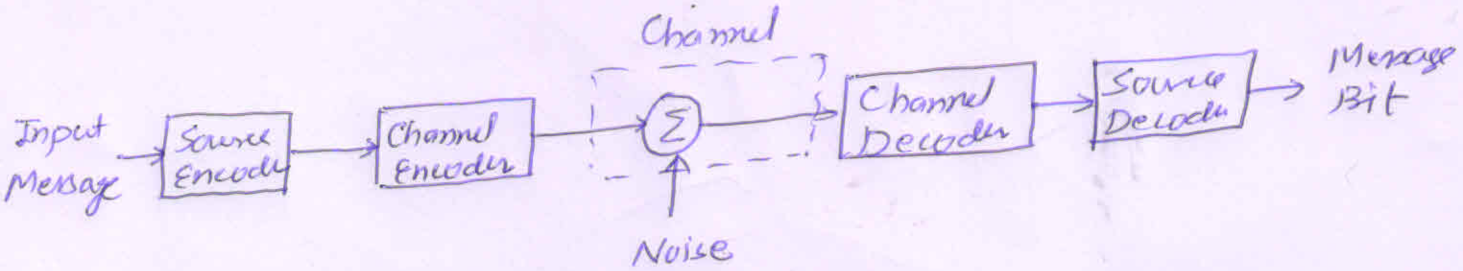
Error Control Coding



Channel Coding : The channel coder Introduce systematic redundancy into the data stream by adding bits to the message bits in such a way as to faciliate the detection and correction of bit error in the origional binary message sequence at the receiver. The channel decoder in the reciever exploits the redundancy to decide which message bits are actually transmitted. the combined objective of the channel Encoder and decoder is to minimise the effect of channel Noise.

Method of Controlling Erros.

(1) Forward Acting Error correction

(2) Error detection with retransmission.

I	II
(1) ~~Err~~ detection and correction by Reciever	(1) detection by reciever
(2) Fast	(2) slow
(3) higher prob. of error	(3) lower prob. of Error

Types of Error

(A) Random Error

(i) Due to Gaussian Noise

(ii) uncorrelated error.

(iii) not effect subsequent interval.

(B) Burst Error

(i) impulsive Noise

(ii) Source: lightning & switching transients

(iii) Effect several ~~~~ successive symbols.

Important terms.

BLOCK CODE: In block codes the binary message or data sequence is divided into sequential blocks each k bit long and each k bit block is converted into an n bit block, where $n>k$. The resultant block code is called (n,k) block code. Each of 2^k data words is mapped to a unique code word. The ratio k/n is called the code rate.

(A) Binary Field.

Addition

$0 \oplus 0 = 0, \quad 1 \oplus 1 = 0, \quad 0 \oplus 1 = 1, \quad 1 \oplus 0 = 1$

multiplication.

~~~~ $0.0=0, \quad 0.1=0, \quad 1\cdot 0 = 0, \quad 1\cdot 1 = 1$

(B) Linear Block code

$a = \{a_1 a_2 a_3 \text{---} a_n\}$ two code word in a code C

$b = \{b_1 b_2 b_3 \text{---} b_n\}$

If sum of $a \oplus b$ is also a code word in C then C is linear Block code.
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Example (4,2) LBC

1100
0011
1111
0000

$$\begin{array}{r} 1100 \\ \oplus 0011 \\ \hline 1111 \\ \hline \end{array} \qquad \begin{array}{r} 1100 \\ \oplus 1111 \\ \hline 0011 \\ \hline \end{array} \qquad \begin{array}{r} 1100 \\ \oplus 0000 \\ \hline 1100 \\ \hline \end{array}$$

(C) Hamming Distance

The hamming distance between the two code is equal to the number of elements in which they differ.

$a = \{1100\}$

$b = \{1111\}$

$d(a,b) = 2$

(D) Hamming Weight $\cdot w(c)$

$w(c)$ is the number of 1's in c.

~~w(1100~~ $w(a) = 2$

$w(b) = 4.$

(E) Minimum Hamming Distance $\cdot d_{min}$.

d_{min} is smallest Hamming distance between any pair of code word in C

Note: d_{min} is the smallest Hamming weight of the non-zero code word in ~~@~~ the C.

Code	Hamming Weight $w(c)$
1100	2
0011	2
1111	4
0000	0

$d_{min} = 2$

(E). Error detection and correction capabilities.

Detect upto 's' error per code word if

$$d_{min} \geq s + 1$$

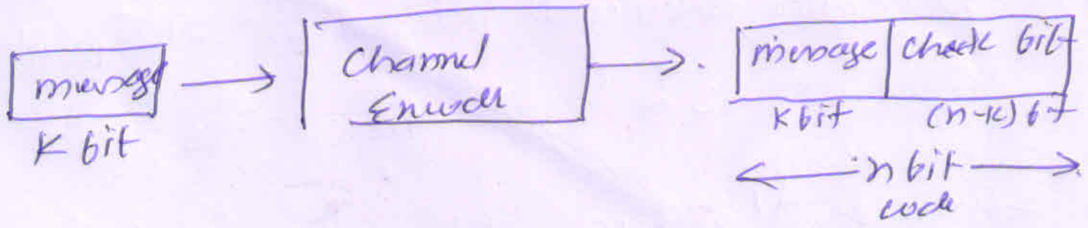
correct upto 't' error per code word if

$$d_{min} \geq 2t + 1$$

(F). Systematic and Non systematic code.

In systematic block code, the data bits appear in specified location of C.

In non systematic code it is not possible to identify message bits and check bits.



Matrix Representation of LBC

(i) Data Matrix. $d = [d_1\ d_2\ d_3 --- d_k]_{1\times K}$

(ii) code Matrix $C = [C_1\ C_2\ C_3 ---- C_n]_{1\times n}$

~~(iii) Generator Matrix~~

parity Equations. (6, 3).

$$C_1 = d_1$$
$$C_2 = d_2$$
$$C_3 = d_3$$
$$\left.\begin{array}{l} C_4 = d_1 \oplus d_2 \\ C_5 = d_1 \oplus d_2 \oplus d_3 \\ C_6 = d_2 \oplus d_3 \end{array}\right] \text{parity Equation.}$$

(iii) parity Matrix P

$$P = \begin{array}{c} \\ d. \end{array}\begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \qquad P^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(iv) Generator Matrix

$$G = [\ I_k \quad P^T\ ]_{K\times n}$$
$$= [\ I_3 \quad P^T\ ]$$
$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(v) code C

$$C = dG.$$

Example $(6, 3)$

$d = ~~1~~\ 110$

$c = ?$

Sol

method I

$~~c~~\ d_1 = 1$

$d_2 = ~~0~~\ 1$

$d_3 = 0$

$c_1 = d_1 = 1$

$c_2 = d_2 = 1$

$c_3 = d_3 = 0$

$c_4 = ~~d~~\ d_1 \oplus d_2 = 1 \oplus 1 = 0$

$c_5 = d_1 \oplus d_2 \oplus d_3 = 1 \oplus 1 \oplus 0 = 0$

$c_6 = d_2 \oplus d_3 = 1 \oplus 0 = 1$

$c = [\underbrace{1\ 1\ 0}_{\text{data}}\ \underbrace{0\ 0\ 1}_{\text{Check bit.}}]$

method II

$c = dG$

$$= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$