Systematic Cyclic Code Generator Matrix.

(7,4) cyclic code

$g(x) = 1 + x + x^3$

$G = [\quad]_{4\times 7}$

$1 \quad x \quad x^2 \quad x^3 \quad x^4 \quad x^5 \quad x^6$

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Non-systematic Cyclic Generator Matrix

Conversion of Non-systematic to Systematic

$G = [I_k \; P^T]$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - & - \\ 0 & 0 & 0 & 1 & - & - & - \end{bmatrix}$$

$$G = \begin{matrix} X \\ X \\ X \\ \checkmark \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 \oplus R_4$

$$G = \begin{matrix} \\ \\ \checkmark \\ \checkmark \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_3$

$$G = \begin{matrix} \\ \checkmark \\ \checkmark \\ \checkmark \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + R_2 + R_4$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

I_k P^T

$$P^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$H = [P \quad I_m]_{m \times n}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

3x7

(7, 4)

Encoding using $(n-k)$ bit Shift Register (systematic)

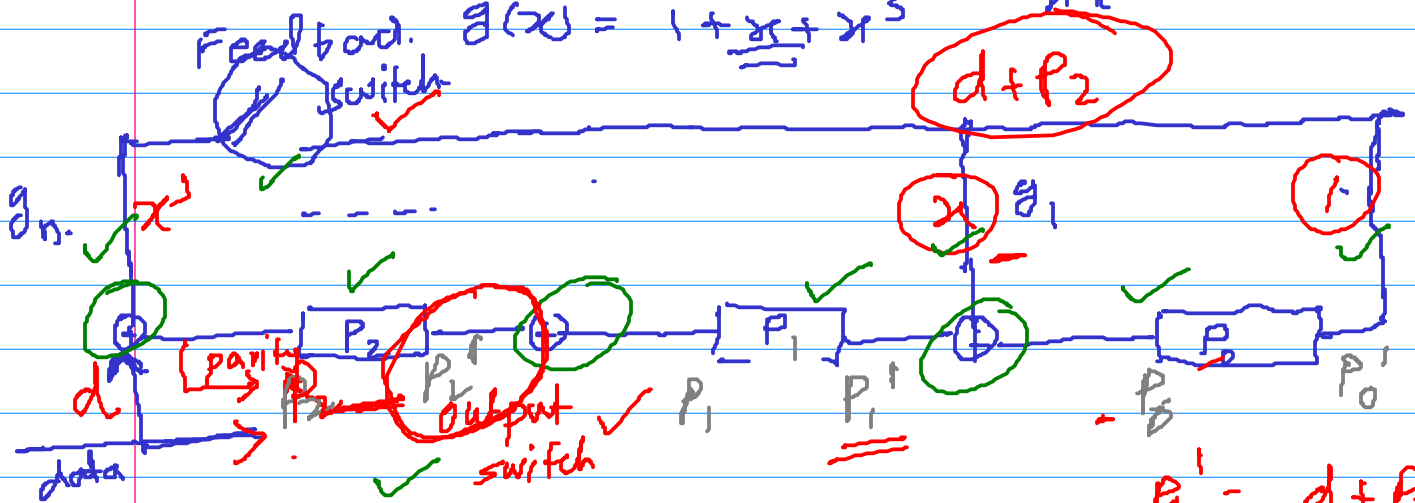
$7 - 4 = 3$

$g(x) = 1 + \_\_\_\_ + x^{n-k}$

S.F: $g(x) = g_0 + g_1 x + g_2 x^2 + \dots + g_{n-k} x^{n-k}$

Feedback switch

$g(x) = 1 + x + x^3$



$P_0' = d + P_2$

$P_1' = d + P_2 + P_0$

$P_2' = P_1$

$\oplus \Rightarrow$ module - 2 adder

XOR

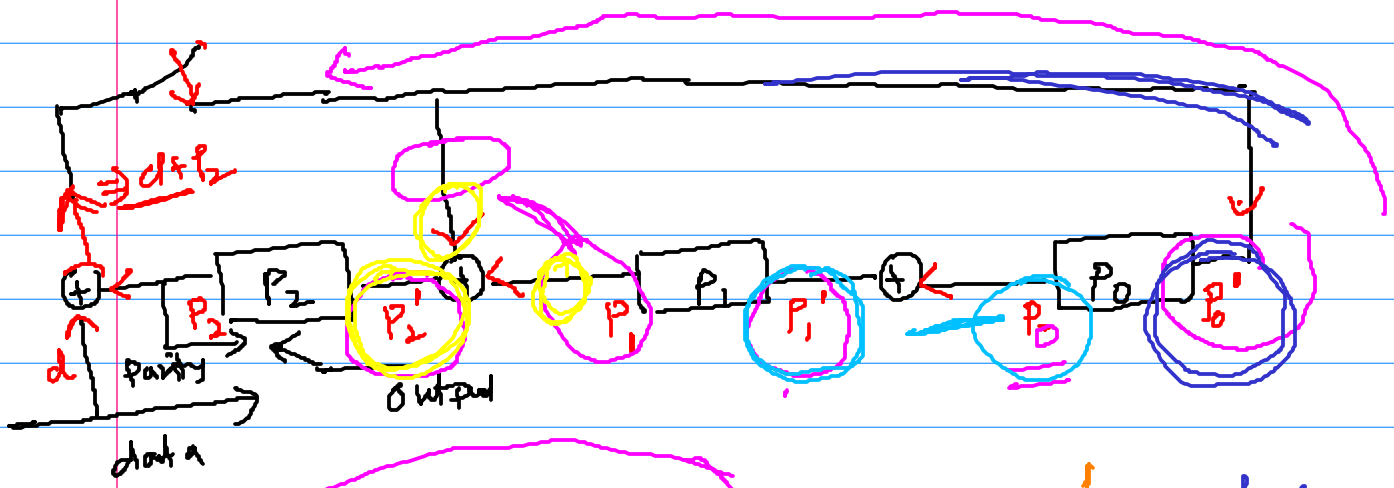
$0 \oplus 0 = 0$ $\quad$ $1 \oplus 0 = 1$

$0 \oplus 1 = 1$ $\quad$ $1 \oplus 1 = 0$

Ex

$g(x) = 1 + x + x^2 + x^5 + x^{n-k}$

$$g(x) = 1 + x^2 + x^3$$



$$p_0' = d + p_2$$
$$p_1' = p_0$$
$$p_2' = p_1 + d + p_2$$

$$p_0' = d + p_2$$
$$p_1' = p_0$$
$$p_2' = d + p_2 + p_1$$