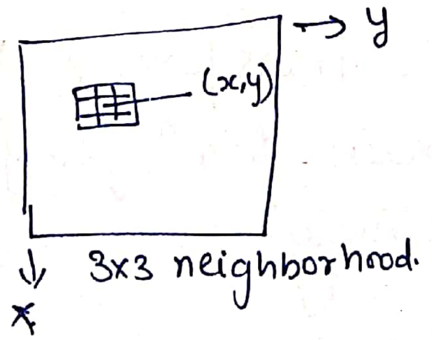
Frequency domain

Frequency :- The no. of times that a periodic function repeat the same sequence of values during a unit variation of the independent variable.

→ Fourier's contribution in this field states that any periodic function can be expressed as the sum of sines &/cosines of different frequencies, each multiplied by different co-efficient. We now call this sum a Fourier Series.

→ Spatial Domain using mask operation that is nothing but the convolution op^ in 2D.



3x3 neighborhood.

$W_{-1,-1}$	$W_{-1,0}$	$W_{-1,1}$
$W_{0,-1}$	$W_{0,0}$	$W_{0,1}$
$W_{1,-1}$	$W_{1,0}$	$W_{1,1}$

3X3 Mask

$$g(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} w_{i,j}\, f(x+i,\, y+j)$$

So using this convolution op^, we are going for spatial domain processing the images.

→ So Convolution op^n in the spatial domain is equivalent to multiplication in the frequency domain.

Similarly in frequency domain convolution is equivalent to multiplication in the Spatial domain.

→ Suppose we have convolution of 2 functions $f(x,y)$ & $h(x,y)$ in spatial domain

$$f(x,y) * h(x,y) \Longleftrightarrow F(u,v)\ H(u,v)$$

Corresponding op^n in the Frequency domain is multiplication of $F(u,v)$ & $H(u,v)$.

* fourier transform of spatial dommain fun^n $f(x,y)$ & $h(x,y)$ respectively.

→ $f(x,y)\ h(x,y) \Longleftrightarrow F(u,v) * H(u,v)$

multiplication of $f(x,y)$ & $h(x,y)$ in spatial domain

Convolution of $F(u,v)$ & $H(u,v)$ in frequency domain.

// Multiplication is term-by-term multiplication
$z[n] = x[n]\,y[n]$ for all n. It will take 2 nos. & perform multiplication.
Convolution is polynomial multiplication which is not same as term by term multiplication. It will take 2 signals & generate 3^{rd} signal.

Frequency Domain Filters.

→ Frequency domain filters are used for smoothing & sharpening of image by removal of high or low frequency components.

Frequency Domain Filters.

- Low Pass
- High Pass
- Band Pass

Low Pass

→ It removes high frequency components

→ It keeps low frequency components

→ Used for Smoothing of image.

$G(u,v) = H(u,v) \, . \, F(u,v)$

$H(u,v)$ → fourier transform of filtering Mask

$F(u,v)$ → fourier transform of original image.

High Pass

→ It removes low frequency components.

→ It keeps high freq. Components.

→ Used for Sharpening the images.

$G(u,v) =$

$H(u,v) = 1 - H'(u,v)$

$H(u,v)$ → fourier transform of high pass filtering

$H'(u,v)$ → fourier transform of low pass filtering

Band Pass

→ It removes very low frequency compo. & very high freq. components.

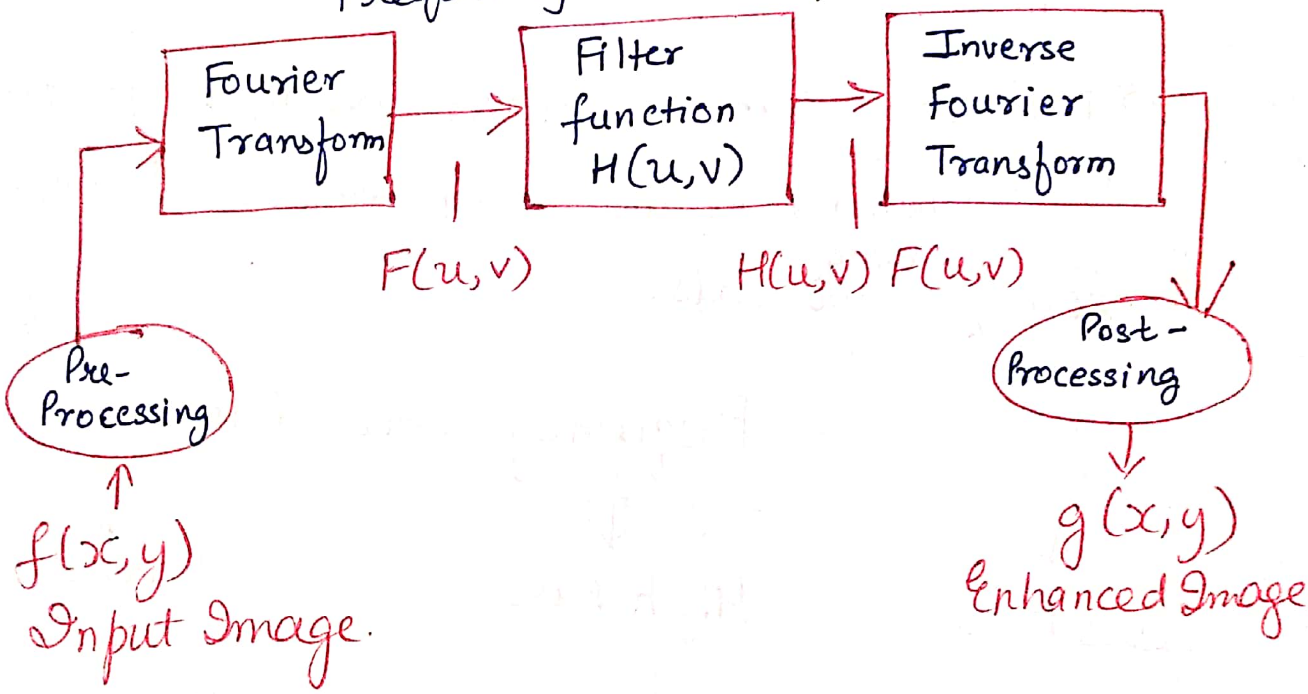
→ It keeps moderate range band of frequencies.

→ It is used to enhance edges while reducing the noise at the same time.

Steps to filter an image in the frequency domain:

1. Compute $F(u, v)$ the DFT of the image.
2. Multiply $F(u, v)$ by a filter funⁿ $H(u, v)$
3. Compute the Inverse DFT of the result

Frequency Domain filtering opⁿ:



filters in Frequency Domain

Low Pass Filter

1. Ideal Low Pass Filter (sharp)
2. Butter Worth low pass filter
3. Gaussian low pass filter (Smooth)

High Pass Filter

1. Ideal High Pass filter
2. Butter worth High pass filter
3. Gaussian High pass filter.

Basic Model for filtering Op^n.

$$G(u,v) = \underbrace{H(u,v)}_{\text{filter fun}^n} \cdot F(u,v)$$

filter fun^n   fourier transform of I/p Image

(We have to select proper filter fun^n H(u,v) which will attenuate the high frequency components & let the low frequency components to be passed to the o/p.)

① Ideal Low Pass Filter :-

$$\rightarrow H(u,v) = \begin{cases} 1 & D(u,v) \leq D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

Distance of the point (u,v) in the frequency domain from the origin of the frequency recantangle.

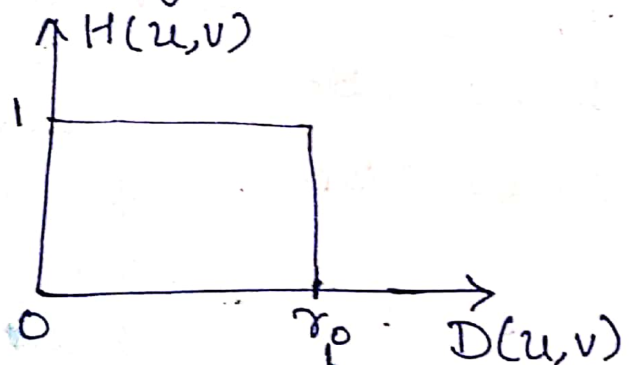
→ It means, If we multiply F(u,v) with H(u,V) than all the frequency components lying within a circle of radius D_0 will be passed to the O/p & all the freq. components lying outside this circle of radius D_0 will not be allowed to be passed.

→ If the fourier transform F(u,v) is the centered Fourier transformation., that means the origin of the fourier transform recantngle is set at the middle of the relangle, then D(u,v) computed as :-

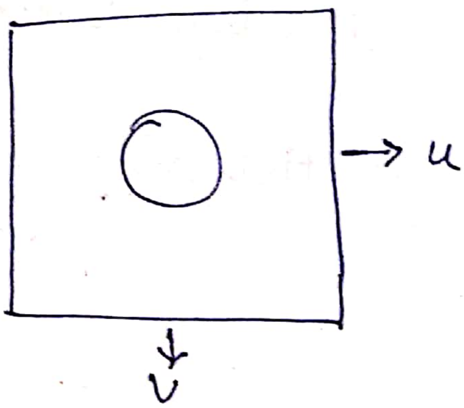
$$D(u,v) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$$

where image size is $(M \times N)$

→ Low Pass Filter suppresses all the frequencies higher than the tops cutoff frequency ~~are~~ or not and leaves smaller frequencies which are unchanged



cutoff frequency.

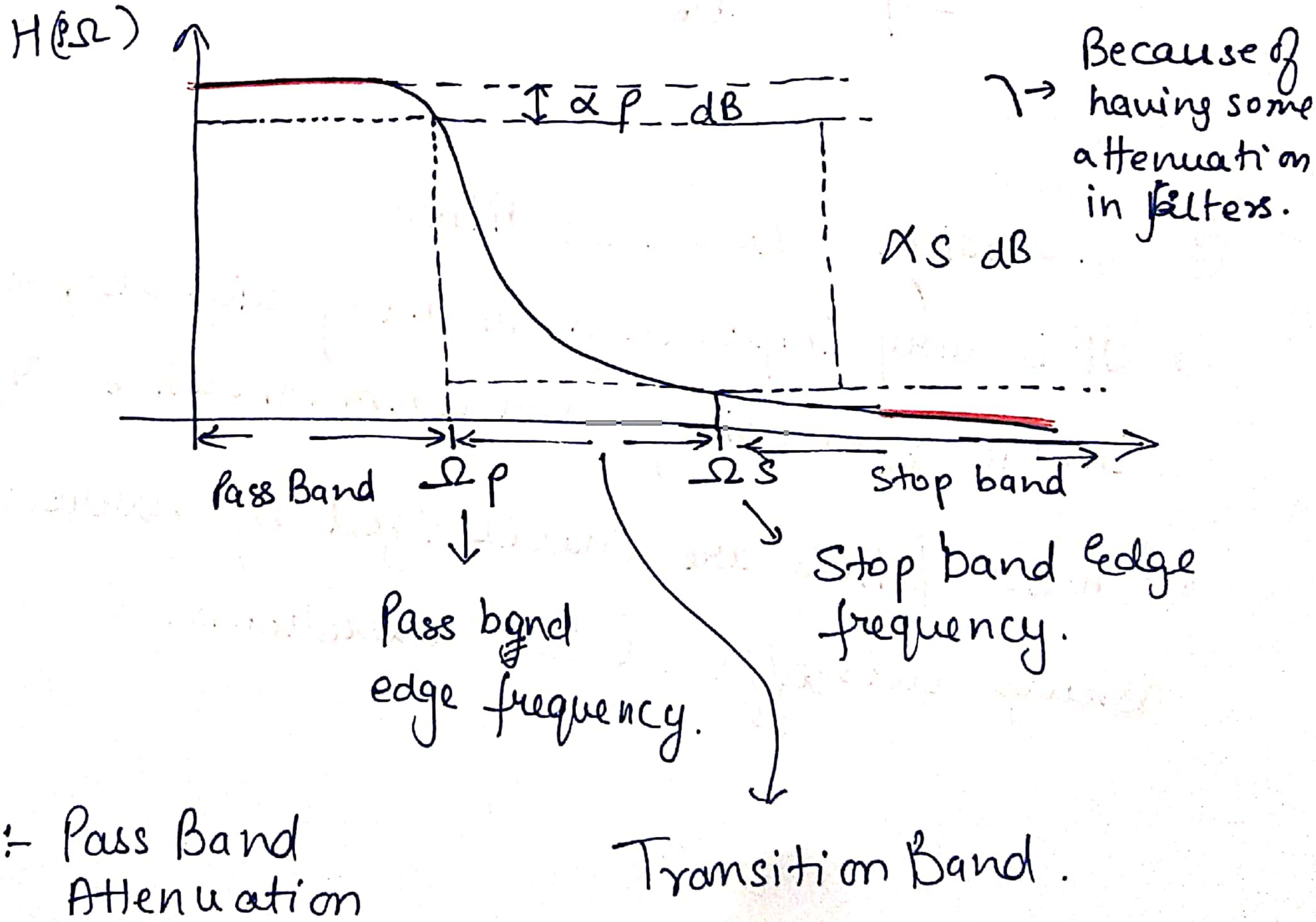


Representation of the ideal low pass filter in the form of image.

Butter Worth Low Pass Filter.

→ As we know that the disaduantage of Idle low Pass filter is Ringing Effect. To avoid this effect we'll use Butterworth Low Pass filter

→ The frequency response of the Butter Worth Low Pass filter does not have a sharp transition as ideal low pass filter.

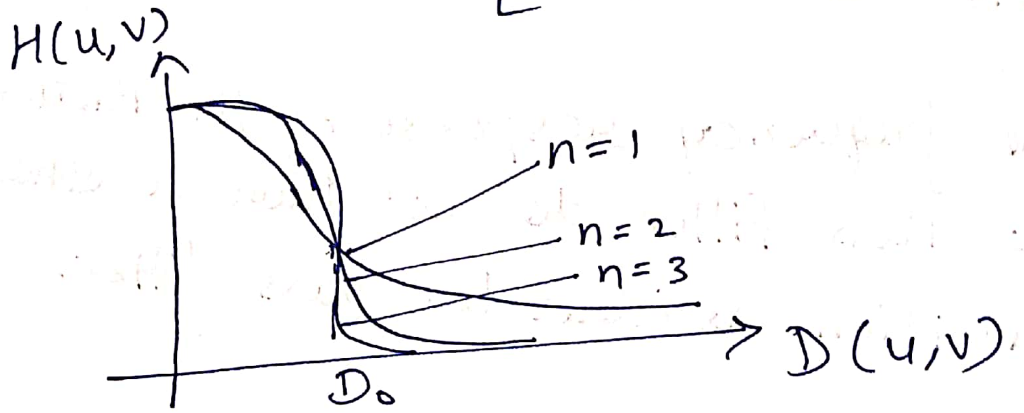


αp :- Pass Band Attenuation

αs :- Stop Band Attenuation.

→ The transfer funⁿ of a Butterworth lowpass filter of order n with cutoff frequency at distance Do from the origin is defined as

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$



③ Gaussian Low Pass filter

→ It is very important in many signals processing image processing and communication application

→ These filters are charaterized by narrow bandwidth sharp cuts-offs & low overshoots.

Gaussian Low Pass Filter

↳ It is very important in many Signal processing, image processing & communication app^.

→ These filters are characterized by narrow Band-widths, Sharp cut-offs & low overshoots.

→ The fourier transform of Gaussian is also a Gaussian

- So the filter has the same response shape in both Spatial & Frequency domains.

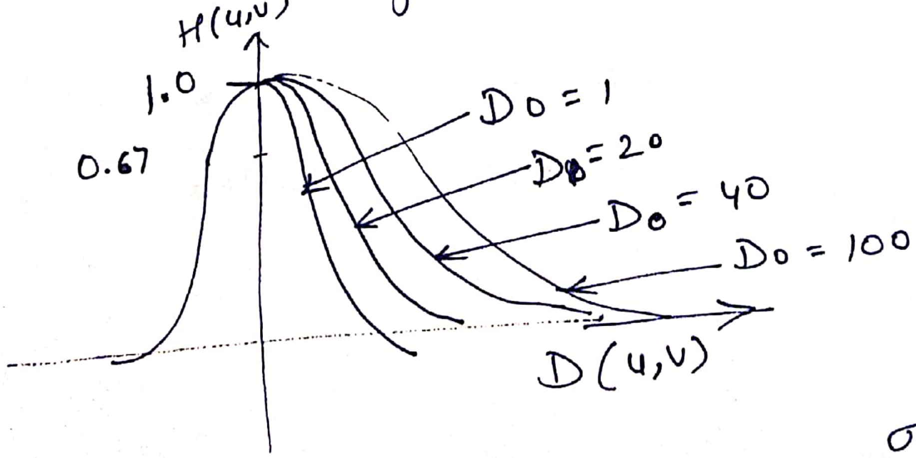
→ Transformation fun^ is given by:-

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}} = e^{-\frac{D^2(u,v)}{2D_0^2}}$$

where $D(u,v)$ is the distance from the origin in the frequency plane.

σ parameter measures the spread of Gaussian curve

Larger the value of the σ, larger the cut off frequency.



$\sigma_0 = r_0$

$D_0 = r_0$

→ As mentioned earlier, Gaussian has the same shape in the spatial & Fourier domain & therefore doesn't incur the ringing effect in the spatial domain of the filtered image.

→ This is the advantage over Idle Low Pass Filter & Butterworth Low Pass Filter.

→ Especially in medical images.

Low Pass Filter
or
Blurring mask has the following Properties :-

1. All the values in blurring masks are (+)ve.
2. The sum of all values is equal to 1.
3. The edge content is reduced by using blurring mask.
4. As the size of mask grow, more smoothing effect will take place.

High Pass Filter
or
Derivative mask has the following Properties :-

1. A derivative mask have (+)ve & (-)ve values both.
2. The sum of all derivative mask values is equal to zero.
3. The edge content is increased by derivative mask.
4. As the size of the mask grows, more edge content is increased.

→ Ideal filters — Sharp filter.

→ Gaussian " — Smooth filter

→ Butterworth filter has a parameter called filter order

↳ For high order value, Butterworth filter approaches the ideal filter.
For low order value, Butterworth filter approaches the Gaussian filter

↳ Thus, the Butterworth filter may be viewed as providing a transition b/w 2 extremes.

Sharpening in the Frequency Domain

→ Edges & Fine details in images are associated with high frequency components

→ High Pass filters only pass the high frequencies, drop the low ones.

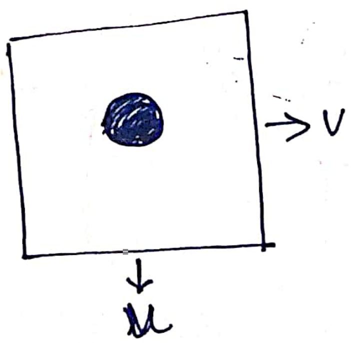
the reverse of low pass filters.

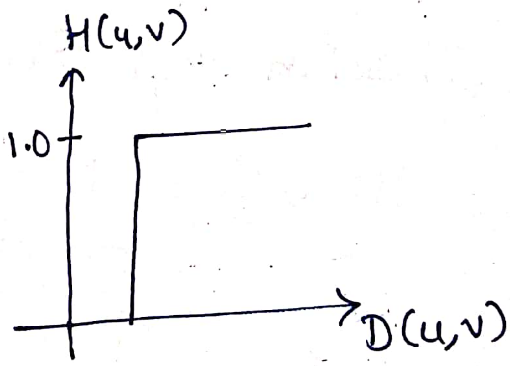
$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

① Ideal High Pass Filters

$$H(u,v) = \begin{cases} 0, & \text{if } D(u,v) \leq r_0 \\ 1, & \text{if } D(u,v) > r_0 \end{cases}$$

$r_0 = D_0$

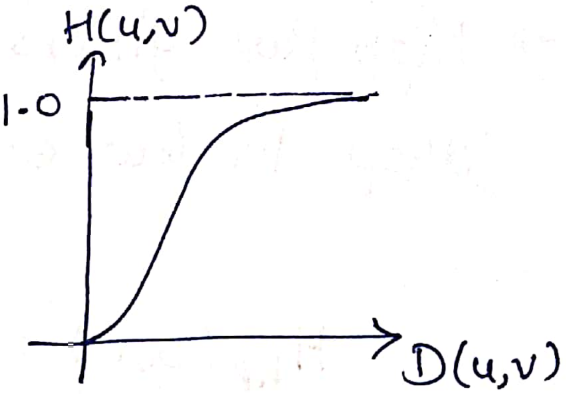


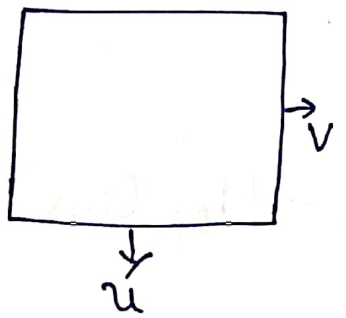


② Butter worth High Pass Filters

$$\rightarrow \quad H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

where n is the order & D_0 is the cutoff distance as before.





③ Gaussian High Pass Filters

$$\rightarrow \quad H(u,v) = 1 - e^{\frac{-D^2(u,v)}{2D_0^2}}$$