Introduction to Information theory & Coding

Source → Encoder → Channel → Decoder → Destination

↑

Noise.

Communication system

Information Source: an object that produce an event, the outcome of which is selected at random according to a probability distribution. (Device which produce message).

- Analog
- Digital or Discrete. (only a finite set of symbols as possible outcome)
  - memory
  - memoryless.

DMS: Discrete Memoryless source.

Information content of a Discrete Memoryless source

The amount of information received from the knowledge of occurance of an event is related to the probability or the likelihood of the occurance of the event. The message associated with an event least likely to occur i.e with least probability contains more information. The amount of information in a message depends only on the uncertainty of the underlying event rather than its content.

Consider a DMS denoted by X with alphabet $\{x_1, x_2, --- x_m\}$ the information content of a symbol x_i; denoted by $I(x_i)$ is defined by

$$I(x_i) = \log_a \frac{1}{P(x_i)} = -\log_a P(x_i)$$

If $a=2$, unit of information is bit

$a=10$ ———————— decit.

$a=e$ ———————— nat.

Note

1. $I(x_i) \geq 0$
2. $I(x_i) = 0$ if $P(x_i) = 1$
3. $I(x_i) > I(x_j)$ if $P(x_i) < P(x_j)$
4. $I(x_i x_j) = I(x_i) + I(x_j)$ if x_i & x_j are independent.

Average Information or Entropy

A DMS produces message $(x_1, x_2, --- x_m)$ with probabilities $(P_1, P_2, --- P_m)$ the associated self information $I(x_1)\ I(x_2) --- I(x_m)$ of source X with

The mean value of $I(x_i)$ over the alphabet of source X with m different symbol is given by.

$$H(x) = E[I(x_j)]$$

$$= \sum_{j=1}^{m} P(x_i) I(x_j)$$

$$= \sum_{i=1}^{m} P(x_i) \log_2 P(x_i) \text{ bits / symbol}$$

H(X): Entropy of Source X. (average Info. content per symbol).

$0 \leq H(X) \leq \log_2 m$

3. Information Rate

If the source emits r symbol per sec, the information rate R of the source is

$$R = r\,H(X) \quad \text{bits/sec}$$

Properties of Entropy

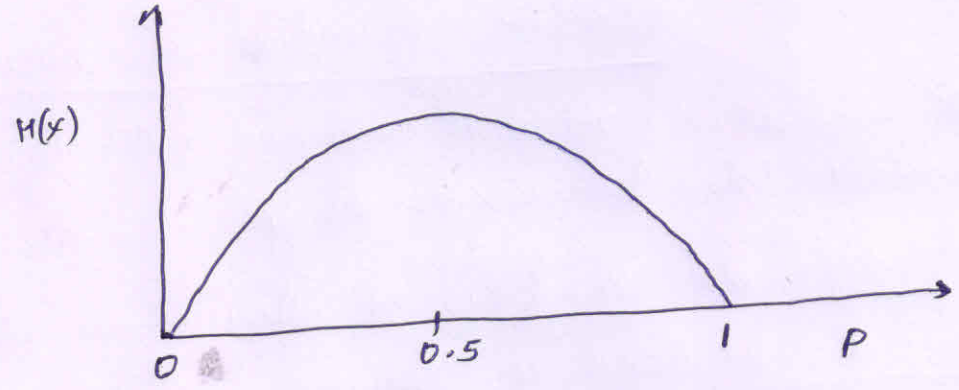
For a Binary Source. $\{x_1, x_2\}$

$P(x_1) = P_1 = P$

$P(x_2) = P_2 = 1 - P.$

$$H(X) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2}$$

$$= -[P \log P + (1-P) \log (1-P)]$$



(i) $H(X) = 0$ if $P = 0$, or $P = 1$

(ii) $H(X)$ is non negative i.e $H(X) \geq 0$.

(iii) $H(X)$ is max. at $P = \frac{1}{2}$.