Assignment 10

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***Abstract*—This document solves a problem involving matrix multiplication.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_10

1 Problem

Find two different 2×2 matrices $\mathbf{A}$ such that $\mathbf{A}^2 = 0$ but $\mathbf{A} \neq 0$

2 Solution

The matrix $\mathbf{A}$ can be given by,

$$\mathbf{A} = \begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{2.0.2}$$

Now,

$$\mathbf{A}^2 = \mathbf{A}\mathbf{A} = 0 \tag{2.0.3}$$

$$\implies \mathbf{A}^2 = \begin{pmatrix} m_1 & n_1 \\ m_2 & n_2 \end{pmatrix} \begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix} = \begin{pmatrix} \mathbf{c_1} & \mathbf{c_2} \end{pmatrix} = 0 \tag{2.0.4}$$

From (2.0.4), we say that the columns of matrix $\mathbf{A}$ must lie in the null space of $\mathbf{A}$. Also atleast one of the columns must be non-zero since given $\mathbf{A} \neq 0$. Thus, the null space of $\mathbf{A}$ contains non zero vectors, hence $rank(\mathbf{A}) < 2$. This implies that the columns of $\mathbf{A}$ are linearly dependent. Hence, $\mathbf{A}$ is a singular matrix.

$$\mathbf{c_1} = m_1 \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + m_2 \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = 0 \tag{2.0.5}$$

$$\mathbf{c_2} = n_1 \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + n_2 \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = 0 \tag{2.0.6}$$

Let's assume,

$$\mathbf{m} = \begin{pmatrix} a \\ a \end{pmatrix} \tag{2.0.7}$$

then,

$$\mathbf{c_1} = \begin{pmatrix} a^2 + an_1 \\ a^2 + an_2 \end{pmatrix} = 0 \tag{2.0.8}$$

thus $a = 0$ or $n_1 = n_2 = -a$. When $a = 0$,

$$\implies \mathbf{c_2} = \begin{pmatrix} n_1 n_2 \\ n_2^2 \end{pmatrix} = 0 \tag{2.0.9}$$

$$\implies n_2 = 0 \tag{2.0.10}$$

Thus we get,

$$\mathbf{A} = \begin{pmatrix} a & -a \\ a & -a \end{pmatrix}; a \neq 0 \tag{2.0.11}$$

and

$$\mathbf{A} = \begin{pmatrix} 0 & n_1 \\ 0 & 0 \end{pmatrix}; n_1 \neq 0 \tag{2.0.12}$$

Now, let's assume,

$$\mathbf{n} = \begin{pmatrix} a \\ a \end{pmatrix} \tag{2.0.13}$$

then,

$$\mathbf{c_2} = \begin{pmatrix} am_1 + a^2 \\ am_2 + a^2 \end{pmatrix} = 0 \tag{2.0.14}$$

thus $a = 0$ or $m_1 = m_2 = -a$. When $a = 0$,

$$\implies \mathbf{c_1} = \begin{pmatrix} m_1^2 \\ m_1 m_2 \end{pmatrix} = 0 \tag{2.0.15}$$

$$\implies m_1 = 0 \tag{2.0.16}$$

Thus we get,

$$\mathbf{A} = \begin{pmatrix} -a & a \\ -a & a \end{pmatrix}; a \neq 0 \tag{2.0.17}$$

and

$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ m_2 & 0 \end{pmatrix}; m_2 \neq 0 \tag{2.0.18}$$

One such matrix also is,

$$\mathbf{A} = \begin{pmatrix} a & a \\ -a & -a \end{pmatrix}; a \neq 0 \tag{2.0.19}$$

Thus two different 2×2 matrices are,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{2.0.20}$$

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix} \tag{2.0.21}$$