Trend ←

y_i

Decomposition

$y_i = \text{Addition}\left(T, S, \right.$

or

$y_i = \text{Multi}\left(T, S\right)$: Multiplicative

Seasonality

Downward Trend

Trend

Milk

Original

RMSE / MSE

train   test 2022
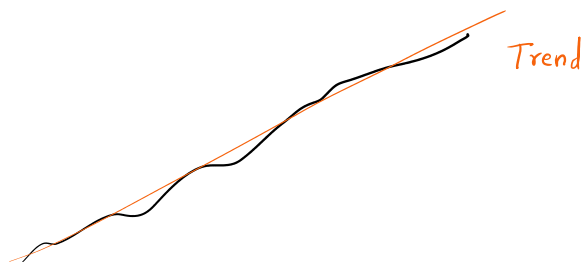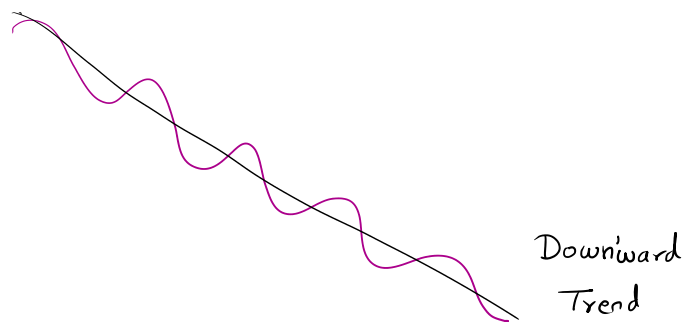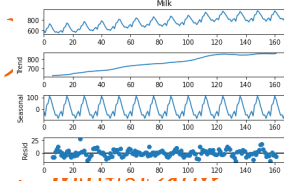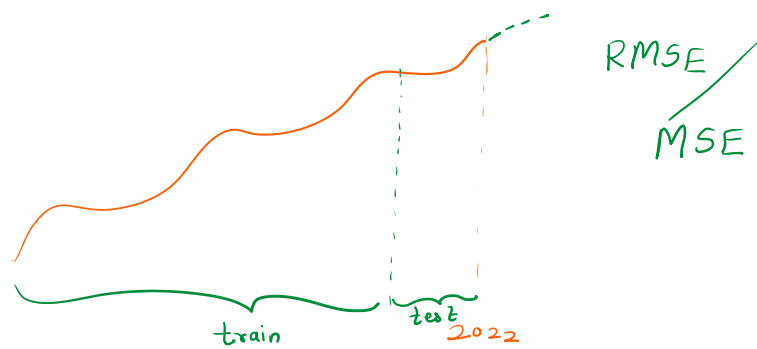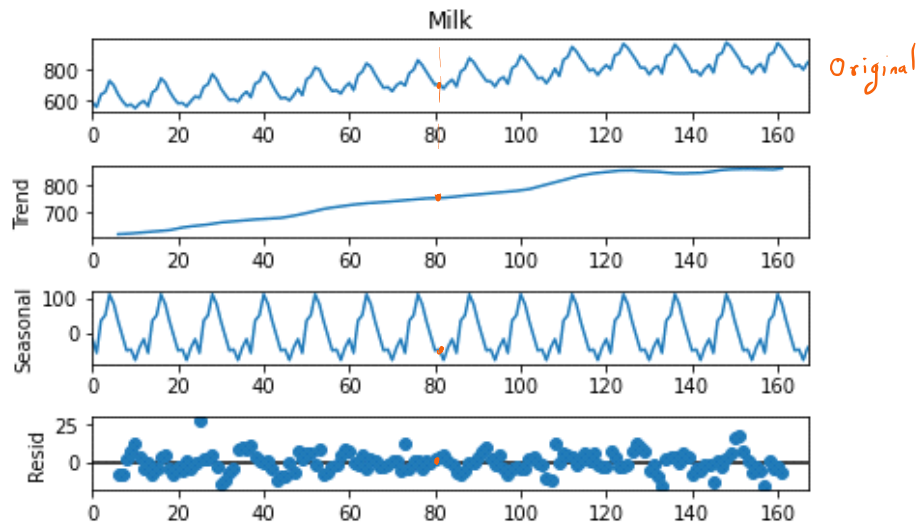
- The k-step ahead forecast is given by combining the level estimate at time t (L$_t$) and trend estimate at time t (T$_t$):
$$F_{t+k} = L_t + kT_t$$
- The level and trend are updated by the equations:
$$L_t = \alpha y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$
$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

α β

y_1 L_1 T_1

y_2 L_2 T_2

$\vdots$

y_{100} L_{100} T_{100}

$$F_{101} = L_{100} + 1 \, T_{100}$$
$_{100+1}$

$\rho : phi$

FRED-NROUST.csv

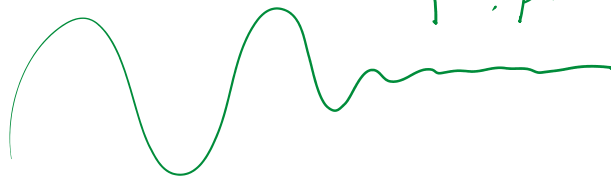| Model	RMSE
SES	0.033868423955341384
Holt's Linear	0.017038778525156884
Holt's Exponential	0.016618469464718837
Additive Trend	0.005611647380117347
Multiplicative Trend	0.005449834442543947
Holt-Winters Additive	0.03363932722649928
Holt-Winters Multiplicative	0.030497159323824734
Damped Holt-Winters Additive	0.014981092470453976
Damped Holt-Winters Multiplicative	0.01597452974533468

BUNDESBANK-BBK01_WT5511.csv

Differencing:-

$$y_t - y_{t-1}$$
1st order   2nd order

y_1

y_2 $y_2 - y_1$

y_3 $y_3 - y_2$ $(y_3 - y_2) - (y_2 - y_1)$

y_4 $y_4 - y_3$ $(y_4 - y_3) - (y_3 - y_2)$

y_5 $y_5 - y_4$ $(y_5 - y_4) - (y_4 - y_3)$

y_6 $y_6 - y_5$ $(y_6 - y_5) - (y_5 - y_4)$

$$y \quad X_1 \quad X_2 \quad X_3 \cdots \cdots X_p$$
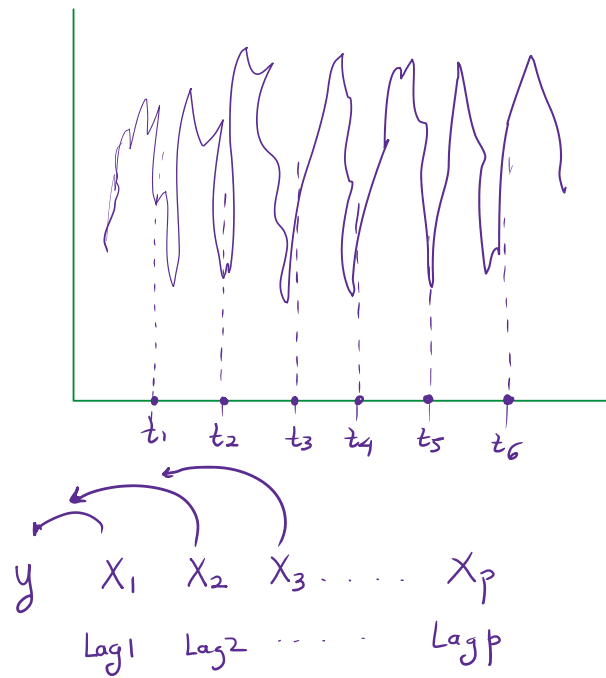
$$\text{Lag1} \quad \text{Lag2} \cdots \cdots \text{Lag} p$$

$$y = b_0 + b_1 \, \text{Lag1} + b_2 \, \text{Lag2} \cdots \cdots + \text{Lag} \; p$$

Auto Regression (AR)

Order of AR : p

Simple Moving Average Model $(MA \; \text{Model})$

$$y_t = \mu + \epsilon_t + \theta \epsilon_{t-1} \quad : \text{order } 1$$

$$y_t = \mu + \epsilon_t + \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} \quad : \text{order } 2$$

$$y_t = \mu + \epsilon_t + \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} + \cdots \cdots + \Theta_q \epsilon_{t-q} \quad : \text{order } q$$

ARMA Model

$$y_t = \mu + \emptyset_1 \, \text{Lag1} + \emptyset_2 \, \text{Lag2} + \cdots \emptyset_p \, \text{Lag} \, p$$
$$+ \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} + \cdots + \Theta_q \epsilon_{t-q} + \epsilon_t \qquad : \text{order} (p, q)$$

Differencing $+$ ARMA $=$ AR$\underset{\uparrow}{I}$MA

Integrated

Order: (p, d, q)

$\uparrow$ order of differencing

Seasonal ARIMA

$(p, d, q) (P, D, Q) [\text{Period}]$

$\underset{\substack{\text{ord} \\ \text{of} \\ \text{SAR}}}{\uparrow} \quad \underset{\substack{\text{ord} \\ \text{of} \\ \text{SD}}}{\uparrow} \quad \underset{\substack{\text{ord} \\ \text{of} \\ \text{SMA}}}{\uparrow}$

FRED-NROUST.csv

Test: Last 8 observations

Package   sktime

```
forecaster = make_reduction(regressor, window_length=12)
```

	X_1	X_2		X_{12}	Y
y_1	y_1	y_2	- - -	y_{12}	y_{13}
y_2	y_2	y_3	. . . .	y_{13}	y_{14}
$\vdots$	y_3	y_4	. . . . .	y_{14}	y_{15}
y_n					