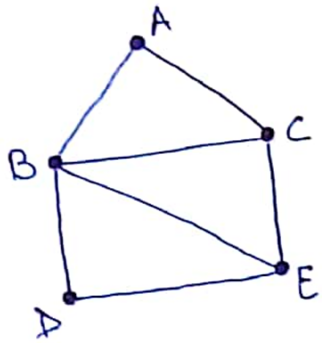
Planar Graph

- A Graph which can be drawn in the plane so that its edges do not cross is said to be planar.

eg:-

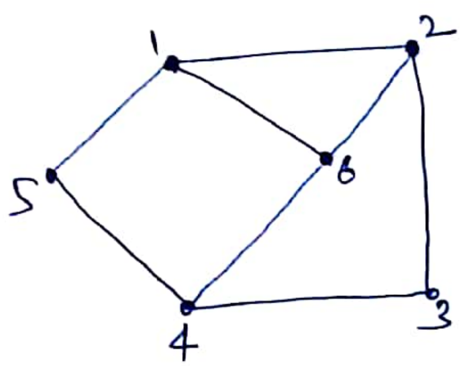


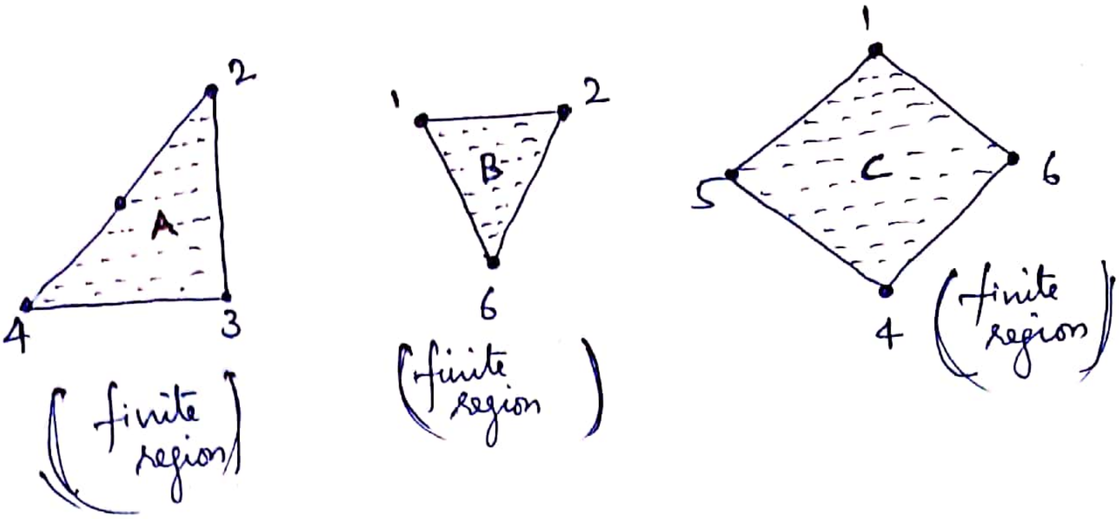
Region or face of a graph

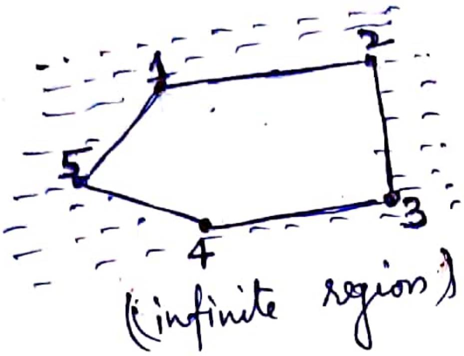
- An area of the plane that is bounded by edges of the planar graph and is not further subdivided into subareas is called a region or face of the planar graph.

finite region — A region is said to be finite if its area is finite.

infinite region — A region is said to be infinite if its area is infinite.







Let f be a face (region) in a planar graph. The length of the cycle (or closed walk) which borders f is called the degree of the region f. It is denoted by deg(f).

Euler's ~~Format~~ Formula for Connected Planar Graphs

If G is a connected planar graph with e edges, v vertices & r regions, then

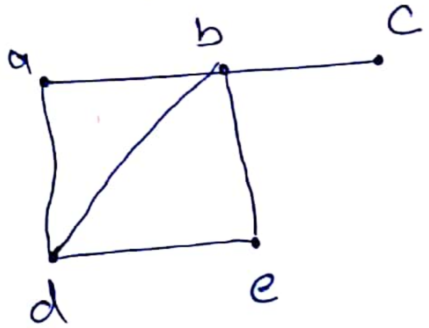
$$v - e + r = 2$$

Operations on Graph

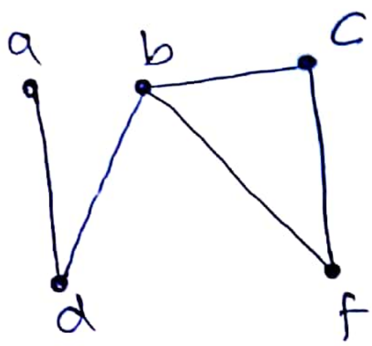
(1) Union

The union of two graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$ and is denote by $G_1 \cup G_2$.

Example Find the union of the following two graphs.

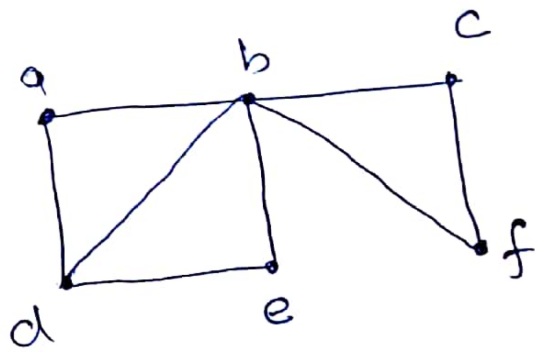


(G_1)



(G_2)

Solⁿ



(2) Intersection :

The intersection of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = (V, E)$ where $V = V_1 \cap V_2$, $E = E_1 \cap E_2$ and is denoted by $G_1 \cap G_2$.

③ Ring Sum of two graph

The ring-sum of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph G where $V=V_1\cup V_2$ and E is the set of edges either G_1 or G_2 but not in both. The ring sum of two graph G_1 & G_2 is denoted by $G_1 \oplus G_2$

$$G_1 \oplus G_2 = (V_1 \cup V_2,\ (E_1\cup E_2)-(E_1\cap E_2))$$

④ Complementary Graph

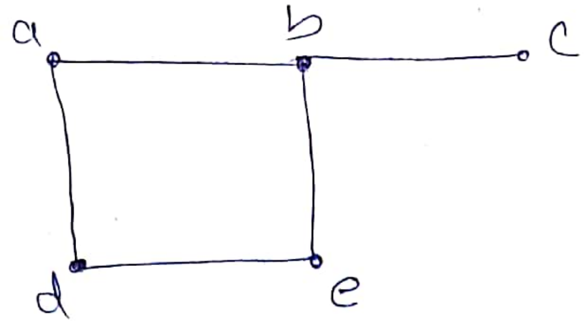
Let $G=(V,E)$ be a Simple graph on n-vertices and $K_n=(V, E_n)$ is a complete graph on n-vertices. Then, complement of the graph G is the graph $G'=(V', E')$ such that

$$V'=V \text{ and } E'=E_n-E$$

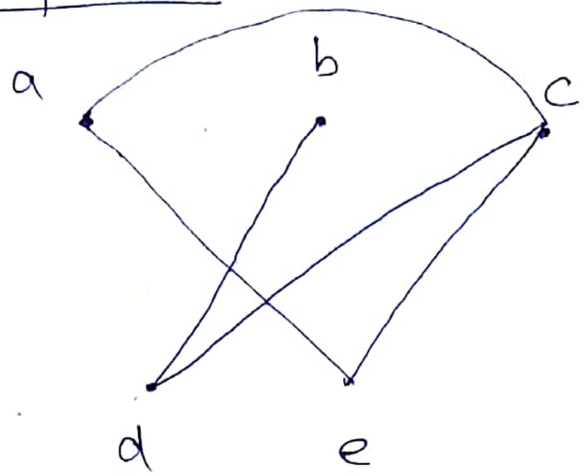
where E_n is the set of edges of K_n.

Q Find the complement of graph G_1.

$$G_1=(V,E)=(\{a,b,c,d,e\}\{\{a,b\}, \{b,c\}, \{b,e\}, \{d,e\}\}).$$



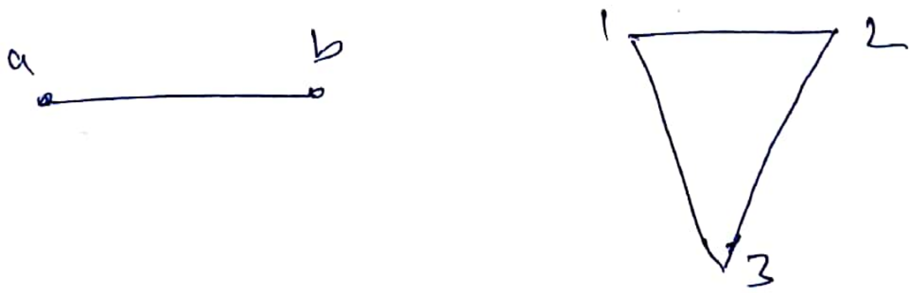
Complement



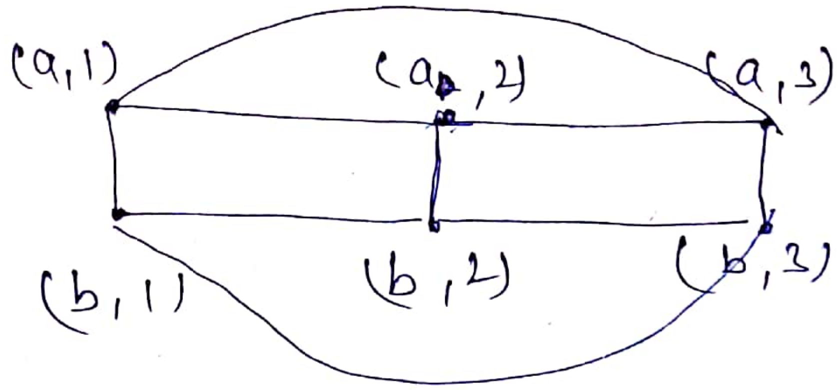
(5) Product of two graph

The product of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G = G_1 \times G_2$ with the set of vertices $V_1 \times V_2$ and any two vertices (u_1, u_2) & (v_1, v_2) are adjacent iff $u_1 = v_1$ & edge b/w u_2 & v_2 ~~beco~~ belongs to E_2 or $u_2 = v_2$ & edge b/w u_1 & v_1 belong to E_1.

Example

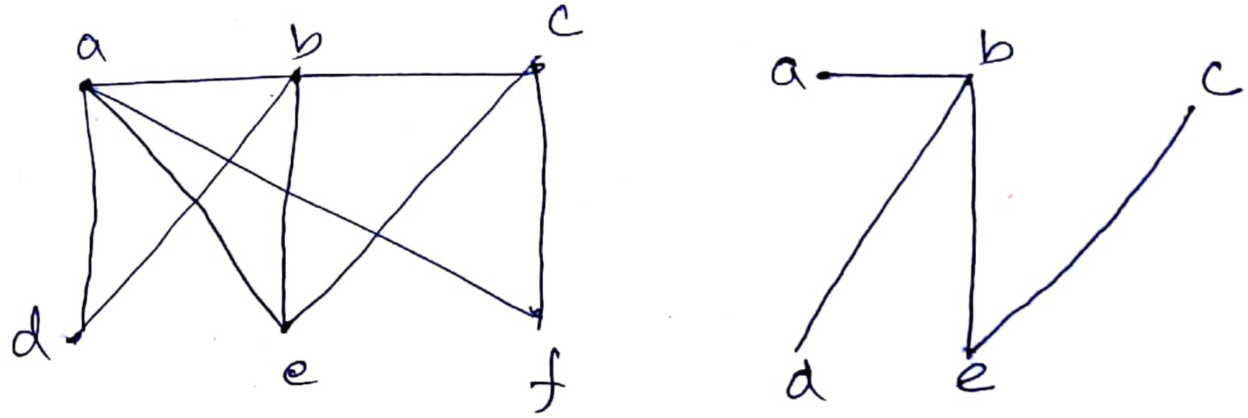


$V_1 \times V_2 = \{(a,1), (a,2), (a,3), (b,1) (b,2) (b,3)\}$



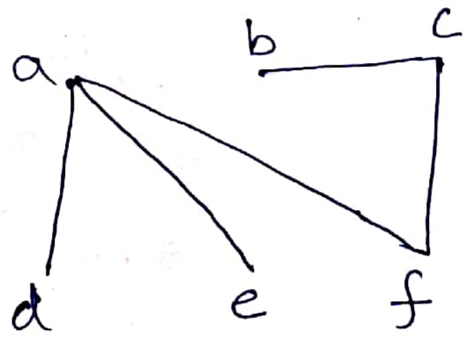
⑥ Difference of two graph

The difference of two graphs G_1 & G_2 is the graph $G = G_1 - G_2$ have all the edges which are in G_1 but not in G_2. $G_1 - G_2$ is also said to be complement of G_2 in G_1.



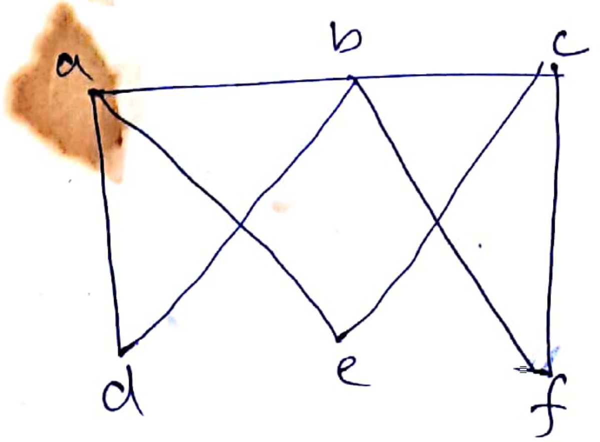
(G_1)

(G_2)



(7) Fusion of vertices

A ~~of~~ pair of vertices u & v in a graph are said to be fused (merged or identified) if two vertices are replaced by a single vertex and every edge that was incident on either u or v or both now incident on this new vertex.



On merging the vertices b & c.