Q Expand $(2a+b)^4$

Sol^n $(2a+b)^4 = C(4,0)(2a)^4 + C(4,1)(2a)^3 b + C(4,2)(2a)^2 b^2$
$+ C(4,3)(2a)b^3 + C(4,4)b^4$
$= 16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4$

Q. Find the general term in expansion of $(x^3 + 1/x)^{10}$

Sol^n $$\left(x^3 + \frac{1}{x}\right)^{10} = (x^3 + x^{-1})^{10}$$

General term in expansion is

$$T_{r+1} = C(10,r)(x^3)^{10-r}(x^{-1})^r$$
$$= C(10,r)\, x^{30-3r}\, x^{-r}$$
$$= C(10,r)\, x^{30-4r}$$

Ex Find the term ~~idependent~~ independent of x in the expansion of $(x^2 + 1/x)^{12}$.

Solⁿ $\left(x^2 + \frac{1}{x}\right)^{12} = (x^2 + x^{-1})^{12}$

The general term in the expansion of $(x^2 + x^{-1})^{12}$ is

$$T_{r+1} = C(12, r)(x^2)^{12-r}(x^{-1})^r$$
$$= C(12, r)\, x^{24-2r}\, x^{-r}$$
$$= C(12, r)\, x^{24-3r}$$

$\because$ ~~It~~ Independent of x means x^0.

$\therefore$ $24 - 3r = 0$

$3r = 24$

$\boxed{r = 8}$

Hence, the coefficient x^0 is $C(12, 8)$.

i.e the term ~~idepend~~ independent of x is $C(12, 8)$ $= 495$.

Ex Prove that $C(n,1) + C(n,3) + \ldots = C(n,0) + C(n,2) + \ldots = 2^{n-1}$

Solⁿ We know that

$$n(C,1) + n(C,3) + n(C,5) + \ldots = n(C,0) + n(C,2) + \ldots$$

Let S denote the common total of these sums.

Adding right hand side to the left, we get

$$C(n,0) + C(n,1) + C(n,2) + C(n,3) + \ldots + C(n,n) = 2S$$

$$\Rightarrow 2^n = 2S$$

$$S = \frac{2^n}{2} = 2^{n-1}$$

Ex In the expansion of $(1+x)^n$, prove that

$$C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{2n!}{(n!)^2}$$

Solⁿ We know that

$$(1+x)^n = C(n,0) + C(n,1)x + \dots + C(n,n)x^n \quad \text{——— (1)}$$

We also know that coefficients of terms equivalent from beginning and end are equal, therefore, we can write

$$(1+x)^n = C(n,n) + C(n,n-1)x + \dots + C(n,0)x^n \quad \text{——— (2)}$$

Multiplying eqⁿ (1) & (2)

$$(1+x)^{2n} = [C(n,0) + C(n,1)x + \dots + C(n,n)x^n][C(n,n) + C(n,n-1)x + \dots + C(n,0)x^n]$$

from right hand side, the coefficients of x^n is

$$C(n,0)^2 + C(n,1)^2 + \dots + C(n,n)^2$$

But the coefficients of x^n in $(1+x)^{2n}$ is given by

$$C(2n,n) = \frac{2n!}{n!\cdot n!} = \frac{2n!}{(n!)^2}$$

$$\therefore \; C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{2n!}{(n!)^2}$$

Q Prove that

$$\frac{C(n,1)}{C(n,0)} + 2\cdot\frac{C(n,2)}{C(n,1)} + 3\cdot\frac{C(n,3)}{C(n,2)} + \dots + n\,\frac{C(n,n)}{C(n,n-1)} = \frac{n(n+1)}{2}$$

Solⁿ We know that

$$\frac{C(n,1)}{C(n,0)} = \frac{n}{1} = n$$

$$2\cdot\frac{C(n,2)}{C(n,1)} = \frac{\frac{\cancel{2}n(n-1)}{1\cdot\cancel{2}}}{\frac{n}{1}} = \frac{n(n-1)}{n} = (n-1)$$

$$3\cdot\frac{C(n,3)}{C(n,2)} = \frac{\cancel{3}\cdot\frac{n(n-1)(n-2)}{1\cdot2\cdot\cancel{3}}}{\frac{n(n-1)}{2\cdot1}} = (n-2)$$

$\vdots$

$$n\cdot\frac{C(n,n)}{C(n,n-1)} = n\cdot\frac{1}{n} = 1.$$

Adding, we get

$$= \frac{C(n,1)}{C(n,0)} + 2\cdot\frac{C(n,2)}{C(n,1)} + 3\cdot\frac{C(n,3)}{C(n,2)} + \dots + n\cdot\frac{C(n,n)}{C(n,n-1)}$$

$$= n + (n-1) + (n-2) + \dots + 1 \le 1+2+3+\dots+(n-1)+n$$

$$= \frac{n(n-1)}{2}$$