

Assignment 15

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Abstract—This document solves a problem involving ordered basis of linear transformation.

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_16

1 PROBLEM

Let V be an n -dimensional vector space over the field F , and let $\mathcal{B} = \{\alpha_1, \dots, \alpha_n\}$ be an ordered basis for V then there is a unique linear operator T on V such that

$$\begin{aligned} T\alpha_j &= \alpha_{j+1}, j = 1, \dots, n-1 \\ T\alpha_n &= 0. \end{aligned}$$

What is the matrix A of T in the ordered basis \mathcal{B} ?

2 COORDINATES OF A VECTOR

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ be the ordered basis of an n -dimensional vector space V over field F and let $\mathbf{v} \in V$. If

$$\mathbf{v} = \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \dots + \beta_n \mathbf{v}_n \quad (2.0.1)$$

then the tuple $(\beta_1, \beta_2, \dots, \beta_n)$ is called the coordinate of the vector \mathbf{v} with respect to the ordered basis \mathcal{B} . It is denoted by the column vector,

$$[\mathbf{v}]_{\mathcal{B}} = (\beta_1, \beta_2, \dots, \beta_n)^T \quad (2.0.2)$$

3 SOLUTION

Given that,

$$T : V \rightarrow V \quad (3.0.1)$$

$$[T(\alpha)]_{\mathcal{B}} = A[\alpha]_{\mathcal{B}} \quad (3.0.2)$$

$$T\alpha_j = \alpha_{j+1} \quad (3.0.3)$$

$$T\alpha_n = 0 \quad (3.0.4)$$

where $j = 1, \dots, n-1$. The matrix A of T in the ordered basis \mathcal{B} is given by,

$$\Rightarrow A = ([T\alpha_1]_{\mathcal{B}} \quad \dots \quad [T\alpha_n]_{\mathcal{B}}) \quad (3.0.5)$$

For $j = 1, \dots, n-1$ we have,

$$T\alpha_j = \alpha_{j+1} \quad (3.0.6)$$

From (2.0.1), (2.0.2) we can write,

$$T\alpha_j = 0\alpha_1 + \dots + 0\alpha_j + 1\alpha_{j+1} + \dots + 0\alpha_n \quad (3.0.7)$$

$$\Rightarrow [T\alpha_j]_{\mathcal{B}} = (0, \dots, 0, 1, 0, \dots, 0)^T \quad (3.0.8)$$

where 1 is in $(j+1)$ th position. Now,

$$T\alpha_n = 0 \quad (3.0.9)$$

$$\Rightarrow [T\alpha_n]_{\mathcal{B}} = 0 \quad (3.0.10)$$

Thus from (3.0.5), (3.0.8) and (3.0.10) we get matrix A in the ordered basis \mathcal{B} as,

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix} \quad (3.0.11)$$