S. No.	data	code word
1	0000	0,000000
2	0001	000 1011
3	0010	
4	0011	
5	0100	
✓6	0101	
✓7	0110	
8	0111	
9	1000	
10	1001	
11	1010	
12	1011	
13	1100	
14	1101	
15	1110	
16	1111	

(7) $d = 0110$, $g(x) = 1 + x + x^3$, $c = ?$

$d(x) = x + x^2$

$c(x) = d(x)\,g(x)$

$= (x + x^2)(1 + x + x^3)$

$= x + x^2 + x^4 + x^2 + x^3 + x^5$

$= x + x^3 + x^4 + x^5$

$c = \underset{1}{0}\ \underset{x}{1}\ \underset{x^2}{0}\ \underset{x^3}{1}\ \underset{x^4}{1}\ \underset{x^5}{1}\ \underset{x^6}{0}$

✓

⑥ $d = 0101$

$d(x) = x + x^3$

$g(x) = 1 + x + x^3$

$\therefore c(x) = d(x)\, g(x)$

$= (x + x^3)(1 + x + x^3)$

$= x + x^2 + x^4 + x^3 + x^4 + x^6$

$= x + x^2 + x^3 + x^6$

$= \underline{0111001}$

④ parity check polynomial. $h(x)$

conditions.

① degree must be equal to k

② $g(x)\, h(x) = 1 + x^n$

$$\boxed{h(x) = \frac{1 + x^n}{g(x)}}$$

example $(7, 4)$ if $g(x) = 1 + x + x^3$, $h(x) = ?$

$$h(x) = \frac{1 + x^7}{1 + x + x^3}$$

$$= \frac{(1+x)(1+x+x^3)(1+x^2+x^3)}{(1+x+x^3)}$$

$$= (1+x)(1+x^2+x^3)$$

$$= 1 + x^2 + x^3 + x + x^3 + x^4$$

$$\boxed{h(x) = 1 + x + x^2 + x^4}$$

Imp (5) Syndrome polynomial $s(x)$

Note $c(x) = d(x)g(x)$

(i) $c(x)/g(x) = d(x)$

(ii) $c(x) \bmod g(x) = 0$

$5/3 = 1$

$5 \,.\, 3 = 2$

5 divide 3 = 1

5 mod 3 = 2

Now received code may be differ from transmitted code

r c

$r = c + e$

$r(x) = c(x) + e(x)$

taking mod $g(x)$ both side.

$r(x) \bmod g(x) = c(x) \bmod g(x) + e(x) \bmod g(x)$

$= 0 + e(x) \bmod g(x)$

$s(x) = \boxed{r(x) \bmod g(x) = e(x) \bmod g(x)}$

Case I If no error $e = 0$

$e(x) = 0$

$e(x) \bmod g(x) = 0$

then $s(x) = 0$

i.e received code contain no error

$r = c$

$r(x)/g(x) = c(x)/g(x) = d(x)$

Case II If error present $e \neq 0$

$e(x) \neq 0$

$e(x) \bmod g(x) \neq 0$

then $S(x) \neq 0$

$1 \; x \; x^2 \; x^3 \; x^4 \; x^5 \; x^6$

Syndrome Evaluation table

error position	error vector	error polynomial $e(x)$	$e(x) \bmod g(x)$ $e(x) \bmod (1+x+x^3)$
0	0000000		
1	1000000	1	$1 \bmod (1+x+x^3) = 1$
2	0100000	x	$x \bmod (1+x+x^3) = x$
3	0010000	x^2	$x^2 \bmod (1+x+x^3) = x^2$
4	0001000	x^3	$x^3 \bmod (1+x+x^3) = 1+x$
5	0000100	x^4	$x+x^2$
6	0000010	x^5	x^2+x+1
7	0000001	x^6	x^2+1

$$x^3+x+1 \overline{)\,1}$$

$$x^3+x+1 \overline{)\,x}$$

$$x^3+x+1 \overline{)\,x^2}$$

$$x^3+x+1 \overline{)\,x^3}$$

$$x^3+x+1 \overline{)\,x^4}$$

$$x^3+x+1 \overline{)\,x^5}$$

$$x^3+x+1 \overline{)\,x^6}$$

$$x^3+x+1 \overline{)\,x^3}$$

x^3+x+1

$x+1$

$$x^3+x+1 \overline{)\,x^4}$$

x^4+x^2+x

x^2+x

$$x^3+x+1 \overline{)\,x^5}$$

$x^5+x^3+x^2$

x^3+x^2

x^3+x+1

x^2+x+1

$$x^3+x+1 \overline{)\,x^6}$$

$x^6+x^4+x^3$

x^4+x^3

x^4+x^2+x

x^3+x^2+x

x^5+x+1

x^2+1

Type Example (7,4), $g(x) = 1+x+x^3$, received code is 1110011, ~~find~~ data word

1. Syndrome evaluation table

Solution (2)

$r = 1110011$

$r(x) = 1\ 1\ 1\ 0\ 0\ 1\ 1$

$1\ x\ x^2\ x^3\ x^4\ x^5\ x^6$

$= 1+x+x^2+x^5+x^6$

$S(x) = r(x) \bmod g(x)$

$$x^3+x+1 \overline{)\ x^6+x^5+x^2+x+1}\ \ (x^3+x^2+x$$

$x^6 \quad +x^4+x^3$

$x^5+x^4+x^3+x^2+x+1$

$x^5 \quad +x^3+x^2$

x^4+x+1

$x^4+x \quad +x^2$

x^2+1

$r(x) \bmod g(x) = 1+x^2$

$S(x) = 1+x^2$

$S(x) \neq 0$ error present, at 7th bit

received $r = 1110011$

transmitted code $c = 1110010$

$d = 2$

$c(x) = 1 + x + x^2 + x^5$

$g(x) = x^3 + x + 1$

$d(x) = c(x) / g(x)$

$$x^3 + x + 1 \;\overline{)\; x^5 + x^2 + x + 1 \;}\; x^2 + 1$$

$x^5 + x^2 + x^3$

$x^3 + x + 1$

$x^3 + x + 1$

0

$d(x) = 1 + x^2$

$d = \underset{1}{1}\ \underset{x}{0}\ \underset{x^2}{1}\ \underset{x^3}{0}$

Ans 1010