Assignment 12

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***Abstract*—This document solves a problem involving vector subspaces.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_12

1 Problem

Is the vector $\begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix}$ in the subspace of $\mathbf{R}^4$ spanned by the vectors $\begin{pmatrix} 2 \\ -1 \\ 3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 1 \\ 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 9 \\ -5 \end{pmatrix}$?

2 Solution

Expressing the given three vectors as columns of a matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 1 \\ 3 & 1 & 9 \\ 2 & -3 & -5 \end{pmatrix} \tag{2.0.1}$$

and

$$\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix} \tag{2.0.2}$$

For the vector $\mathbf{b}$ to be in the subspace of $\mathbf{R}^4$ spanned by the three vectors.

$$\mathbf{Ax} = \mathbf{b} \tag{2.0.3}$$

must have a solution.

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 1 \\ 3 & 1 & 9 \\ 2 & -3 & -5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix} \tag{2.0.4}$$

Forming the augmented matrix and row reducing it by elementary row operations,

$$\begin{pmatrix} 2 & -1 & 1 & 3 \\ -1 & 1 & 1 & -1 \\ 3 & 1 & 9 & 0 \\ 2 & -3 & -5 & -1 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 - R_1]{R_2 \leftarrow 2R_2 + R_1, R_3 \leftarrow R_3 - \frac{3}{2}R_1} \tag{2.0.5}$$

$$\begin{pmatrix} 2 & -1 & 1 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & \frac{5}{2} & \frac{15}{2} & \frac{-9}{2} \\ 0 & -2 & -6 & -4 \end{pmatrix} \xleftrightarrow[R_4 \leftarrow R_4 + 2R_2]{R_3 \leftarrow 2R_3 - 5R_2} \begin{pmatrix} 2 & -1 & 1 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & -14 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{2.0.6}$$

From (2.0.6), it is clear that the system does not have a solution. Hence the vector $\begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix}$ does not lie in the subspace of $\mathbf{R}^4$ spanned by the given three vectors.