Subject Code : 4CS2-01. (18EJICS177) Vaibhav Saran

SET- 4.

Q1 $a_n = -3a_{n-1} + 10a_{n-2}$, $n \geq 2$. $a_0 = 1$, $a_1 = 4$.

$\Rightarrow a_n + 3a_{n-1} - 10a_{n-2} = 0$ — (1)

Characteristic eqⁿ of (1) : $\Rightarrow x^2 + 3x - 10 = 0$.

$\Rightarrow x^2 + 5x - 2x - 10 = 0$

$\Rightarrow x(x+5) - 2(x+5) = 0$

$\Rightarrow (x+5)(x-2) = 0$.

$x = -5, 2$.

$r_1 = -5$ & $r_2 = 2$ are the characteristic roots.

∴ the solution to recurrence relation will have the form

$a_n = a(-5)^n + b2^n$ — (A)

To find a & b , put $n=0$ & $n=1$ to get a system of 2 eqⁿ.

$a_0 = 1 = a(-5)^0 + b2^0$

$\Rightarrow a + b = 1. \quad \text{— } 2$

$a_1 = 4 = a(-5)^1 + b2^1$

$\Rightarrow -5a + 2b = 4 \quad \text{— } 3$

Solving 2 & 3

2 × 5

$$\Rightarrow \begin{aligned} \cancel{5a} + 5b &= 5 \\ -\cancel{5a} + 2b &= 4 \\ \hline 7b &= 9 \\ b &= \frac{9}{7} \end{aligned}$$

put b in 2

$\Rightarrow a + \frac{9}{7} = 1.$

$\Rightarrow a = 1 - \frac{9}{7} = -\frac{2}{7}$

$\therefore$ the solution is $a_n = -\frac{2}{7}(-5)^n + \frac{9}{7}2^n.$

Q2 (i) Ring Sum:

Given two graphs. $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$

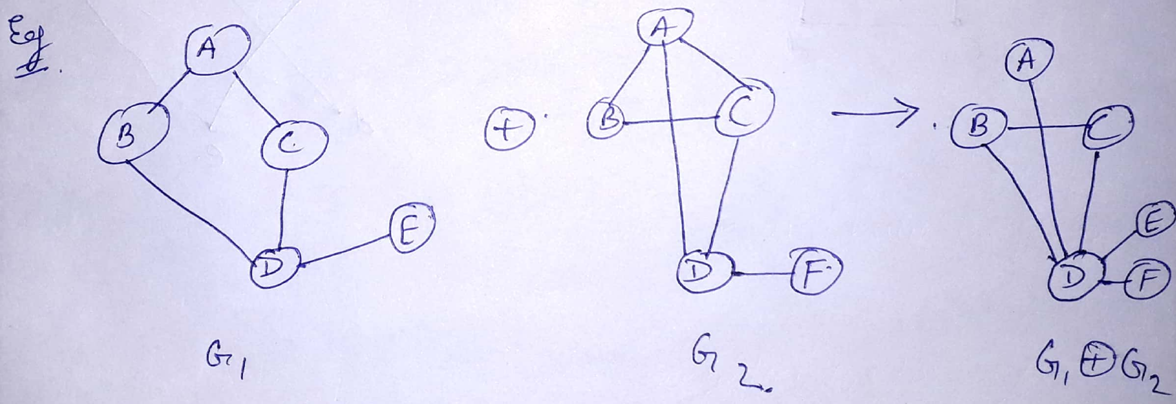
We define the ring sum.

$$G_1 \oplus G_2 = (V_1 \cup V_2, (E_1 \cup E_2) - (E_1 \cap E_2))$$

with isolated points dropped.

So an edge is in $G_1 \oplus G_2$ iff it is an edge of G_1 or an edge of G_2 but not both...

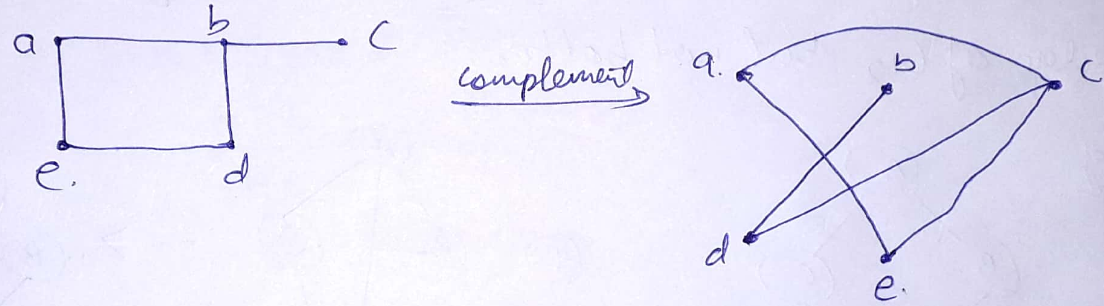
Eg.



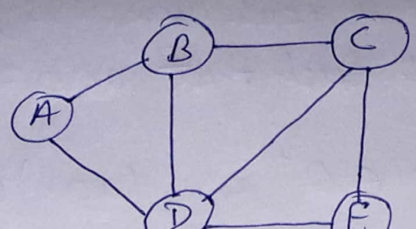
(ii) Complementary Graph:

Let $G = (V, E)$ be a simple graph on n- vertices and. $K_n = (V, E_n)$ is a complete graph on n vertices. Then, complement of the graph. G is the graph. $G' = (V', E')$ such that $V' = V$ and. $E' = E_n - E$, where. E_n is the set of edges of K_n.

for eg



Q3. Given :



We find chromatic numbers using Welsh-Powell algorithm

deg(A) = 2.
deg(B) = 3
deg(C) = 3.
deg(D) = 4.
deg(E) = 2.

Step 1 : order the vertices acc. to decreasing degree.
D, B, C, A, E.

Step 2 : paint D with colour. C_1.

Step 3 : paint B, C with colour. C_2.

Step 4 : paint A, E with colour C_3

All the vertices have been assigned a colour and the chromatic number is = 3. Ans

Q4 Q^+ is the set of all positive ~~real~~ rational numbers, for all $a, b \in Q^+$ we have the operation o such that $a o b = ab/2$

Associativity: let $a, b, c \in Q^+ \Rightarrow (a o b) o c = a o (b o c)$

Now,

$$(a o b) o c = \frac{ab}{2} o c = \frac{\frac{ab}{2} \times c}{2} = \frac{abc}{4}. \quad -①$$

$$a o (b o c) = a o \frac{bc}{2} = \frac{a \times \frac{bc}{2}}{2} = \frac{abc}{4} \quad ②$$

① = ② ⇒ It holds associativity

Identity element: let $a \in Q^+$ & $e \in Q^+$ such that $e o a = a$

$$\Rightarrow \frac{ea}{2} = a.$$

$$\Rightarrow ea = 2a.$$

$$\Rightarrow ea - 2a = 0 \Rightarrow a(e-2) = 0$$

$$\Rightarrow e = 2$$

$\therefore e = 2$ is the identity element in Q^+.

Inverse:

Let $a \in Q^+$ & $b \in Q^+$ such that $a o b = e$.

$\Rightarrow \frac{ab}{2} = e$. $\because e = 2$.

$\Rightarrow \frac{ab}{2} = 2$.

$\Rightarrow b = \frac{4}{a}$. $(\because a \neq 0)$

$\therefore$ for every $a \in Q^+$ there exists $\frac{4}{a} \in Q^+$ such that

$a o b = \frac{\cancel{a}}{\cancel{2}} \times \frac{\cancel{4}^2}{\cancel{a}} = 2$

Commutativity: Let $a, b \in Q^+$

$\boxed{a o b = b o a}$

Since $a o b = \frac{ab}{2} = \frac{ba}{2} = b o a$.

$\therefore$ It is commutative.

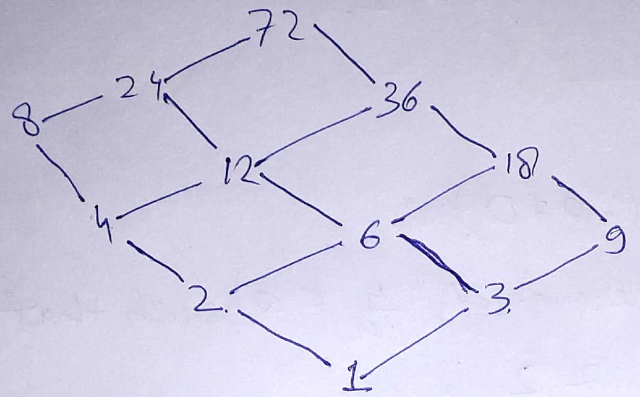
Hence (Q^+, o) is an abelian Group

H.R.

(18EJICS177) Vaibhav Saran (8)

Q5 $D_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$

*(steps on next pg).

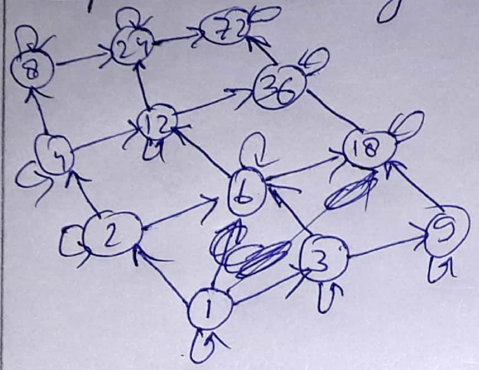


In the above hasse dig we observe that join and meet exist for all pair of elements, hence the ~~given~~ hasse dig. is a lattice.

Ans

* Steps to make Hasse dig.

Step 1 : Drawing the relation.



Step 2 : Remove all self loops and transitivities and as our relation is upwards arrows can also be removed as a result we get the following Hasse dig.

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