

Assignment 7

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Abstract—This document performs the QR decomposition on a matrix.

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_7

1 PROBLEM

Find the QR decomposition of $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$

2 SOLUTION

Let \mathbf{c}_1 and \mathbf{c}_2 be the column vectors of the given matrix.

$$\mathbf{c}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{c}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (2.0.2)$$

The column vectors can be represented as,

$$\mathbf{c}_1 = k_1 \mathbf{u}_1 \quad (2.0.3)$$

$$\mathbf{c}_2 = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \quad (2.0.4)$$

where,

$$k_1 = \|\mathbf{c}_1\| \quad (2.0.5)$$

$$\mathbf{u}_1 = \frac{\mathbf{c}_1}{k_1} \quad (2.0.6)$$

$$r_1 = \frac{\mathbf{u}_1^T \mathbf{c}_2}{\|\mathbf{u}_1\|^2} \quad (2.0.7)$$

$$\mathbf{u}_2 = \frac{\mathbf{c}_2 - r_1 \mathbf{u}_1}{\|\mathbf{c}_2 - r_1 \mathbf{u}_1\|} \quad (2.0.8)$$

$$k_2 = \mathbf{u}_2^T \mathbf{c}_2 \quad (2.0.9)$$

From (2.0.3) and (2.0.4),

$$\begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 \end{pmatrix} = \mathbf{Q} \mathbf{R} \quad (2.0.11)$$

Where \mathbf{R} is an upper triangular matrix and

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \quad (2.0.12)$$

Using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{3^2 + 1^2} = \sqrt{10} \quad (2.0.13)$$

$$\mathbf{u}_1 = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix} \quad (2.0.14)$$

$$r_1 = \left(\frac{3}{\sqrt{10}} \quad \frac{1}{\sqrt{10}} \right) \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \sqrt{10} \quad (2.0.15)$$

$$\mathbf{u}_2 = \begin{pmatrix} \frac{-1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix} \quad (2.0.16)$$

$$k_2 = \left(\frac{-1}{\sqrt{10}} \quad \frac{3}{\sqrt{10}} \right) \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \sqrt{10} \quad (2.0.17)$$

Now putting the values from (2.0.13) to (2.0.17), we obtain the QR decomposition of given matrix,

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{10}} & \frac{-1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \sqrt{10} & \sqrt{10} \\ 0 & \sqrt{10} \end{pmatrix} \quad (2.0.18)$$