Date :

Assignment-9

Q1) What is a fractal? Explain characteristics & classification of fractals.

→ Fractals are infinitely complex patterns that are self similar accross different scales. Natural objects are often fractals, eg: tree, mountains, etc.

* Characteristics

i) Fractals possess infinite details at each point.

ii) Self-similarity is observed between the object parts.

* Classification

① Self-similar fractals:

i) Such fractals exhibit self-similarity in the shape.

ii) They have components which are scaled-down versions of the object itself.

iii) We can construct the object by applying scaling factors to all parts or by using same scaling factor for al

iv) If random scaling factor is used, the fractal is called statistically self-similar.

② Self-affine fractals:-

i) They use different scaling factors for all 3 directions.

ii) We can add random variation to get statistically self-affine fractals.

iii) This class of fractals is best suited to model terrain, water, e

③ Invariant fractals:

i) Non-linear transformation is applied to construct such fractals.

operations either ... in S-1 ...

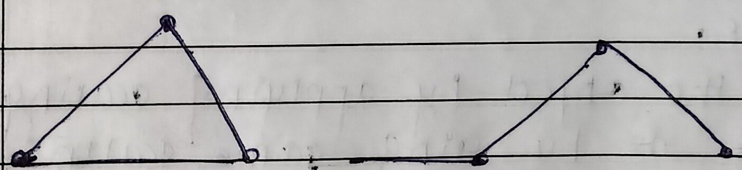
v) The pipeline mode was executed in S-1 ...

Date :

ii) A self-squaring function in complex space produces self-squaring fractals like the mandelbrot
iii) self inverse fractals are generated using inversion procedure.

Q2) Write short notes on:-

① Koch curve.
→ i) The curve begins with a straight line
ii) It divides the line into 3 segments
iii) The middle segment is replaced by 2 sides of an equilateral Δ with apex pointing outside.
iv) Now we have 4 segments. The same procedure is repeated on these 4 segments.



initiator   Generator.

② B-spline curve.
→ i) they are the mostly widely used class of curves for approximating the shape of the curve.
ii) Degree of a polynomial is independent of a no. of control points.
iii) they have local control over the curves & surfaces.
iv) However, the derivation & generation of B-spline curves are more complex than Bezier curves.
v) B-spline curves donot interpolate any on the control points.

Date :

vi) Blending functions of B-spline curve are described using cox-deBoor recursive formula:

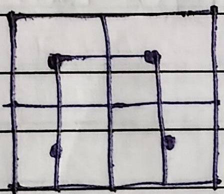
$$B_{i,1}(t) = \begin{cases} 1 & \text{if } t_i \le t \le t_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{i,d}(t) = \frac{t-t_i}{t_{i+d-1}-t_i} B_{i,d-1}(t) + \frac{t_{i+d}-t}{t_{i+d}-t_{i+1}} B_{i+1,d-1}(t)$$

(3) Hilbert curve:

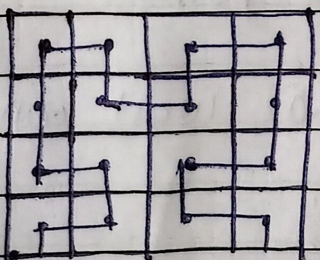
→ i) It is also called as the Hilbert space-filling curve.

ii) It is constructed by following successive approximations. If a square is divided into 4 quadrants, we can draw the 1st approximation to the hilbert's curve by connecting centre pts of each quadrant.



1st approx^n.

iii) 2nd approximation is drawn by further subdividing each of the quadrants & connecting their centres, before moving to the next major quadrant



iv) At each subdivision, scale changes by 2 but the length changes by 4. Hence, hilbert curves' topological dimension is 1 but fractal dimension is 2.

Date :

(4) Bezier curve & its properties:

→ i) It is determined by defining a polygon.

ii) They have a number of properties that make them highly useful & convenient for curve & surface design.

iii) They are also easy to implement. Hence, they are widely used in various CAD systems

iv) It can be fitted to any number of control points But, as the number increases, degree of the polynomial also increases.

* Properties:

1. The basic functions are real.
2. The curve always passes through the 1st & last control points ie curve has same end points as the guiding polynomial.
3. The curve follows the shape of the defining polygon.
4. The curve lies entirely within the convex hull formed by the 4 control points.

Q2) Differentiate between Bezier curve & B-spline curves.

→

Bezier curve	B-spline curve
i) Passes from the 1st & last control points	i) Does not interpolate any control point.
ii) Does not have local control on a curve.	ii) It has local control.
iii) changing 1 control point affects the position of all points on the curve.	iii) changing one control point affects only few points on the curve.

Date :

iv) Less flexible	iv) more flexible
v) Lesser computations are required	v) It requires more number of computations.
vi) Degree is determined by the number of control points.	vi) Degree of the curve is independent of the no. of control point.

Q4) Write short note on blending function of Bezier curve.

→ i) The Bezier blending functions $BEZ_{k,n}(u)$ are derived from Bernstein polynomials. They are specified as

$$BEZ_{k,n}(u) = C(n,k)\, u^k (1-u)^{n-k}$$

ii) where $C(n,k)$ are binomial co-efficients. Binomial coefficients are given by

$$C(n,k) = \frac{n!}{k!\,(n-k)!}$$

iii) equivalently, we can define Bezier blending functions with recursive calculation as

$$BEZ_{k,n}(u) = (1-u)\, BEZ_{k,n-1}(u) + u\,BEZ_{k-1,n-1}(u),\ n > k \geq 1$$

The position vector eq^n represents a set of 3 parametric equations for individual curve co-ordinates.

$$x(u) = \sum_{k=0}^{n} x_k\, BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k\, BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^{n} z_k\, BEZ_{k,n}(u)$$

Date : ____________

The successive binomial co-efficients can be calculated as

$$c(n,k) = \frac{n-k+1}{k} c(n,k-1)$$

Q5) Write short note on interpolation & approximation.

→ i) The curve is specified by a set of control points whose position determines the shape of the curve.

ii) If the curve passes through all the control points, it is called interpolation.

iii) Interpolation curves are used in animation, specifying camera motion & also in digitizing the co-ordinates.

iv) If the curve does not pass through the control points & approximate the shape, it is called as approximation or extrapolation. Such curves are used to estimate the shape of the object surface.

v) Manipulating spline curves using control point representation is easy. Designer can adjust the shape of the curve by repositioning the control points, which is highly preferred in interactive graphics design.