Assignment 9

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***Abstract*—This document solves whether two system of linear equations are linear equivalent or not.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_9

1 Problem

Let $\mathbb{F}$ be the field of complex numbers. Are the following two systems of linear equations equivalent? If so, express each equation in each system as a linear combination of the equations in the other system.

$$\mathbf{x_1} - \mathbf{x_2} = 0$$
$$2\mathbf{x_1} + \mathbf{x_2} = 0$$

and

$$3\mathbf{x_1} + \mathbf{x_2} = 0$$
$$\mathbf{x_1} + \mathbf{x_2} = 0$$

2 Solution

The given system of linear equations are,

$$A_1 : \quad \mathbf{x_1} - \mathbf{x_2} = 0 \tag{2.0.1}$$
$$A_2 : \quad 2\mathbf{x_1} + \mathbf{x_2} = 0 \tag{2.0.2}$$

Forming augmented matrix and applying row reduction,

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix} \tag{2.0.3}$$
$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{3}} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.4}$$
$$\tag{2.0.5}$$

Thus, the solution of the system is,

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$

The second system of linear equations is,

$$B_1 : \quad 3\mathbf{x_1} + \mathbf{x_2} = 0 \tag{2.0.7}$$
$$B_2 : \quad \mathbf{x_1} + \mathbf{x_2} = 0 \tag{2.0.8}$$

Forming augmented matrix and applying row reduction,

$$\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 3R_2 - R_1} \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} \tag{2.0.9}$$
$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{2}} \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1 - R_2}{3}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.10}$$

The solution of the system is,

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.11}$$

From (2.0.6) and (2.0.11), both the linear system of equations have same solution set. Hence the two system of linear equations are equivalent. Now,

$$A_1 = (1)B_1 + (-2)B2 \tag{2.0.12}$$
$$A_2 = (\frac{1}{2})B_1 + (\frac{1}{2})B_2 \tag{2.0.13}$$

Second system of equations,

$$B_1 = (\frac{1}{3})A_1 + (\frac{4}{3})A_2 \tag{2.0.14}$$
$$B_2 = (\frac{-1}{3})A_1 + (\frac{2}{3})A_2 \tag{2.0.15}$$

Thus each equation in each system can be expressed as linear combination of the equations in the other system if both systems are equivalent.