# $(n,k)$ Cyclic linear Block code

Code set

$$\underbrace{\{0000}_{C_1}, \underbrace{1100}_{C_2}, \underbrace{1001}_{C_3}, \underbrace{0011}_{C_4}, \underbrace{0110}_{C_5}\}$$

$C_2 \qquad 1100$

$\sigma_R(C_2) \quad \underline{0110}$

$C_4 \qquad 0011$

$\sigma_R(C_4) \quad \underline{1001}$

$C_1 \qquad 0000$

$\sigma_R(C_1) \quad \vec{0000} = C_1$

Algebric properties.

① $(1+x)^2 = (1+x)(1+x)$
$$= 1 + x + x + x^2$$
$$= 1 + x^2$$

i.e. factor $1+x^2$ are $(1+x)$ and $(1+x)$

② $(1+x^2)^2 = (1+x^2)(1+x^2)$
$$= 1 + x^2 + x^2 + x^4$$
$$= 1 + x^4$$

③ $1 + x^6 = (1 + x^3)(1 + x^3$

④ $(1+u)(1+x+x^3)(1+x^2+x^3)$

$(1+x)(1+x+x^3)(1+x^2+x^3)$

$= (1+x+x^3+x+x^2+x^4)(1+x^2+x^3)$

$= (1+x^2+x^3+x^4)(1+x^2+x^3)$

$= 1+x^2+x^3+x^2+x^4+x^5+x^3+x^5+x^6+x^4+$
$\qquad\qquad x^6+x^7$

$= 1+x^2+x^2+x^3+x^3+x^4+x^4+x^5+x^5+x^6+x^6+x^7$

$= 1+x^7$

$\text{degree}=1 \qquad \text{degree}=3 \qquad \text{degree}=3$

i.e. factor $(1+x^7)$ are $(1+x)$, $(1+x+x^3)$ & $(1+x^2+x^3)$

① data polynomial ( 7 [ 4 ] ) code

$K=4$

(i) $d = 1101$
$\qquad\quad 1\ x\ x^2\ x^3$

$d(x) = 1+x+x^3$

(ii) $d = 1011$
$\qquad\quad 1\ x\ x^2\ x^3$

$d(x) = 1+x^2+x^3$

(iii) $d = 0011$

$d(x) = x^2+x^3$

$d = 0011$
$\quad\ 1\ x\ x^2\ x^3$

$A(x) = x^2+x^3$

(iv) $d(x) = x+x^2$

$d = \underset{1}{0}\ \underset{x}{1}\ \underset{x^2}{1}\ \underset{x^3}{0}$

(v) $d(x) = x$

$d = \underset{1}{0}\ \underset{x}{1}\ \underset{x^2}{0}\ \underset{x^3}{0}$

$(n, k)$

③ Generator Polynomial $g(x)$

properties. ① $g(x)$ must be factor of $(1+x^n)$

② degree of $g(x)$ must be $(n-k)$

i.e. for $(7, 4)$

(i) $g(x)$ must be factor of $(1+x^7)$ ✓

(ii) degree of $g(x) = 7-4 = 3$ ✓

$$g(x) = 1 + x + x^3$$

③ code polynomial $\qquad \qquad C = dG$

$$\boxed{c(x) = d(x)\, g(x)}$$

Example 1.  $(7, 4)$, $g(x) = 1 + x + x^3$, $d = 1010$,

find $C = ?$

Solution

$d = 1010$

$$d(x) = \overset{1 \times x^2 \ x^3}{1 + x^2}$$

$$g(x) = 1 + x + x^3$$

$$\therefore c(x) = d(x)\, g(x)$$

$$= (1 + x^2)(1 + x + x^3)$$

$$= 1 + x + x^3 + x^2 + x^3 + x^5$$

$$c(x) = 1 + x + x^2 + x^5$$

Non systematic form

$d = 1010$

$$C = \underset{1 \ x \ x^2 \ x^3 \ x^4 \ x^5 \ x^6}{1\ 1\ 1\ 0\ 0\ 1\ 0}$$

**Example 2** for $(7,4)$ cyclic code with $g(x) = 1+x+x^3$ let code word $c = 1100101$, find data

solution

$$c = 1100101$$

$$c(x) = \frac{1 \; x \; x^2 \; x^3 \; x^4 \; x^5 \; x^6}{1+x+x^4+x^6}$$

given $g(x) = 1+x+x^3$

$x = \cancel{\text{d/G}}$

$d = \cancel{\frac{c}{G}}$

$$\therefore \quad c(x) = d(x)\,g(x)$$

$$d(x) = \frac{c(x)}{g(x)}$$



long division:

$$x^3+x+1 \,)\, \overline{x^6 + x^4 + x + 1}$$

$$\quad\quad \underline{x^6 + x^4 \qquad\qquad + x^3}$$

$$\quad\quad \underline{\pm \quad \pm \qquad\qquad \mp}$$

$$\quad\quad\quad x^3 + x + 1$$

$$\quad\quad\quad \underline{x^3 + x + 1}$$

$$\quad\quad\quad\quad\quad 0$$

quotient: $x^3 + 1$

$$= \frac{1+x+x^4+x^6}{1+x+x^3}$$

$$= \frac{x^6 + x^4 + x + 1}{x^3 + x + 1}$$

$$d(x) = x^3 + 1$$

$$d = \frac{1}{1} \; \frac{0}{x} \; \frac{0}{x^2} \; \frac{1}{x^3}$$

$$= \underline{1001}$$