

$$f(x) = a_2 x^2 + a_1 x + a_0 \quad (\text{Quadratic})$$

## ② Exponential Function:-

It is defined as  $f: \mathbb{R} \rightarrow \mathbb{R}^+$  (Positive Real No's)

where  $f(x) = a^x$   
 where  $a \neq 1$  and  $a \in$  set of all positive Real No's i.e.  $\mathbb{R}^+$

Property:-

- ① Intercept of  $y$  in a graph is 1.
- ③. If value of  $a > 1$  or  $x_1 > x_2$ , then func<sup>n</sup> is in increasing order.

## ③. Logarithmic function:-

$$f(x) = a^x = y$$

Taking log both sides-

$$\log y = x \log a$$

$$x = \frac{\log y}{\log a}$$

$$x = \log_a y$$

- ④. Invertible Function :-  
 It is also known as inverse function.  
 If  $f: A \rightarrow B$   
 then  $f^{-1}: B \rightarrow A$

Q.  $f(x) = \{(1, x) (2, x) (3, y) (4, z)\}$   
 $f^{-1}(x) = \{(x, 1) (x, 2) (y, 3) (z, 4)\}$   
 It is not a func<sup>n</sup> as  $x$  has 2 images.

⑤. Floor func<sup>n</sup> :-  
 $\lfloor x \rfloor$   
 $\lfloor 3.14 \rfloor = 3$   
 $\lfloor \sqrt{5} \rfloor = 2$

⑥. Ceiling Function :-  
 $\lceil x \rceil = \lceil 3.14 \rceil = 4$

⑦. Absolute function :-

Always  
positive  
values.  
 $ABS(x) = +x$   
 $ABS(-4) = 4$   
 $ABS(5.4) = 5.4$

⑧. Integer Value function :-  
 $INT(3.14) = 3$

⑨. Modular Function :-

Remainder  $\text{MOD}(x, y)$   
 $= x \text{ mod } y$

$\text{MOD}(5, 2)$   
 $5 \text{ Mod } 2 = 1$

Q. Composition of Function :-

Consider a func<sup>n</sup>  $f: A \rightarrow B$  and  $g: B \rightarrow C$   
 i.e. codomain of  $f$  is the domain of  $g$   
 then it define a new func<sup>n</sup> from  
 $f: A \rightarrow C$  called composition of  $f$  and  $g$ .  
 It can be written  $f \circ g(x)$ .

$f \circ g(a) = f(g(a))$

Q. Let the func<sup>n</sup>  $f$  and  $g$  be defined by  
 $f(x) = 2x + 1$ ,  $g(x) = x^2 - 2$ . Find  
 the composition func<sup>n</sup>  $g \circ f(x)$ .

Sol<sup>n</sup>.  
 $f(x) = 2x + 1$   
 $g(x) = x^2 - 2$   
 $f \circ g(x) = f(g(x))$   
 $g \circ f(x) = g(f(x))$   
 $= g(2x + 1)$   
 $= (2x + 1)^2 - 2$   
 $= 4x^2 + 4x + 1 - 2$   
 $= 4x^2 + 4x - 1$



Prove that  $\log_a a^x = x$  and  $a^{\log_a x} = x$   
 inverse of exponential function.  $g(x) =$

Sol<sup>n</sup>  
 $f(x) = \log_a a^x$   
 $g(x) = \log_a x$

$$\log(x) = f(g(x))$$

$$= f(\log_a x)$$

$$= \log_a a^{\log_a x}$$

$$= x$$

$$\text{get } f(x) = g(f(x))$$

$$= g(\log x)$$

$$= \log_a a^{\log x}$$

$$= x$$

8. Half life of a radioactive substance

is time required for 1 half of the  
 time decay. Amount of carbon (C),  
 an isotope of carbon, present at a  
 feature time (t) in months is given  
 by-

$$A(t) = 100 e^{-0.0338t}$$

Find half life of material of  
 substance.

136.615  
 0 = 136  
 2250

$\log_e = \text{natural log or } \ln$

Sol<sup>n</sup>  
 $\Rightarrow$

$$A(t) = 100 e^{-0.0338t}$$

$$A(0) = 100$$

$$A(t) = \frac{1}{2} \times A(0) = 50$$

$$100 e^{-0.0338t} = 50$$

$$e^{-0.0338t} = \frac{1}{2}$$

$$-0.0338t = \log_e \frac{1}{2}$$

$$+ 0.0338t = + \log_e 2$$

$$t = \frac{\ln 2}{0.0338} = 20.5$$

9. Find the no. of positive integers  $\leq 1016$   
 and divisible by 13.

Sol<sup>n</sup>  
 $\Rightarrow$   
 $n = \left\lfloor \frac{1016}{13} \right\rfloor = 78$

$$n = 78$$

10. Find the no. of positive integers  $\leq 3000$   
 and not divisible by 7 or 8.

Sol<sup>n</sup>  
 $\Rightarrow$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A| = \left\lfloor \frac{3000}{7} \right\rfloor = 428$$

$$|B| = \left\lfloor \frac{3000}{8} \right\rfloor = 375$$



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$$|A \cap B| = \left\lfloor \frac{3000}{56} \right\rfloor = 53$$

$$|A \cup B| = 428 + 375 - 53 = 750$$

not  
No. divisible by 7 & 8 =  $3000 - 750$   
 $n = \underline{\underline{2250}}$