Systematic Cyclic Code Generator Matrix.

(7,4) cyclic code

$g(x) = 1 + x + x^3$

$G = \begin{bmatrix} \quad \end{bmatrix}_{4\times 7}$

Non-systematic Cyclic Generator Matrix

$$\begin{array}{c} \begin{array}{ccccccc} 1 & x & x^2 & x^3 & x^4 & x^5 & x^6 \end{array} \\ G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Conversion of Non-systematic to Systematic

$G = [I_k \quad P^T]$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - & - \\ 0 & 0 & 0 & 1 & - & - & - \end{bmatrix}$$

$$G = \begin{array}{c} \times \\ \times \\ \times \\ \checkmark \end{array} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 \oplus R_4$

$$G = \begin{array}{c} \\ \\ \checkmark \\ \checkmark \end{array} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_3$

$$G = \begin{array}{c} \\ \checkmark \\ \checkmark \\ \checkmark \end{array} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + R_2 + R_4$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\underbrace{}_{I_k} \qquad \underbrace{}_{P^T}$$

$$P^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} P & I_m \end{bmatrix}_{m \times n}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3 \times 7}$$

(7,4)

Encoding using (n-k) bit Shift Register (Systematic)

$7 - 4 = 3$

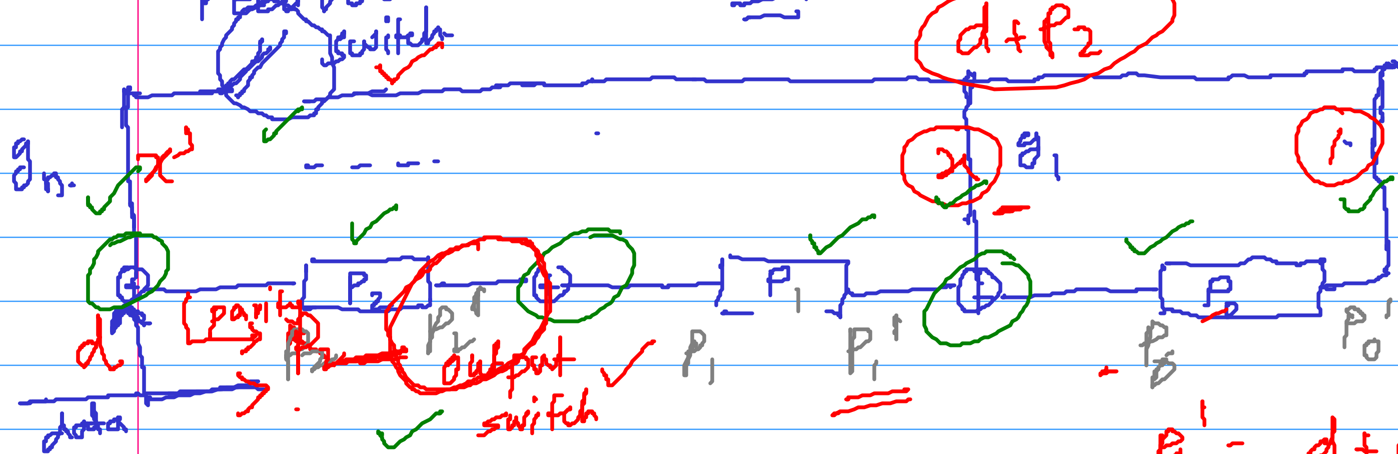
k bit data → n bit code

$g(x) = 1 + \_\ \_\ \_ + x^{n-k}$

$$\left\{ \begin{array}{l} g(x) = 1 + x + x^3 \\ g(x) = 1 + x^2 + x^3 \end{array} \right.$$

S.F. $g(x) = g_0 + g_1 x + g_2 x^2 + \dots + g_{n-k} x^{n-k}$

$g(x) = 1 + x + x^3$



$P_0' = d + P_2$

$P_1' = d + P_2 + P_0$

$P_2' = P_1$

$\oplus \Rightarrow$ modulo - 2 adder

XOR

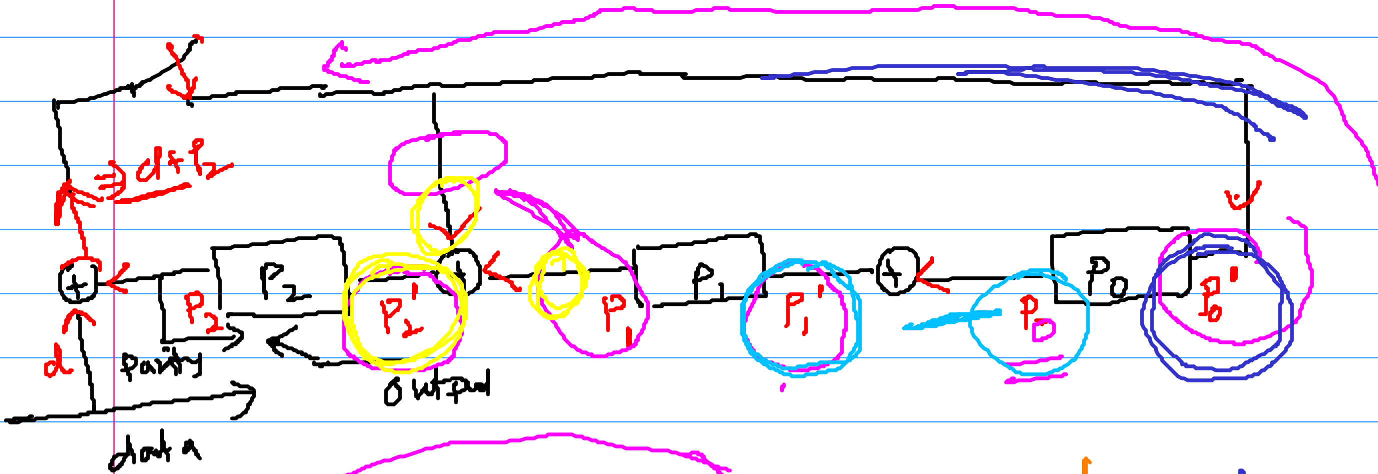
$0 \oplus 0 = 0$ $\quad$ $1 \oplus 0 = 1$

$0 \oplus 1 = 1$ $\quad$ $1 \oplus 1 = 0$

Ex

$g(x) = 1 + x + x^2 + x^5 + x^7$ $\quad$ ($n-k$)

$$g(x) = 1 + x^2 + x^3$$



$$P_0' = d + P_2$$
$$P_1' = P_0$$
$$P_2' = P_1 + d + P_2$$

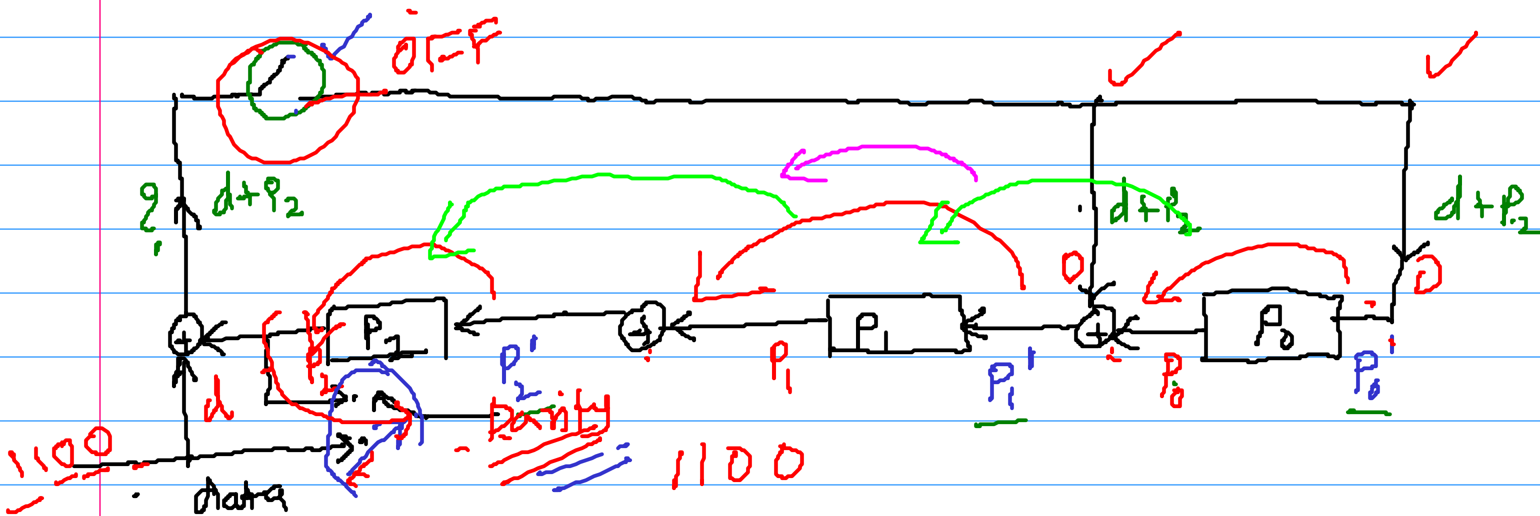
$$P_0' = d + P_2$$
$$P_1' = P_0$$
$$P_2' = \underline{d + P_2} + P_1$$

(n-k) Bit Shift Register Based Cyclic Code Encoder

(7,4) , $g(x) = 1 + x + x^3$

Nos. of flip flop Required $= n-k = 7-4 = 3$

Nos. of adder (Mod-2) $= 3$



$P_0' = d + P_2$

$P_1' = d + P_0 + P_2$

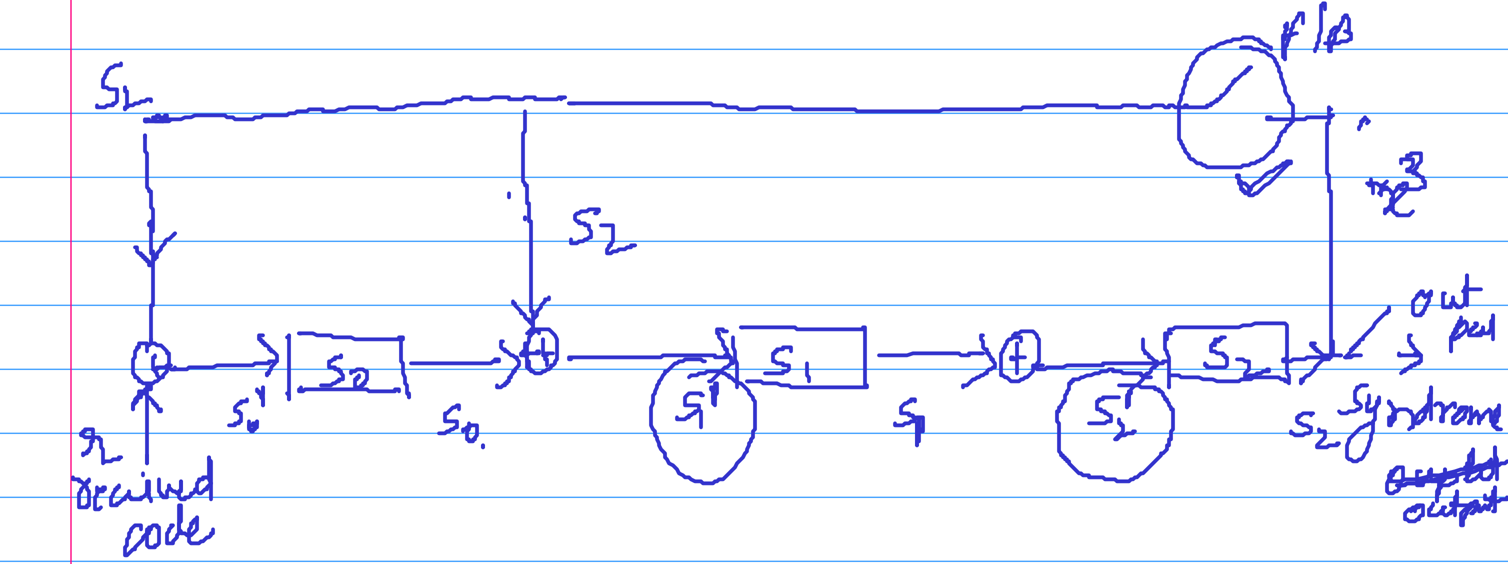
$P_2' = P_1$

data	parity bit before shift			Parity bit after shift		
d	$P_2 = P_2'$	$P_1 = P_1'$	$P_0 = P_0'$	$P_2' = P_1$	$P_1' = d + P_0 + P_2$	$P_0' = d + P_2$
	0	0	0	0	0	0
1	0	0	0	0	1	1
1	0	1	1	1	0	1
0	1	0	1	0	0	1
0	0	0	1	0	1	0

Clock	data	Register bit after shift			F/B switch	Output switch	O/P code.
	d	p_2'	p_1'	p_0'	on/off	on/off	c
1	1	0	1	1	ON	message	1
2	1	1	0	1	ON	message	1
3	0	0	0	1	ON	message	0
4	0	0	1	0	ON	message	0
5					OFF	parity	0 p_2'
6					OFF	parity	1 p_1'
7					OFF	parity	0 p_0'

Decoder using (n-k) bit Shift Register.

(a) Syndrome calculator



$$
\begin{cases}
S_0' = r + S_2 \\
S_1' = S_0 + S_2 \\
S_2' = S_1
\end{cases}
$$

$r = 1001101$

		register output before shift			Register output after shift		
C	x	S_0	S_1	S_2	S_0'	S_1'	S_2'
		0	0	0	0	0	0
1	1	0	0	0	1	0	0
2	0	1	0	0	0	1	0
3	0	0	1	0	0	0	1
4	1	0	0	1	0	1	0
5	1	0	1	0	1	0	1
6	0	1	0	1	1	0	0
7	1	1	0	0	1	1	0

$S_2 = 0$

$S_1 = 1$

$S_0 = 1$