# Chi-Square Test

Independence of Attributes

# Contingency Table Analysis:
# A Chi-Square Test for Independence

| **Second Classification Category** | **First Classification Category** | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | **Row Total** |
| 1 | $O_{11}$ | $O_{12}$ | $O_{13}$ | $O_{14}$ | $O_{15}$ | $\mathbf{R_1}$ |
| 2 | $O_{21}$ | $O_{22}$ | $O_{23}$ | $O_{24}$ | $O_{25}$ | $\mathbf{R_2}$ |
| 3 | $O_{31}$ | $O_{32}$ | $O_{33}$ | $O_{34}$ | $O_{35}$ | $\mathbf{R_3}$ |
| 4 | $O_{41}$ | $O_{42}$ | $O_{43}$ | $O_{44}$ | $O_{45}$ | $\mathbf{R_4}$ |
| 5 | $O_{51}$ | $O_{52}$ | $O_{53}$ | $O_{54}$ | $O_{55}$ | $\mathbf{R_5}$ |
| **Column Total** | $\mathbf{C_1}$ | $\mathbf{C_2}$ | $\mathbf{C_3}$ | $\mathbf{C_4}$ | $\mathbf{C_5}$ | **n** |

# Contingency Table Analysis: A Chi-Square Test for Independence

A and B are independent if: $P(A \cap B) = P(A) \times P(B)$.
If the first and second classification categories are independent: $E_{ij} = (R_i)(C_j)/n$

Null and alternative hypotheses:

H0: The two classification variables are independent of each other

H1: The two classification variables are not independent

Chi-square test statistic for independence:

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Degrees of freedom: df=(r-1)(c-1)

Expected cell count: $E_{ij} = \frac{R_i C_j}{n}$