Combinators

Combinatorics deals with counting and enumeration of specified objects, patterns or designs. Techiniques of counting are important in mathematics and Computer Science.

Permutation and Combination

Permutations : A permutation of 'n' objects taken 'r' at a time is an arrangement of 'r' of the objects $(r \leq n)$

A permutation of n objects taken r at a time is also called r-permutation or an r-arrangement.

Representation of permutations

$P(n, r)$ means the number of ~~permution~~ permutations of n objects taken r at a time.

$P(n, r)$ ~~or~~ ^nP_r or $P\binom{n}{r}$ or $[n]_r$ or $n_{(r)}$

Formula of Permutations

$$^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

Note : $\lfloor 0 = 1$

(a) <u>When repetition of objects is allowed.</u>

The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.

~~(b) When repetition~~

(b) <u>Permutations when the objects are not distinct</u>

The number of permutations of n objects of which P_1 are of one kind, P_2 are of second kind, ... P_k are of k^{th} kind and the rest if any, are of different kinds is $\frac{n!}{P_1!\, P_2! \cdots P_k!}$

Combinations

A combination is a selection of some or all of ~~at~~ a number of different objects where the order of selection is immaterial. The number of selections of r objects from given n objects is ~~dd~~ denoting by ^nC_r, and is given by $^nC_r = \frac{n!}{r!\, n-r!}$

Remarks

1. Use permutations if a problem calls for the number of arrangements of objects and different orders are to be counted.
2. Use combinations if a problem calls for the number of ways of selecting objects and the order of selection is not to be counted.

Some Important facts

Let n and r be positive integers such that $r \le n$. Then

(i) ${}^nC_r = {}^nC_{n-r}$

(ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(iii) $n\,{}^{n-1}C_{r-1} = (n-r+1)\,{}^nC_{r-1}$

Questions: There are 4 black, 3 green and 5 red balls. In how many ways can they be arranged in a row?

Soln: Total number of balls $= 4+3+5 = 12$.

The number of ways in which the balls can be arranged in a row $= \frac{12!}{4!\,3!\,5!} = 27,720$

Ques: How many words of three distinct letters can be formed form the letters of word "LAND"?

Soln: The number of three distinct letter words that can be formed from 4 letters of word LAND is $P(4,3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4! = 24$

Ques: A box contains 10 light bulbs. Find the number n of ordered samples of:

(a) Size 3 with replacement, and

(b) Size 3 without replacement

Soln: (a) $n = 10^r = 10^3 = 10 \times 10 \times 10 = 1000$

(b) $P(10, 3) = 10 \times 9 \times 8 = 720$

Circular Permutation

Let n distinct be given. If n objects are to be arranged round a circle we take an objects and fix it in one position.

Now the remaining permutations of n different objects $= n-1!$

When order of permutation is anticlockwise or clockwise of objects around a circle as same circular permutations. Every arrangement with n objects round a circle is counted twice in $(n-1)!$ circular permutations

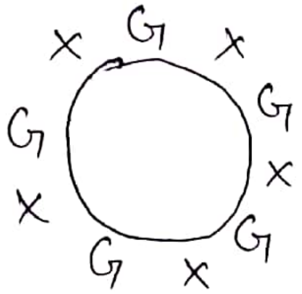
Total number of different permutations of n distinct objects is $= \frac{(n-1)!}{2}$

Ex: In how many ways can a party of 9 persons arrange themselves around a circular table?

Solⁿ: One person can sit at any place in circular table. The other 8 persons can arrange themselves in $\underline{8!}$ ways.

Ex: In how many ways 5 gents and 4 ~~to~~ ladies dine at a round table. if no two ladies are to sit together?

Solⁿ



Firstly, 5 gents can sit round the circular table in 5 positions. They can be arranged in $(5-1)! = 4!$ ways.

The ladies can sit 4 out of 5 seats. This can be done by $P(5,4)$ ways.

$\therefore$ Required number of ways $= 4!\ P(5,4)$

$$= \frac{4!\ 5!}{5-4!}$$

$$= \frac{4!\ 5!}{1!}$$

$$= 2{,}880$$

Ques In a small village, there are 87 families, of which 52 families have atmost 2 children. In a rural development program 20 families are to be chosen for assistance, of which atleast 18 families must have atmost 2 children. In how many ways can the choice be made?

Solⁿ 18 families having atmost 2 children and 2 selected from other type of families.

$$^{52}C_{18} \times {}^{35}C_2$$

Scanned with CamScanner

19 families having atmost 2 children and 1 selected from other type of families.

$$^{52}C_{19} \times {}^{35}C_1$$

All selected 20 families having atmost 2 children

$$^{52}C_{20}$$

Hence, total number of possible choices is

$$\boxed{^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}}$$