

Assignment 18

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Abstract—This document solves a problem based on polynomial vector spaces.

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_18

1 PROBLEM

Consider the vector space V of real polynomials of degree less than or equal to n . Fix distinct real numbers a_0, a_1, \dots, a_k . For $p \in V$

$$\max \{ |p(a_j)| : 0 \leq j \leq k \}$$

defines a norm on V

- 1) only if $k < n$
- 2) only if $k \geq n$
- 3) if $k + 1 \leq n$
- 4) if $k \geq n + 1$

2 SOLUTION

Options 2 and 4 are correct as verified in the table

Properties	Norm $\forall x \in V$
1. Positivity	$\ x\ \geq 0, \ x\ = 0 \iff x = 0$
2. Scalar Multiplication	$\ \alpha x\ = \alpha \ x\ , \alpha \in F$
3. Triangle Inequality	$\ x + y\ \leq \ x\ + \ y\ $

TABLE 1: Properties of Norm

For $p \in V$ then the norm, $\max \{ p(a_j) : 0 \leq j \leq k\} = 0 \iff p(a_j) _{0 \leq j \leq k} = 0$	
Conditions	Explanation
only if $k < n$	A polynomial doesn't necessarily have $k + 1$ distinct real roots, i.e., for some j , $ p(a_j) \neq 0$ thus p is not identically zero. Thus condition fails.
only if $k \geq n$	p is a polynomial of degree $\leq n$, it can't have more than n roots and is only possible when, $p(x) = 0 \implies p(a_j) _{0 \leq j \leq k} = 0$ hence p is identically zero. Thus condition satisfies.
if $k + 1 \leq n$	Not a norm for $k < n$. Hence incorrect.
if $k \geq n + 1$	Norm for $k \geq n$. Hence correct.

TABLE 2: Verifying Property 1 of Norm