Tree

A connected graph that contains no cycle or circuits is called a Tree.

Forest

A graph with no cycle such that whose connected components are trees, or In other words, the set of disjoints trees called a forest.

Leaf or Terminal Node

A vertex of degree one in a tree is called a leaf or terminal

Properties of a Tree

1) The number of vertices is one more than the number of edges in a Tree

$$|V| = |E| + 1$$

2) A tree with two or more vertices has atleast two leaves.

3) A K_2 i.e. complete graph with two vertices is a tree.

4)

Minimally Connected Graph

A connected graph G is said to be minimally connected graph if removal of any edge e from it disconnects the graph.

Spanning Tree

Spanning Tree of graph G of n vertices and e edges as the following property —

1. It passes through all vertices of Graph G.
2. It connected subgraph of G
3. It does not have any closed circuit

Rank of a Graph

No. of edges of Graph G which are including in spanning tree are called branches of a tree.

The rank of a Graph (R) = $(n-1)$

$n \rightarrow$ no. of vertices

Nullity of a Graph

The edges of Graph G which are not included in tree are called nullity of a graph.

$$T = e - (n-1)$$

$e \rightarrow$ no. of edges

$n \rightarrow$ no. of vertices.

Minimal Spanning Tree

Minimal Spanning tree of a graph G is that Spanning tree that as Smallest length Sum among all the Spanning tree.

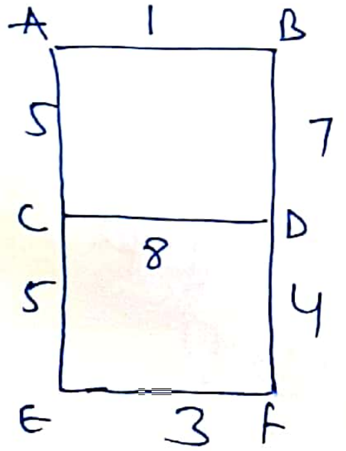
There are two ~~types of~~ methods for finding the minimal Spanning tree.

1) <u>Kruskal's Algorithm</u>

<u>Step 1</u> — Arrange the edges of the graph in order of increasing weight.

<u>Step 2</u> — Draw an isolated ~~vertex~~ vertices.

<u>Step 3</u> — Starting only with vertices of G and proceeding sequencelly, at each edges which does not Result in a cycle, until (n−1) edges. ~~are~~.



A − B → 1
E − F → 3

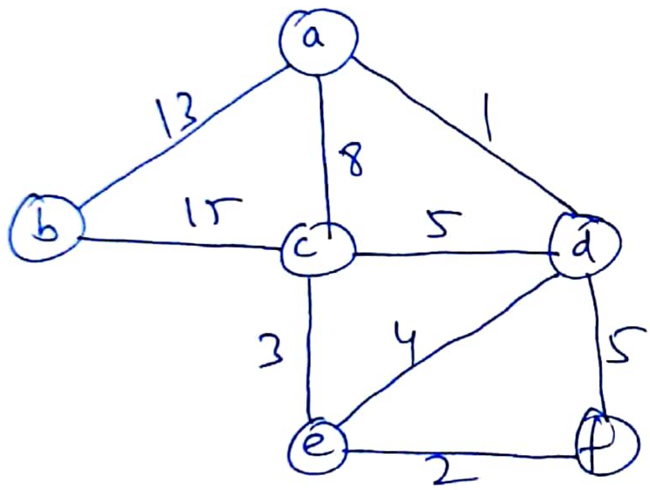
Prim's Algorithm

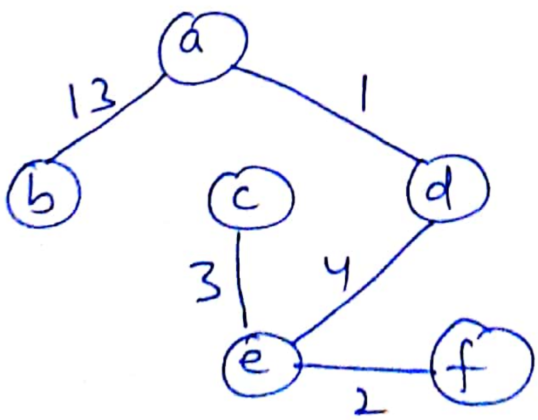
Prim's Algorithm finds a minimum cost spanning tree by selecting edges from the graph one-by-one as follows:

(i) It starts with a tree, T, consisting of the starting vertex x.

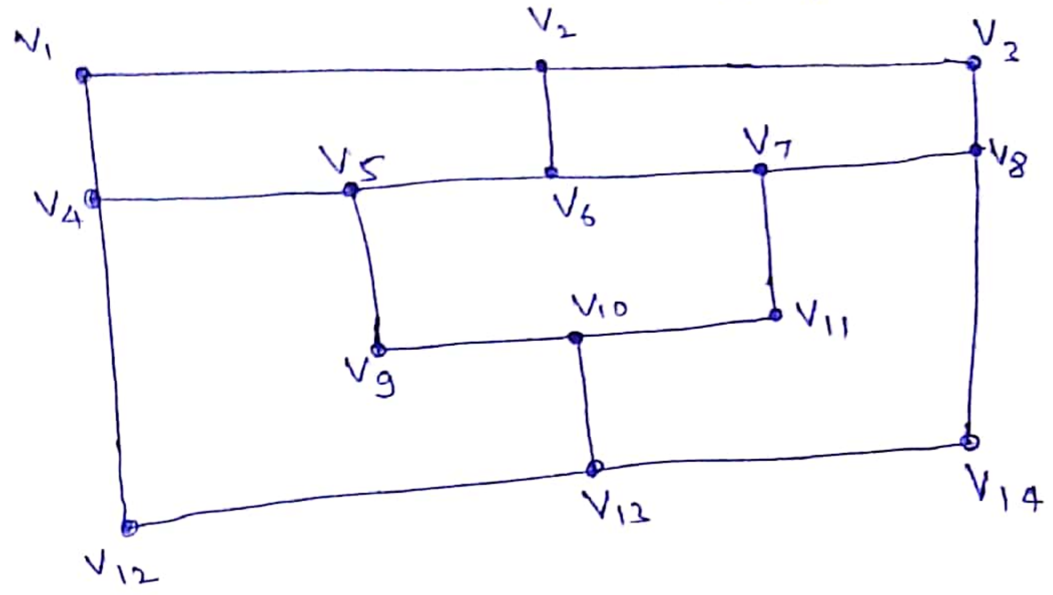
(ii) Then, it adds the shortest edge emerged from x that connects T to the rest of the graph.

(iii) It then moves to added vertex and repeats the process.





Example : Find the rank & nullity of a graph G.



$n = 14 \quad e = 18$

$\therefore$ Rank of Graph $= n-1 = 14-1 = 13$

Nullity of Graph $= e-(n-1) = 18-13 = 5$

Number of edges $=$ Rank + Nullity
$= 13 + 5$
$= 18.$

Minimal Spanning Tree

The minimal spanning tree T of graph G is that spanning tree that has smallest length sum among all spanning tree.