[redacted]

Q Prove that following statements are equivalent:

(i) $P = n$ is even.

(ii) $Q = (n-1)$ is odd.

(iii) $R = n^2$ is even.

Sol^n $p \rightarrow q$

$n = 2K$.

$q \rightarrow n - 1 = 2K - 1$

odd odd

$\Rightarrow$ true.

$q \rightarrow r$

$n - 1 = 2K + 1$

$n = 2K + 2$

$n = 2(K+1)$

$n^2 = \cancel{8}4(K+1)^2$

even. even.

$\Rightarrow$ True

$r \rightarrow p$.

$n^2 = 4K^2$.

$n = 2K$.

Language of Logic

An elementary statement is a statement with a subject and verb. and sometimes an object but no connectives.
Eg Ram plays football.

prepo~~compo~~sitions & Compound Statements.

A preposition is a declarative statement which is true or false but not both many perepositien are composite i.e. composed of sub preposition and various connectives, such composite preposition are known as. compound statements

Basic Logical Operations:

① Conjunction → $p \vee q$

P	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F.

p : perposit^n stat.
q : " "
$\vee$: conju / OR

② Disjunction → $p \wedge q$.

P	Q.	$p \wedge q$
T	T	T
F	T	F
T	F	F
F	F	F.

3.) Negation: $\sim p$ or $\neg p$.

p	$\sim p$
T	F
F	T

4.) Biconditional Statement: $p \Leftrightarrow q$.
(X-OR)

$(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \Leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Converse, Inverse, Contrapositive.

Statements: (Conditional)

① If it is raining then sky is grey.

Inverse : If it is not raining then sky is not grey.

$\sim p \rightarrow \sim q$ (inverse)

Converse : If sky is grey It is raining.

$q \rightarrow p$ (converse)

Contrapositive : If sky is not grey then it is not raining

$\sim q \rightarrow \sim p$ (contrapositive).

Inverse: When you make the conditional statement negative.

Converse: when you flip the conditional statement around.

Contrapositive: when you make the converse statement negative then it is contrapositive.

(2) If two angles are congruent then they have same measure.

Inverse: If two angles are not congruent then they are not of same measure.

Converse: If two angles have same measure they are congruent.

Contrapositive: If two angles do not have same measure they are not congruent.

Tautology

Q $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

from column 3 and 5 we can conclude.

Q find the truth table of $(p \wedge q) \rightarrow r$.
Show that $\sim(p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$

p	q	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$\sim q$	$p \leftrightarrow \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

$\Rightarrow \sim(p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$

~~Contingency:~~

~~Tautology:~~

Q find $(p \wedge q) \rightarrow r$

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Note: A compound preposition that is always true, no matter what the truth values of the preposition that occurs in it is called Tautology.

Note: A compound ~~statement~~ preposition i.e. always false; is called contradiction.

Note: A compound preposition i.e. neither tautology nor contradiction is called a contingency

Algebra Laws of preposition

① Identity.
$p \wedge T \Leftrightarrow p$.
$p \vee F \Leftrightarrow p$.

② Domination Law.
$p \vee T \Leftrightarrow T$
$p \wedge F \Leftrightarrow F$

③ Idempotent Law.
$p \vee p \Leftrightarrow p$
$p \wedge p \Leftrightarrow p$.

④ Double Negation Law.
$\sim(\sim p) \Leftrightarrow p$.

⑤ Commutative Law.
$p \wedge q \Leftrightarrow q \wedge p$
$p \vee q \Leftrightarrow q \vee p$

⑥ Associative Law.
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

⑦ Distributive law
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

⑧ DeMorgans Law.
$\sim(p \vee r) \Leftrightarrow \sim p \wedge \sim r$
$\sim(p \wedge r) \Leftrightarrow \sim p \vee \sim r$.

⑨ Absorption Law
$p \wedge (p \vee q) \Leftrightarrow p$
$p \vee (p \wedge q) \Leftrightarrow p$.

⑩ Implication Law / Equivalence Rule.
$(p \to q) \Leftrightarrow \sim p \vee q$

⑪ Contrapositive Law.
$(p \to q) \Leftrightarrow \sim q \to \sim p$.

⑫ Tautology
$p \vee \sim p \Leftrightarrow T$

⑬ Contradiction.
$p \wedge \sim p \Leftrightarrow F$

⑭ Equivalence Law
$(p \leftrightarrow q) \Leftrightarrow (p \to q) \wedge (q \to p)$

Q. $(p \wedge q) \to q$.

Solⁿ $(p \wedge q) \to q$

$\sim(p \wedge q) \vee q$.
$\sim p \vee (\sim q \vee q)$
$\sim p \vee T$.
T.
H.P.

Q. $\sim(q \to p) \vee (p \wedge q) \Leftrightarrow q$.

$\sim(\sim q \vee p) \vee (p \wedge q)$.
$(q \wedge \sim p) \vee (q \wedge p)$.
$q \wedge (\sim p \vee p)$
$q \wedge T$
q.

Normal Form of Compound Preposition.

→ CNF (Conjuctive Normal Form)
→ DNF (Disjunctive Normal Form).

A compound composition is in DNF if it is ~~turned~~ term or disjunction of two or more terms (OR of AND)
A compound ~~pr~~ composition is in CNF if it is ~~clas~~ clause or a conjunction of 2 or more clause. (AND of OR).

maxterm → "and of OR"

minterm → "OR of AND"

Q. $\sim(p \vee q) \leftrightarrow (p \wedge q)$

$(\because p \leftrightarrow q) \Leftrightarrow (p \to q) \wedge (q \to p)$

$= [\sim(p \vee q) \to (p \wedge q)] \wedge [(p \wedge q) \to \sim(p \vee q)]$

$= [\sim(\sim(p \vee q)) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$= [(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$= [(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \wedge \sim q) \vee (\sim p \wedge \sim q)]$

$= [(p \vee q \vee p) \wedge (p \vee q \vee q)] \wedge [(\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim$

$= [(p \vee q) \wedge (p \vee q)] \wedge [(\sim p \vee \sim q) \wedge (\sim p \vee \sim q)]$

$= (p \vee q) \wedge (\sim p \vee \sim q)$

It is ("and of OR")
so it is maxterm and CNF.

$(p \vee q \wedge \sim p) \vee (p \vee q \wedge \sim q)$

$((p \vee q) \wedge \sim p) \vee ((p \vee q) \wedge \sim q)$

$((p \wedge \sim p) \vee (q \wedge \sim p)) \vee ((p \wedge \sim q) \vee (q \wedge \sim q))$

It is "or of and"
so it is minterm & DNF.

Argument Validation.

Quantifiers.

→ Universal Quantifier $\forall x \in A$

→ Quantifier. $\exists x \in A$.

1.) If two sides of triangle are equal then opposite angles are equal.

2.) Two sides of $\triangle$ are not equal.

$\therefore$ the opposite angle are not equal.

Conclusion

P_1 : $P \to Q$.

P_2 : $\sim P$.

C : $\sim Q$.

P	Q	$P \to Q$	$\sim P$	$\sim Q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Ambiguous

P_1 P_2

$\therefore$ The statement is invalid.

classmate

Date ______

Page ______

Share

p I take bus or I walk.

q If I walk I get tired.

r : I do not get tired.

s : I take bus

Soln

A : I take bus

W : I walk.

G : I get tired.

P_1 : $A \vee W$

P_2 : $W \to G$

P_3 : $\sim G$

C : A

A	W	G	P_1: $A \vee W$	P_2: $W \to G$	P_3: $\sim G$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	T	T	F
T	F	F	T	T	T
F	T	T	T	T	F
F	T	F	T	F	T
F	F	T	F	T	F
F	F	F	F	T	T