Advanced Exponential Forecasting

Series with No Trend and Seasonality

- Moving Average and Simple Exponential Smoothing should be used for forecasting the series with no trend and seasonality

Series with Additive Trend

- For series with trend, we can use Holt's method, also known as double exponential smoothing
- Similar to Simple Exponential Smoothing, the level of the series is estimated from the data and is updated as more data would become available
- Level is estimated using maximum likelihood method

Holt's Linear Trend Method

- The k-step ahead forecast is given by combining the level estimate at time t (L_t) and trend estimate at time t (T_t):

$$F_{t+k} = L_t + kT_t$$

- The level and trend are updated by the equations:

$$L_t = \alpha y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$
$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

  – Where α and β are smoothing constants whose values range from 0 to 1 and are set by the user or chosen iteratively by R function holt()

- Level equation shows L_t, Level at time t as weighted average of the observation at time t y_t and within sample one step ahead forecast at time t , $(L_{t-1}+T_{t-1})$
- Trend Equation shows T_t , trend estimate at time t as weighted average of (L_t-L_{t-1}) and T_{t-1}, the previous trend estimate

More Details: http://robjhyndman.com/hyndsight/hw-initialization/

Exponential Trend Method

- The k-step ahead forecast is given by combining the level estimate at time t (L_t) and trend estimate at time t (T_t):

$$F_{t+k} = L_t \times {T_t}^k$$

- The level and trend are updated by the equations:

$$L_t = \propto y_t + (1 - \propto)(L_{t-1} \times T_{t-1})$$

$$T_t = \beta\left(\frac{L_t}{L_{t-1}}\right) + (1-\beta)T_{t-1}$$

  - Where α and β are smoothing constants whose values range from 0 to 1 and are set by the user or chosen iteratively by R function holt()

- Level equation shows L_t, Level at time t as weighted average of the observation at time t y_t and within sample one step ahead forecast at time t , $(L_{t-1} \times T_{t-1})$
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Damped Trend Methods

- It has been observed that Holt's Linear Trend and Exponential Trend tend to over-forecast for longer forecast horizons
- Gardner and McKenzie (1985) suggested a parameter that dampens the trend line to a flat line some time in the future
- Methods with damped trend have been proven to be more successful when forecasts are to be predicted by automatic process
- There are two types of damped trend methods:
  - Additive Damped Trend
  - Multiple Damped Trend

Additive Damped Trend

- In association with the smoothing parameters α and β, damped methods also include a damping parameter φ; 0 <φ< 1 as:

$$F_{t+k} = L_t + (\varphi + \varphi^2 + \cdots + \varphi^k)T_t$$
$$L_t = \propto y_t + (1 - \propto)(L_{t-1} - \varphi T_{t-1})$$
$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)\varphi T_{t-1}$$

- If φ=1 then the method is Holt's Linear Method

Multiplicative Damped Trend

- Taylor(2003) introduced a damping parameter to the exponential trend

$$F_{t+k} = L_t \times T_t^{(\varphi+\varphi^2+\cdots+\varphi^k)}$$
$$L_t = \alpha y_t + (1-\alpha)L_{t-1} \times T_{t-1}^{\varphi}$$
$$T_t = \beta\left(\frac{L_t}{L_{t-1}}\right) + (1-\beta)T_{t-1}^{\varphi}$$

Holt-Winters Seasonal Method

- This method comprises of the forecast equation and three smoothing equations each for level, trend and seasonal component
- We use m to denote the period of season
- The additive method of Holt-Winters can be preferred when the seasonal variations are roughly constant through the series
- The multiplicative method is preferred when the seasonal variations are changing proportional to the level of the series.

Holt-Winters Additive Method

- The component form of the model:

$$F_{t+k} = L_t + kT_t + S_{t-m+k_m{}^+}$$
$$L_t = \propto (y_t - S_{t-m}) + (1 - \propto)(L_{t-1} + T_{t-1})$$
$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$
$$S_t = \gamma(y_t - L_t) + (1 - \gamma)S_{t-m}$$

Where

S_t : Seasonal Estimate at time t

${k_m}^+$: [(k-1) mod m)]+1 which ensures that the estimates of the seasonal indices used for forecasting come from the final year

Holt-Winters Multiplicative Method

- The component form of the model: (Additive Trend)

$$F_{t+k} = (L_t + kT_t)S_{t-m+k_m^+}$$

$$L_t = \propto \left(\frac{y_t}{S_{t-m}}\right) + (1 - \propto)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma\left(\frac{y_t}{L_t}\right) + (1 - \gamma)S_{t-m}$$

Where

S_t : Seasonal Estimate at time t

${k_m}^+$: [(k-1) mod m)]+1 which ensures that the estimates of the seasonal indices used for forecasting come from the final year

Taxonomy of Exponential Methods

	Seasonal Component		
Trend Component	N (None)	A (Additive)	M (Multiplicative)
N (None)	**(N,N)**	(N,A)	(N,M)
A (Additive)	**(A,N)**	**(A,A)**	**(A,M)**
A_d (Additive Damped)	(A_d,N)	(A_d,A)	**(A_d,M)**
M (Multiplicative)	**(M,N)**	(M,A)	(M,M)
M_d (Multiplicative Damped)	**(M_d,N)**	(M_d,A)	(M_d,M)

- (N,N) : Simple exponential smoothing
- (A,N) : Holt's linear method
- (M,N) : Exponential trend method
- (A_d,N) : Additive damped trend method
- (M_d,N) : Multiplicative damped trend method
- (A,A) : Additive Holt-Winters method
- (A,M) : Multiplicative Holt-Winters method
- (A_d,M) : Holt-Winters damped method