# Assignment 11

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***Abstract*—This document solves a problem involving vector spaces.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_11

## 1 Problem

If $\mathbb{C}$ is the field of complex numbers, which vectors in $\mathbf{C}^3$ are linear combinations of $\begin{pmatrix}1\\0\\-1\end{pmatrix}$, $\begin{pmatrix}0\\1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\1\\1\end{pmatrix}$ ?

## 2 Solution

Expressing the given vectors as the columns of a matrix,

$$\mathbf{A} = \begin{pmatrix}1 & 0 & 1\\0 & 1 & 1\\-1 & 1 & 1\end{pmatrix} \tag{2.0.1}$$

The row reduced echelon form of the matrix on performing elementary row operations can be given as,

$$\mathbf{R} = \mathbf{CA} \tag{2.0.2}$$

where $\mathbf{C}$ is the product of elementary matrices,

$$\mathbf{C} = \begin{pmatrix}0 & 1 & -1\\-1 & 2 & -1\\1 & -1 & 1\end{pmatrix} \tag{2.0.3}$$

Thus we get,

$$\mathbf{R} = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} \tag{2.0.4}$$

From (2.0.4), $rank(\mathbf{A}) = 3$. Thus $\mathbf{A}$ is a full rank matrix. Hence the columns of $\mathbf{A}$ are linearly independent i.e., the given vectors are linearly independent and forms the basis for $\mathbf{C}^3$.

Hence any vector $\mathbf{Y} \in \mathbf{C}^3$ can be written as the linear combinations of $\begin{pmatrix}1\\0\\-1\end{pmatrix}$, $\begin{pmatrix}0\\1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\1\\1\end{pmatrix}$.