Maths – Assignment-3

(1) $\left(x - \frac{4}{x}\right)^6$.

Using Binomial Expansion.

$= {}^6C_0 (x)^6 \left(-\frac{4}{x}\right)^6 + {}^6C_1 x^5 \left(\frac{-4}{x}\right)^5 + {}^6C_2 x^4 \left(\frac{-4}{x}\right)^4$

$+ {}^6C_3 x^3 \left(\frac{-4}{x}\right)^3 + {}^6C_4 x^2 \left(\frac{-4}{x}\right)^2 + {}^6C_5 x^1 \left(\frac{-4}{x}\right)$

$+ {}^6C_6 x^0$

$= \frac{4096}{x^6} - \frac{6144}{x^4} + \frac{3840}{x^2} - 1280 + 240x^2$

$- 24x^4 + x^6$) Ans.

(2) $T_{r+1} = {}^nC_r \, a^{n-r} b^r$.

assuming $x^{16} \rightarrow (r+1)^{th}$ term.

$T_{r+1} = {}^{12}C_r (2x^2)^{12-r} \left(-\frac{x}{2}\right)^r$.

$= {}^{12}C_r \, 2^{12-2r} (-1)^r x^{24-r}$ — (1)

$24 - r = 16$

$r = 8$.

r in (1)

coeff of $x^{16} = {}^{12}C_8 (2)^{12-16} (-1)^8 x^{16}$.

$= \frac{495}{16} x^{8,16}$

$\hookrightarrow 30.9$

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(5) By using characteristic roots,

$$a_n = 5a_{n-1} - 6a_{n-2}$$

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

we know, if

$$a_n + \alpha a_{n-1} + \beta a_{n-2} = 0$$

then, it can be represented as,

$$x^2 + \alpha x + \beta x = 0.$$

then,

$$a_n = a(r_1)^n + b(r_2)^n$$

where r_1 & r_2 are roots of x,

solving this,

$$x^2 - 5x + 6 = 0.$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x-2) - 3(x-2) = 0$$

$$(x-3)(x-2) = 0$$

$$x = 2, 3$$

so,

$$a_n = a(2)^n + b(3^n)$$

given is $a_0 = 1$ $a_1 = 4$.

so,

$$a_0 = a + b = 1 \quad -(1)$$

$$a_1 = 2a + 3b = 4 \quad -(2)$$

solving (1) & (2)

$$2a + 3b = 4$$

$$-2a + 2b = -2$$

$$b = 2.$$

$$\Rightarrow a = -1$$

$$\Rightarrow \boxed{a_n = 2(3)^n - 2^n.}$$

(3) Total no. of chances = 6.

(a) no. of ways = $6! = 720$.

(b) dog first → $5 \times 4 \times 3 \times 2 \times 1 = 120$

(c) boy followed by dog → 5 possibilites

B D $4 \times 3 \times 2 \times 1 = 24$

∴ 5 times

$5 \times 24 = 120$ ways

(d) man boy Dog. _ _ _ _

$2 \times 1 \times 1 \times 3 \times 2 \times 1$

$= 12.$

this can be done in 4 ways.

$= 12 \times 4 = 24$ ways.

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