

Assignment 15

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Abstract—This document solves a problem involving linear transformations. Consider,

Download latex-tikz codes from

https://github.com/Vaibhav11002/EE5609/tree/master/Assignment_15

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \mathbf{x} \quad (2.1.2)$$

$$\implies \mathbf{T} \neq 0 \quad (2.1.3)$$

Now,

$$\mathbf{T}^2 : \mathbf{R}^2 \rightarrow \mathbf{R}^2 \quad (2.1.4)$$

$$\mathbf{T}^2(\mathbf{x}) = \mathbf{T}(\mathbf{T}(\mathbf{x})) \quad (2.1.5)$$

$$= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \quad (2.1.6)$$

$$\implies \mathbf{T}^2(\mathbf{x}) = \mathbf{0} \quad (2.1.7)$$

Thus \mathbf{T}^2 is a zero transformation,

$$\implies \mathbf{T}^2 = \mathbf{0} \quad (2.1.8)$$

Thus from (2.1.3), (2.1.8) it is clear that $\mathbf{T}^2 = \mathbf{0}$ but $\mathbf{T} \neq 0$.

1 PROBLEM

Let \mathbf{V} be a vector space over the field \mathbf{F} and \mathbf{T} is a linear operator on \mathbf{V} . If $\mathbf{T}^2 = 0$, what can you say about the relation of the range of \mathbf{T} to the null space of \mathbf{T} ? Give an example of linear operator \mathbf{T} on \mathbf{R}^2 such that $\mathbf{T}^2 = 0$ but $\mathbf{T} \neq 0$.

2 SOLUTION

Given,

$$\mathbf{T} : \mathbf{V} \rightarrow \mathbf{V} \quad (2.0.1)$$

Now, \mathbf{T}^2 is also a linear operator on \mathbf{R}^2 as,

$$\mathbf{T}^2(c\alpha) = \mathbf{T}(\mathbf{T}(c\alpha)) = \mathbf{T}(c\mathbf{T}(\alpha)) \quad (2.0.2)$$

$$= c\mathbf{T}(\mathbf{T}(\alpha)) = c\mathbf{T}^2(\alpha) \quad (2.0.3)$$

Let some vector $\mathbf{y} \in \text{Range}(\mathbf{T})$ then there exists $\mathbf{x} \in \mathbf{V}$ such that,

$$\mathbf{T}(\mathbf{x}) = \mathbf{y} \quad (2.0.4)$$

Now given that,

$$\mathbf{T}^2(\mathbf{x}) = \mathbf{0} \quad (2.0.5)$$

$$\implies \mathbf{T}(\mathbf{T}(\mathbf{x})) = \mathbf{0} \quad (2.0.6)$$

$$\mathbf{T}(\mathbf{y}) = \mathbf{0} \quad (2.0.7)$$

$\therefore \mathbf{y}$ lies in the Null space of \mathbf{T} . Hence \mathbf{T} is singular. Thus, the range of \mathbf{T} must be contained in Null space of \mathbf{T} i.e., $\text{Range}(\mathbf{T}) \subseteq \text{NullSpace}(\mathbf{T})$

2.1 Example

$$\mathbf{T} : \mathbf{R}^2 \rightarrow \mathbf{R}^2 \quad (2.1.1)$$