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Q Find maxima and minima of the f^n

$$u = a^2x^2 + b^2y^2 + c^2z^2$$

where $x^2+y^2+z^2=1$

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Imp.

Q1. To prove $\int_0^1 \sqrt{(1-x^4)}\,dx = \frac{[\Gamma(1/4)]^2}{6\sqrt{2\pi}}$

Q2 prove that $\int_0^{\pi/2} \tan^m x\,dx = \pi/2 \sec\left(\frac{m\pi}{2}\right)$.

V.V.V Imp

Q3. Evaluate : $\int_0^1 x^4 (1-x^2)^{5/2}\,dx = \frac{3\pi}{512}$

Q4. Prove that $\int_0^\infty \sqrt{x}\,e^{-x^2}\,dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\,dx = \frac{\pi}{2\sqrt{2}}$

Q5 prove that $\int_{-\infty}^{\infty} \cos\left(\frac{\pi x^2}{2}\right) dx = 1.$

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Q. Evaluate. $\int_{x=0}^{\log 2}\int_{y=0}^{x}\int_{z=0}^{x+\log y} e^{x+y+z}\,dz\,dx\,dy$

$\int_0^{\log 2}\int_0^{x} \cancel{(e^{x+y+z})} \cancel{(e^{x+y+z})} \left(e^{x+y+z}\right)_0^{x+\log y} dx\,dy$ $\boxed{e^{\log y} = y}$

$\int_0^{\log 2}\int_0^{x}\left[e^{x+y+x+\log y} - e^{x+y}\right] dx\,dy$

$= \int_0^{\log 2}\int_0^{x}\left[\underset{I}{y}\cdot \underset{II}{e^{2x+y}} - e^{x+y}\right] dx\,dy$

$= \int_0^{\log 2}\left[\left(y\cdot e^{2x+y}\right)_0^{x} - \int_0^{x} 1\cdot e^{2x+y}\,dy - e^{(x+y)}\Big|_0^{x}\right] dx$

$= \int_0^{\log 2}\left[x\cdot e^{3x} - \left(e^{2x+y}\right)_0^{x} - (e^{2x} - e^{x})\right] dx.$

$= \int_0^{\log 2}\left[x\cdot e^{3x} - e^{3x} + \cancel{e^{2x}} - \cancel{e^{2x}} + e^{x}\right] dx$

$= \int_0^{\log 2}\left[x\cdot e^{3x} - e^{3x} + e^{x}\right] dx.$

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$$= \left(\frac{x e^{3x}}{3}\right)_0^{\log 2} - \int_0^{\log 2} \frac{1 \cdot e^{3x}}{3}\,dx - \left(\frac{e^{3x}}{3}\right)_0^{\log 2} + \left(e^{x}\right)_0^{\log 2}$$

$$= \log 2 \cdot \frac{e^{3\log_x 2}}{3} - \left(\frac{e^{3x}}{9}\right)_0^{\log 2} - \frac{e^{3\log 2}}{3} + \frac{1}{3}$$

$$+ e^{\log 2} - 1$$

$$= 8/3 \log 2 - \left[\frac{e^{3\log 2}}{9} - 1/9\right] - 8/3 + \frac{1}{3} + 2 - 1$$

$$= 8/3 \log 2 - \frac{1}{9}\left[e^{3\log 2} - 1\right] - \frac{8+1+3}{3}$$

$$8/3 \log 2 - 1/9 \left[e^{3\log 2} - 1\right] - 12/3$$

$$8/3 \log 2 - 1/9 [7] - 12/3$$

$$8/3 \log 2 - 7/9 - \frac{12}{3}$$

$$\boxed{8/3 \log 2 - \frac{19}{9}}$$

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line integral → integral along any curve is known as line integral.

$$\int_C \vec{F}\cdot d\vec{r} = \int_C [f_1 dx + f_2 dy + f_3 dz]$$

workdone

$$\oint_C \vec{F}\cdot d\vec{r}$$

$\vec{r} = xi + yj + zk.$
$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$
$\vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}.$

closed curve circulation

Q evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (x^2 + y^2)\hat{i} + xy\hat{j}$

C — curve $\boxed{y = x^2}$ in xy-plane from (0,0). to (3,9)

$$\int_0^3 (x^2 + y^2)\,dx + xy\,dy.$$

$$\int_0^3 (x^2 + x^4)\,dx + x\cdot x^2\, 2x\,dx.$$

$$= \int_0^3 (x^2 + 3x^4)\,dx$$

$$= \left.\frac{x^3}{3} + \frac{3x^5}{5}\right]_0^3$$

$$= \frac{(3)^3}{3} + \frac{3(3)^5}{5} = \frac{27 + 729}{}$$

$$\frac{405 + 2187}{15} = \frac{2191}{15} = 146.06$$

In parametric form:→

$$x = \phi_1(t) \quad y = \phi_2(t).$$

$$\int_C \vec{f} \cdot d\vec{r} = \int_C \left(\vec{f} \cdot \frac{d\vec{r}}{dt}\right) dt.$$

$$\int_C f \cdot dr = \int_{x=0}^{3} \left[x^2 + 3x^4\right] dx$$

Q2. Evaluate $\int_C \vec{f} \cdot d\vec{r}$ $\quad \vec{f} = xy\hat{i} + yz\hat{j} + zxk$

$c \rightarrow$ curve $\vec{r} = ti + t^2 j + t^3 k$

$\vec{r} = x\hat{i} + y\hat{j} + zk.$ $\quad$ t varies from -1 to 1

$x = t, \quad y = t^2 \quad z = t^3$

$\vec{f} = t^3 i + t^5 j + t^4 k.$

$$\frac{d\vec{r}}{dt} = \hat{i} + 2t\hat{j} + 3t^2 k.$$

$$\vec{F}\cdot\frac{d\vec{r}}{dt} = t^3 + 2t^6 + 3t^6 = t^3 + 5t^6$$

$$\int_C \left(\vec{F}\cdot\frac{dr}{dt}\right)dt = \int_{-1}^{1} [t^3 + 5t^6]\,dt$$

$$\frac{t^4}{4} + 5\frac{t^7}{7}$$

$$= \left[\frac{1}{4} + \frac{5}{7}\right] - \left[\frac{(-1)^4}{4} + 5\frac{(-1)^7}{7}\right]$$

$$\frac{8+20}{28} - \left[\frac{1}{4} - \frac{5}{7}\right]$$

$$1 - \left[\frac{7-20}{28}\right]$$

$$1 - {}^{-13}/_{28} = \frac{28-13}{28} = \frac{15}{28}$$

$$\boxed{= 10/7}\ \text{Ans}$$

Q(3). Evaluate $\int_C \vec{F}\cdot d\vec{r}$, $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$

$C \to$ rectangular in xy-plane bounded by $x=0$, $x=a$

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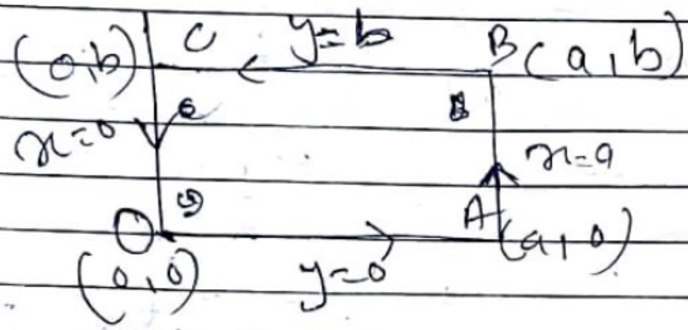
$y=0 \quad y=b.$



$$\vec{F}\cdot d\vec{r} = (x^2+y^2)dx - 2xy\,dy.$$

① line integral along OA → x=0 y=0

dy=0

$$\int_{OA} F\cdot dr = \int_{x=0}^{a} x^2 dx = a^3/3$$

② line integral along AB x=a y=0

dx=0

$$\int_{AB} F\cdot dr = -\int_{0}^{b} 2ay\,dy.$$

$$2a\int_{b}^{0} y\,dy$$

$$=2a\left[y^2/2\right]_b^0 = \boxed{-\frac{2ab^2}{2}}$$

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3 - line integral along BC.

$x = a$ to 0 $y = b$ $dy = 0$

$\int_a^0 (x^2 + y^2$

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Find volume of solid generated by revolution of cissoid about its asymptote.

$y^2 (2a-x) = x^3$ or $x = a \sin^2 t,\ y = 2a \sin^3 t$.

Solution. Eq of curve $y^2 = \frac{x^3}{2a-x}$

here $x = 2a$ is asymptote

So the volume is;

$$v = \int_{-\infty}^{\infty} \pi (2a-x)^2 \, dy$$

$$v = 2\int_{0}^{\infty} \pi (2a-x) . dy.$$

Now diff w.r.t 'x'

$$y^2 = \frac{x^3}{2a-x}.$$

$$2y \, dy/dx = \frac{3x^2 (2a-x) - x^3(1)}{(2a-x)^2}$$

$$2y \, dy/dx = \frac{6ax^2 - 2x^3}{(2a-x)^2}$$

$$dy/dx = x^2 \frac{(3a-x)}{(2a-x)^2} \cdot \sqrt{2a-x}$$

$$dy = \sqrt{x} \frac{(3a-x)}{(2a-x)^{3/2}} \cdot dx$$

put this in (1)

$$v = 2\int_0^{2a} x(2a-x)^2 \sqrt{x} \frac{(3a-x)}{(2a-x)^{3/2}}\, dx$$

$$v = 2\pi \int_0^{2a} \sqrt{x}\sqrt{2a-x}\,(3a-x)\, dx.$$

let $x = 2a \sin 2\theta$.

$dx = 4a \sin\theta \cdot \cos\theta\, d\theta.$

$$v = 2\pi \int_0^{\pi/2} \sqrt{2a}\sin\theta \sqrt{2a}\cos\theta \cdot 3a - 2a\sin\theta \times 4a\sin\theta\cos\theta\, d\theta.$$

$$v = 16\pi a^3 \int_0^{\pi/2} \sin^2\theta \cos^2\theta\,(3 - 2\sin 2\theta)\, d\theta.$$

$$v = 16\pi a^3 \left\{ 3 \times \frac{\Gamma\frac{2+1}{2}\,\Gamma\frac{2+1}{2}}{2\,\Gamma\frac{2+2+2}{2}} - 2 \times \frac{\Gamma\frac{4+1}{2}\,\Gamma\frac{2+1}{2}}{2\,\Gamma\frac{2+4+2}{2}} \right.$$

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$$16\pi a^3\left[\frac{3\,\Gamma 3/2\,\Gamma 3/2}{2\,\Gamma 3} - 2/2\,\frac{\Gamma 5/2\,\Gamma 3/2}{\Gamma 4}\right]$$

$$v = 16\pi a^3\left[\frac{3/2\times 3/2\ \sqrt{\pi}\times 1/2\times\sqrt{\pi}}{2\times 1} - \frac{3/2\times 1/2\times 1/2\times 2}{3\times 2\times 1}\right]$$

$$v = 16\pi a^3\left[\frac{3\pi}{2\times 4\times 2} - \frac{3\pi}{8}\times\frac{1}{2}\right]$$

$$v = 16\pi a^3\left[\frac{3\pi-\pi}{16}\right]$$

$$v = \pi a^3 \times 2\pi.$$

$$v = 2\pi^2 a^3$$

Q Prove that surface generated by revolution of following tractrix about its asymptote is equal to surface area of sphere of radius

$$x = a\cos t + a/2 \log \tan^2 (t/2);\ y = a\sin t$$

Solution $S = \int 2\pi y \, d\theta.$

$$dx/dt = \frac{a\cos^2 t}{\sin t};\ \frac{dy}{dt} = a\cos t$$

Asymptote $y = 0$ i.e x-axis

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\frac{a^2\cos^4 t}{\sin^2 t} + a^2\cos^2 t} = a\cot t.$$

Curve is symmetrical about both asymptote so,

when $t=0$, $x=-\infty$ and $t=\pi/2$, $x=0$

$$S = 2\int_0^{\pi/2} 2\pi y \, \frac{ds}{dt} \, dt$$

$$= 4\pi \int_0^{\pi/2} a\sin t \cdot a\cot t \, dt$$

$$= 4\pi a^2 \int_0^{\pi/2} \sin t \cdot \cot t \, dt$$

$$= 4\pi a^2 \int_0^{\pi/2} \sin t \cdot \frac{\cos t}{\sin t} \, dt$$

$$= 4\pi a^2 \left[\sin t\right]_0^{\pi/2} = 4\pi a^2$$

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{Surface area $= 4\pi a^2$ = Surface area of sphere.

$\boxed{\text{hence proved}}$