Factorization

Let's say we have a number 42

Questions

Can we decompose this number into product of it's another smaller numbers?

Answers

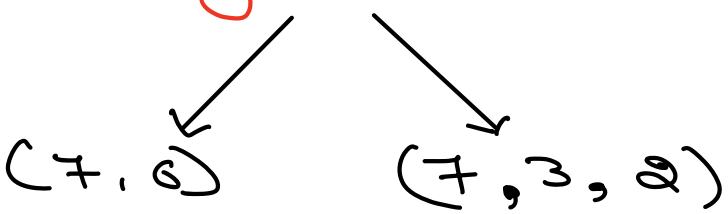
Yes! Ex:

① $42 = 7 \times 6$

② $42 = 7 \times 3 \times 2$

Definitions

These smaller numbers that we used for decomposing the larger Number are called factors



You Already knew this! Right?

Can we do the same for a matrix ?

i.e.

Can we decompose a large matrix into products of smaller Matrix ?

if you said : Yes 

Let's see how and Why ?

$$A_{n \times m} \Rightarrow U_{n \times d} \text{ and } I_{m \times d}$$

such that

$$A_{n \times m} = U_{n \times d} \times I^{\text{Transpose}}_{m \times d}$$

Note : d is a hyper-parameter and is also known as Rank or latent / Hidden dimension

Well this Looks good ! but Why ?

Answers Let's go back to Recommendation System.

Remember the data was represented using SPARSE MATRIX :

$A_{n \times m}$: where $n, m \rightarrow$ O(millions)

For example, Let's say

$n = 10{,}000{,}000$ (Users)

$m = 10{,}000{,}000$ (items)

This makes our matrix with size

$n \times m$

$10^7 \times 10^7 \Rightarrow 10^{14}$ Records

Now, Not all of these 10^{14} records !! are useful, infact most of these are sparse i.e. 0 or null

You know where we are going with this Right?

Can we decompose this huge matrix?

Lets pick a random value of

$d = 100$

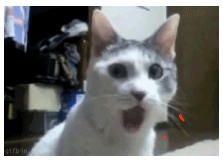
$$A_{n \times m} = U_{n \times d} \cdot I_{d \times m}$$

10^{14} records $= 10^7 \times 100 + 100 \times 10^7$

$10^9 + 10^9 \Rightarrow 2 \times 10^9$

i.e. a reduction of:

$$\frac{10^{14}}{2\times10^{9}} = \frac{1}{2}10^{5}$$!!



i.e. Lots of memory and compute power saved to process and store same amount of information

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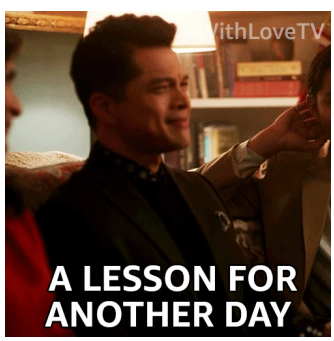
So, How do we find these U and H matrix such that $A = U \cdot H^{Transpose}$

Well that's →



For now, Let's go to a challenging Exercise to understand How prediction works for a given Interaction matrix after it's decomposed into Factors

QUIZ!

Let's say we are given Interaction matrix A, and we have found U and I matrix as following:

$U_{n \times 2}$ $I^T_{2 \times}$

$A =$

	m_1	m_2	m_3
u_1	2	4	5
u_2	3	1	2
u_3	?	?	5
u_4	1	2	1

u_1	0.8	0.5
u_2	0.2	0.9
u_3	1.2	0.8
u_4	1.1	0.7

m_1	m_2	m_3
2.3	0.7	0.5
1.5	2.1	3.1

Where the values stored in matrix A are Ratings given by User$_i$ to movie j.

?: represents movies, user haven't seen yet.

Questions

1. What is the value of Latent dimension?
2. Calculate what Rating the user U_3 will provide to movie m_1 and m_2 respectively?
3. Which movie among the two shall be recommended to the user U_3 first?

Hint : $A = U \cdot I^T$

Solution: Fill the values of ? Here:

2.3	0.7	0.5
1.5	2.1	3.1

U_3

0.8	0.5
0.2	0.9
1.2	0.8
1.1	0.7

?	?	