

Assignment 9

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Abstract—This document solves whether two system of linear equations are linear equivalent or not.

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_9

1 PROBLEM

Let \mathbb{F} be the field of complex numbers. Are the following two systems of linear equations equivalent? If so, express each equation in each system as a linear combination of the equations in the other system.

$$x_1 - x_2 = 0$$

$$x_1 + x_2 = 0$$

and

$$3x_1 + x_2 = 0$$

$$x_1 + x_2 = 0$$

2 SOLUTION

The given system of linear equations can be written as,

$$\mathbf{Ax} = 0 \quad (2.0.1)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.2)$$

$$\mathbf{Bx} = 0 \quad (2.0.3)$$

$$\Rightarrow \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.4)$$

Consider \mathbf{A} , performing row operation using elementary matrices,

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \quad (2.0.5)$$

$$\leftrightarrow \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix} \quad (2.0.6)$$

$$\leftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \quad (2.0.7)$$

$$\leftrightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.8)$$

$$\leftrightarrow \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.9)$$

$$\leftrightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} \quad (2.0.10)$$

$$\leftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & \frac{2}{3} \end{pmatrix} \quad (2.0.11)$$

$$\leftrightarrow \begin{pmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 0 & \frac{2}{3} \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 0 \end{pmatrix} = \mathbf{B} \quad (2.0.12)$$

Thus we get \mathbf{B} from matrix \mathbf{A} by multiplying it with matrix \mathbf{C} , where \mathbf{C} is product of elementary matrices given as,

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \\ = \begin{pmatrix} \frac{1}{3} & \frac{4}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \quad (2.0.13)$$

Thus, since matrix \mathbf{B} can be obtained from matrix \mathbf{A} by a finite sequence of elementary row operations on \mathbf{A} , the given two system of linear equations are equivalent. Thus, '

$$\mathbf{B} = \mathbf{CA} \quad (2.0.14)$$

$$\Rightarrow \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{4}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \quad (2.0.15)$$

From (2.0.15), it can be seen that each equation in system \mathbf{B} is a linear combination of equation in system \mathbf{A} .