Assignment 18

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***Abstract*—This document solves a problem based on polynomial vector spaces.**

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_18

1 Problem

Consider the vector space V of real polynomials of degree less than or equal to n. Fix distinct real numbers $a_0, a_1, \cdots, a_k$. For $p \in V$

$$max\left\{\left|p(a_j)\right| : 0 \leq j \leq k\right\}$$

defines a norm on V when

2 Solution

A norm on vector space has the property that

$$||x|| \geq 0 \tag{2.0.1}$$
$$||x|| = 0 \iff x = 0 \tag{2.0.2}$$

For $max\left\{\left|p(a_j)\right| : 0 \leq j \leq k\right\}$ to be a norm on V it should satisfy (2.0.2) that is,

$$max\left\{\left|p(a_j)\right|\right\} = 0 \iff p(x) = 0 \tag{2.0.3}$$

i.e., $p(x)$ should be identically zero. Also,

$$\implies p(a_0) = 0 \tag{2.0.4}$$
$$p(a_j) = 0 \tag{2.0.5}$$
$$\vdots \tag{2.0.6}$$
$$p(a_k) = 0 \tag{2.0.7}$$

Now, if $k < n$ then $p(x)$ can be given as,

$$p(x) = (x - a_0)(x - a_1) \cdots (x - a_k)q(x) \tag{2.0.8}$$

where $p(x)$ is of order m($\leq n$) and $q(x)$ is of the order (m-k-1).

Thus from (2.0.8), $p(x)$ is not identically zero.

When $k \geq n$, then from (2.0.8), it can be seen that the order of $p(x)$ becomes $> n$ which is not possible as V is vector space of real polynomials with degree less than or equal to n.

Thus, for $k \geq n$, $p(x) = 0$ i.e., it is identically zero. Hence, $max\left\{\left|p(a_j)\right| : 0 \leq j \leq k\right\}$ defines a norm on V only if $k \geq n$.