Assignment 10

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***Abstract*—This document solves a problem involving matrix multiplication.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_10

1 Problem

Find two different 2×2 matrices $\mathbf{A}$ such that $\mathbf{A}^2 = 0$ but $\mathbf{A} \neq 0$

2 Solution

The matrix $\mathbf{A}$ can be given by,

$$\mathbf{A} = \begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{2.0.2}$$

Now,

$$\mathbf{A}^2 = \mathbf{A}\mathbf{A} = \mathbf{0} \tag{2.0.3}$$

$$\implies \mathbf{A}^2 = \begin{pmatrix} \mathbf{A}\mathbf{m} & \mathbf{A}\mathbf{n} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{2.0.4}$$

From (2.0.4), we say that the the null space of $\mathbf{A}$ contains columns of matrix $\mathbf{A}$. Also atleast one of the columns must be non-zero since given $\mathbf{A} \neq 0$. Thus, the null space of $\mathbf{A}$ contains non zero vectors, $rank(\mathbf{A}) < 2$. Hence, $\mathbf{A}$ is a singular matrix. This implies that the columns of $\mathbf{A}$ are linearly dependent.

$$\mathbf{A}\mathbf{x} = 0 \tag{2.0.5}$$

$$\begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \tag{2.0.6}$$

$$x_1\mathbf{m} + x_2\mathbf{n} = 0 \tag{2.0.7}$$

$$\mathbf{n} = \frac{-x_1}{x_2}\mathbf{m} \tag{2.0.8}$$

$$\implies \mathbf{n} = k\mathbf{m} \tag{2.0.9}$$

where $\mathbf{m} \neq 0$ as $\mathbf{A} \neq 0$

Now from (2.0.4),

$$\mathbf{A}\mathbf{m} = 0 \tag{2.0.10}$$

$$m_1\mathbf{m} + m_2\mathbf{n} = 0 \tag{2.0.11}$$

$$(m_1 + km_2)\,\mathbf{m} = 0 \tag{2.0.12}$$

Thus we get, $m_1 = -km_2$

$$\mathbf{A} = \begin{pmatrix} -km_2 & -k^2m_2 \\ m_2 & km_2 \end{pmatrix}; m_2 \neq 0 \tag{2.0.13}$$

(2.0.9) can be written as,

$$\implies \mathbf{m} = \frac{1}{k}\mathbf{n} \tag{2.0.14}$$

$$\implies \mathbf{m} = c\mathbf{n} \tag{2.0.15}$$

where $\mathbf{n} \neq 0$ as $\mathbf{A} \neq 0$

From (2.0.4),

$$\mathbf{A}\mathbf{n} = 0 \tag{2.0.16}$$

$$n_1\mathbf{m} + n_2\mathbf{n} = 0 \tag{2.0.17}$$

$$(cn_1 + n_2)\,\mathbf{n} = 0 \tag{2.0.18}$$

Thus we get, $n_2 = -cn_1$

$$\mathbf{A} = \begin{pmatrix} cn_1 & n_1 \\ -c^2n_1 & -cn_1 \end{pmatrix}; n_1 \neq 0 \tag{2.0.19}$$

From (2.0.13), (2.0.19) two different 2×2 matrices $\mathbf{A}$ can be given as,

$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \tag{2.0.20}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \tag{2.0.21}$$