\mathbf{v}$ with respect to the ordered basis $\mathcal{B}$. It is denoted by the column vector,

$$[\mathbf{v}]_{\mathcal{B}} = (\beta_1, \beta_2, \ldots, \beta_n)^T \tag{2.0.2}$$

3 Solution

Given that,

$$T : V \to V \tag{3.0.1}$$
$$[T(\alpha)]_{\mathcal{B}} = A[\alpha]_{\mathcal{B}} \tag{3.0.2}$$
$$T\alpha_j = \alpha_{j+1} \tag{3.0.3}$$
$$T\alpha_n = 0 \tag{3.0.4}$$

where $j = 1, \ldots, n-1$. The matrix A of T in the ordered basis $\mathcal{B}$ is given by,

$$\implies A = \begin{pmatrix} [T\alpha_1]_{\mathcal{B}} & \cdots & [T\alpha_n]_{\mathcal{B}} \end{pmatrix} \tag{3.0.5}$$

For $j = 1, \ldots, n-1$ we have,

$$T\alpha_j = \alpha_{j+1} \tag{3.0.6}$$

From (2.0.1), (2.0.2) we can write,

$$T\alpha_j = 0\alpha_1 + \ldots + 0\alpha_j + 1\alpha_{j+1} + \ldots + 0\alpha_n \tag{3.0.7}$$
$$\implies [T\alpha_j]_{\mathcal{B}} = (0, \ldots, 0, 1, 0, \ldots, 0)^T \tag{3.0.8}$$

where 1 is in $(j+1)$th position. Now,

$$T\alpha_n = 0 \tag{3.0.9}$$
$$\implies [T\alpha_n]_{\mathcal{B}} = 0 \tag{3.0.10}$$

Thus from (3.0.5), (3.0.8) and (3.0.10) we get matrix A in the ordered basis $\mathcal{B}$ as,

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \tag{3.0.11}$$