Unit - 4

Turing Machine

Introduction.

→ Alan Turing is father of a model which has computing capability of general purpose computer. Hence this model is popularly known as Turing m/c.

→ This m/c has following features :-

(i) It has external memory which remembers arbitrarily long sequence of i/p.

(ii) It has unlimited memory capability.

(iii) The model has facility by which the input at left or right on the tape can be read easily.

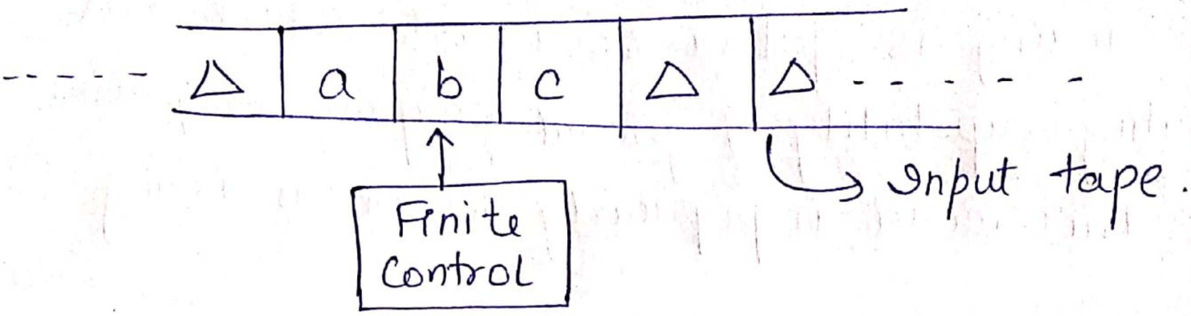
(iv) The machine can produce certain output based on its input sometimes it may be required that the same input has to be used to generate the output. So in this machine this distinction b/w input & output has been removed. Thus a common set of alphabets can be used for turing machine.

→ Turing m/c is extended version of Pushdown Automata

→ It receive their i/p written on the same tape which they also use for storage.

→ It control the head position to where reading & writing on the tape is performed.

Model of Turing M/C



(i) Input Tape :- It have infinite no. of cells, each cell contain i/p symbol and thus the input string can be placed on a tape.

The empty tape is filled by blank Δ characters.

(ii) Finite Control :- It reads the current input symbol. The tape head can move to left or right both

(iii) Finite Set of States :- States through which m/c has to undergo.

(iv) Finite Set of Symbols called external symbols which are used in building the logic of turing machine.

Definition of Turing M/C.

→ Turing m/c is a collection of following components.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \Delta \text{ or } B, F)$$

where,

Q is a finite set of states

Σ is a finite set of I/P

Γ " " " " external symbols

δ is transition / mapping fun^n. → triple ~~fun~~

~~q_0~~ $\delta(q_0, a) \rightarrow (q_1, A, L)$

q_1 → next state, A → external symbol, L → direction for moving tape head.

q_0 is Initial state where $q_0 \in Q$.

Δ or B is a blank symbol ~~used~~ used as end marker for i/p

F is a set of final states where turing m/c halts.

Representation of Turing M/C.

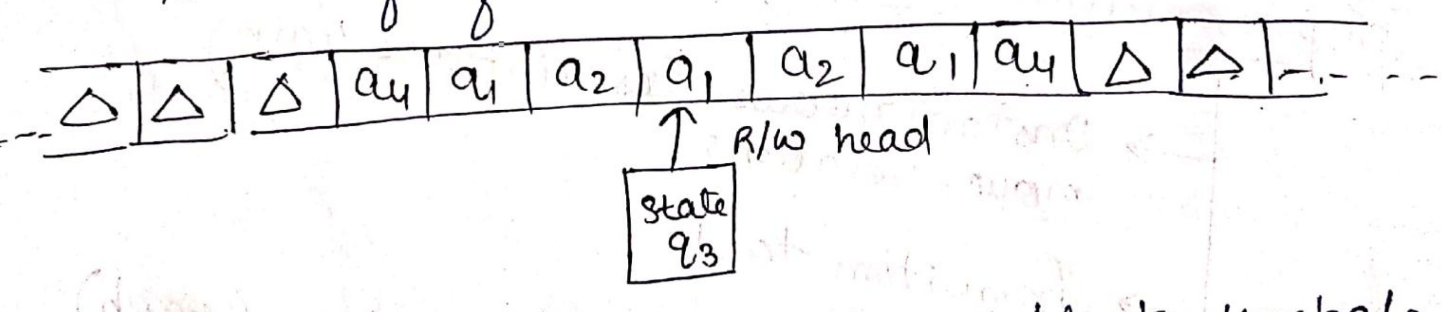
- → Instantaneous Descriptions using move - relations
- → Transition table.
- → Transition Diagram (Transition Graph).

(I) Representation By Instantaneous Descriptions..

→ An ID (Instantaneous Description) of a Turing m/c is defined in terms of the entire i/p string and the current state because the i/p string to be processed is not sufficient to define a ID of a Turing m/c for the R/W head can move to the left also.

→ **Definition :-**

An ID of a Turing m/c M is a string $\alpha\beta\gamma$, where β is the present state of M, the entire i/p string is split as $\alpha\gamma$, the 1st symbol of γ is the current symbol a under R/W head & γ has all the subsequent symbols of the i/p string & α string is the substring of the i/p string formed by all the symbols to the left of a.



In above diagram, The non blank symbols to the left of a_1 form the string $\underbrace{a_4 a_1 a_2}_{\alpha}$, which is written to the left of q_3.
The sequence of nonblank symbols to the right of a_1 is $\underbrace{a_2 a_4 a_4}_{\gamma}$.

Thus the ID is given as.

$a_4\ a_1\ a_2 \quad q_3 \quad a_1 \quad q_2\ a_4\ a_4$

left sequence — $a_4\ a_1\ a_2$

Present state — q_3

Symbol under R/W head — a_1

Right Sequence. — $q_2\ a_4\ a_4$

Moves in a Turing M/c

→ As in the case of Pushdown automata, $\delta(q, x)$ induces a change in ID of Turing M/c. We call this change in ID a <u>moves</u>.

→ Suppose $\delta(q, x_i) = (p, y, L)$. The i/p string to be processed is $x_1 x_2 x_3 \ldots x_n$ & the present state / symbol under R/W head is x_i.
So the ID before processing x_i is ~~$x_1 x_2 \ldots x_{i-1}$~~

$$x_1 x_2 \ldots\ldots x_{i-1}\, q\, x_i \ldots x_n .$$

After processing of x_i, the resulting ID is

$$x_1 \ldots\ldots x_{i-2}\ p\ x_{i-1}\ y\ x_{i+1} \ldots x_n$$

This change of ID is represented by

$$x_1 x_2 \ldots x_{i-1}\, q\, x_i \ldots x_n \vdash x_1 \ldots x_{i-2}\, p\, x_{i-1}\, y\, x_{i+1} \ldots x_n$$

If $\delta(q, x_i) = (p, y, R)$, then the change of ID is represented by

$$x_1 x_2 \ldots x_{i-1}\, q\, x_i \ldots x_n \vdash x_1 x_2 \ldots x_{i-1}\, y\, p\, x_{i+1} \ldots x_n$$

We can denote an ID by I_j for some j.
$I_j \vdash I_k$ defines a relation among IDs.
$\vdash^*$ denotes the reflexive - transitive closure of the relation $\vdash$.

(II) Representation By Transition Table

→ We give the definition of δ in the form of a table called transition table.

If $\delta(q, a) = (\gamma, \alpha, \beta)$, we write $\alpha\beta\gamma$ under a-column & q-row.

So if we get $\alpha\beta\gamma$ in the table, it means that

- α — is written in the current cell
- β — gives the movement of the head (L or R)
- γ — denotes the new state into which the Turing m/c enters.

→ eg

Present State	Tape symbols		
	b/△	0	1
→ q_1	$1Lq_2$	$0Rq_1$	–
q_2	bRq_3	$0Lq_2$	$1Lq_2$
q_3	–	bRq_4	bRq_5
q_4	$0Rq_5$	$0Rq_4$	$1Rq_4$
q_5	$0Lq_2$	–	–

(α, β, γ)

Table – Write . dir . NS.

Diagram – NS Write Dir.

III Representation By Transition diagram.

→ In transition diagram, states are represented by vertices. Directed edges are used to represent transition of states.

The labels are triples of the form (α, β, γ) where, $\alpha, \beta \in \Gamma$ & $\gamma \in \{L, R\}$.

When there is a directed edge from q_i to q_j with label (α, β, γ), it means that

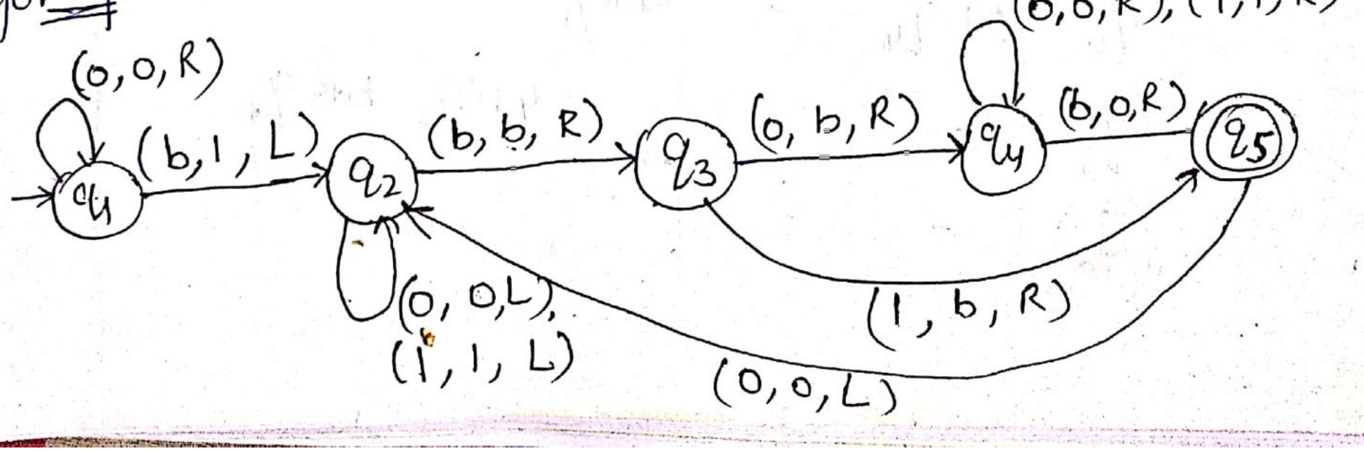
$$\delta(\underbrace{q_i}_{\text{current state}}, \underbrace{\alpha}_{\text{R/W head scans the present symbol.}}) = (\underbrace{q_j}_{\text{new state}}, \underbrace{\beta}_{\beta \text{ is written in the cell under R/W head}}, \underbrace{\gamma}_{\text{R/W head moves L or R.}})$$

→ Every edge in transition diagram can be represented by a 5-tuple $(q_i, \alpha, \beta, \gamma, q_j)$.

→ Initial state $\rightarrow q_1$

→ final state q_f

for eg Transition diagram table given (II) will be.



Language acceptability by Turing m/c

→ A string $\underline{w}$ in Σ^* is said to be accepted by M (Turing m/c) if $q_0 w \vdash^* \alpha_1 p \alpha_2$ for some $p \in F$ (final state) &

$\alpha_1, \alpha_2 \in \Gamma^*$.

→ Turing m/c M does not accept w if the m/c M either halts in a nonaccepting state or doesn't halt.

→ **eg** Consider the Turing m/c M described by the Transition table given. Describe the Processing of (a) 011, (b) 0011 (c) 001 using IDs. Which of the above strings are accepted by M?

Present State	Tape Symbol 0	1	x	y	b
$\rightarrow q_1$	xRq_2	–	–	–	bRq_5
q_2	$0Rq_2$	yLq_3	–	yRq_2	–
q_3	$0Lq_4$	–	xRq_5	yLq_3	–
q_4	$0Lq_4$	–	xRq_1	–	–
q_5	–	–	–	yRq_5	bRq_6
q_6	–	–	–	–	–

Consider the transition table of Turing m/c M & find out whether the following strings are acceptable or not.

(i) 011

$q_1 \underline{0} 1 1$

$x q_2 \underline{1} 1$

$q_3 \underline{x} y 1$

$x q_5 \underline{y} 1$

$x y q_5 \underline{1}$

There is no transition from q_5 on 1. So M/C halts & hence 011 is not acceptable.

(ii) 0011

$q_1 \underline{0} 0 1 1$

$x q_2 \underline{0} 1 1$

$x 0 q_2 \underline{1} 1$

$x q_3 \underline{0} y 1$

$q_4 \underline{x} 0 y 1$

$x q_1 \underline{0} y 1$

$x x q_2 \underline{y} 1$

$x x y q_2 \underline{1}$

$x x q_3 \underline{y} y$

$x q_3 \underline{x} y y$

$x x q_5 \underline{y} y$

$x x y q_5 \underline{y}$

$x x y y q_5 \underline{\Delta}$

$x x y y \Delta q_6 \underline{\Delta}$

There is no transition from q_6 on Δ So M/C halts & Hence 0011 string accepted by M as q_6 is final state

(iii) 001

$q_1 \underline{0} 0 1$

$x q_2 \underline{0} 1$

$x 0 q_2 \underline{1}$

$x q_3 \underline{0} y$

$q_4 \underline{x} 0 y$

$x q_1 \underline{0} y$

$x x q_2 \underline{y}$

$x x y q_2 \underline{\Delta}$

M/C halts as q_2 is not final state so, 001 string is not accepted by M/C.