

# $\chi^2$ -test for Independence of Attribute

$H_0$ : The row attribute & column attribute are independent of each other

$H_1$ : The row & col<sup>n</sup> attributes dependent on each other

row \ col					

column attribute

row attribute

AirBags Type	Driver & Passenger	Driver only	None
Compact	2	9	5
Large	4	7	0
Midsize	7	11	4
Small	0	5	16
Sporty	3	8	3
Van	0	3	6

$H_0$ : AirBags & Type are indep.

$H_1$ : ————— are dep.

AirBags Type	Driver & Passenger	Driver only	None	All
Compact	$O_{11}$ 2	$O_{12}$ 9	$O_{13}$ 5	16 = $R_1$
Large	$O_{21}$ 4	$O_{22}$ 7	$O_{23}$ 0	11 = $R_2$
Midsize	$O_{31}$ 7	$O_{32}$ 11	$O_{33}$ 4	22 = $R_3$
Small	$O_{41}$ 0	$O_{42}$ 5	$O_{43}$ 16	21 = $R_4$
Sporty	$O_{51}$ 3	$O_{52}$ 8	$O_{53}$ 3	14 = $R_5$
Van	$O_{61}$ 0	$O_{62}$ 3	$O_{63}$ 6	9 = $R_6$
All	16 = $C_1$	43 = $C_2$	34 = $C_3$	93 = $n$

$O_{ij}$ : Observed Frequency of  $i$ th row &  $j$ th col<sup>n</sup> cell

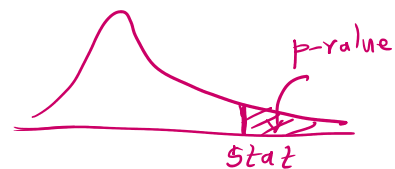
$E_{ij}$ : Expected Frequency of  $i$ th row &  $j$ th cell cell

$$E_{11} = \frac{R_1 C_1}{n} = \frac{16 \times 16}{93}$$

$$E_{ij} = \frac{R_i C_j}{n}$$

$$E_{32} = \frac{43 \times 22}{93}$$

$$E_{62} = \frac{43 \times 9}{93}$$



$$r = 6, c = 3$$

Degrees of freedom:  $df = (r-1)(c-1)$

$$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

```
In [7]: test_statistic, p_value, df, expected_frequencies = chi2_contingency(ctab)
...: print("P-Value =", p_value)
P-Value = 0.000272287749055816 < 0.05
```

$\therefore$  We reject  $H_0$  at 5% l.o.s.

— AirBags & Type

$\therefore$  We reject  $H_0$  at 5% l.o.s.  
Conclusion: Attributes of AirBag & Type  
may be dependent.

```
df_bar = pd.melt(ctab.reset_index(), id_vars="Type")
sns.barplot(x="AirBags",
            y="value",
            hue="Type",
            data=df_bar)
plt.show()
```

