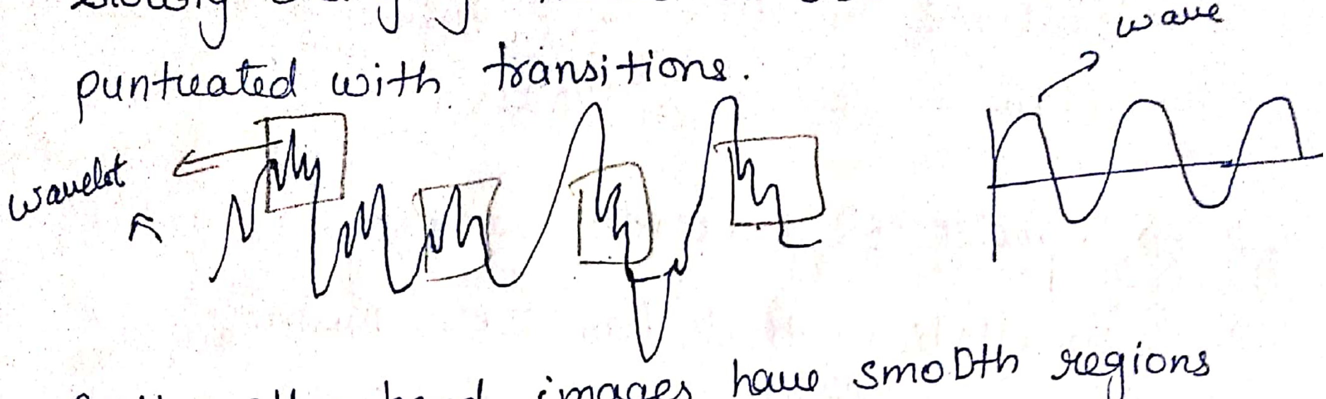
Basics of Wavelet Transformation

→ Real world data or signals frequently exhibits slowly changing trends or oscillations puntuated with transitions.

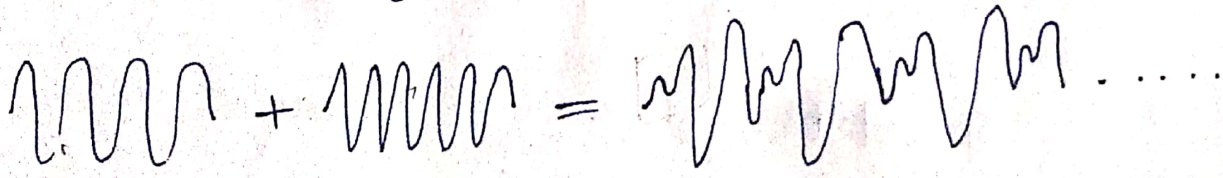


→ On the other hand images have smooth regions interupted by edges or abrupt changes in contrast.

→ These abrupt (sudden) changes are often the most interesting parts of data. Both perceptically & in terms of the information they provide.

→ The fourier transformation Is powerful tool for data analysis. However, it doesn't represent abrupt changes efficiently.

→ The reason for that is that the fourier transformation represent data as a sum of sine waves which are not localized in the time or space

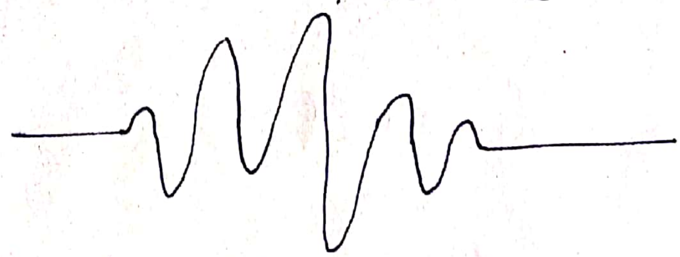


These sine waves oscillates forever.

→ Therefore to accurately analyze signals & images, that have abrupt changes.

→ We need to use new class of fun^ that are well localized in time & frequency. This bring us to the topic of wavelets.

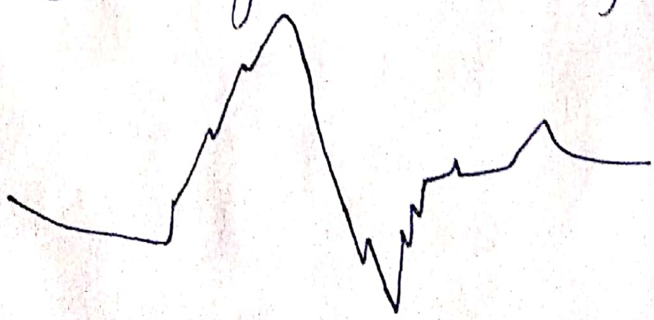
→ Wavelet is a rapidly decaying wave like oscillation that has zero mean.



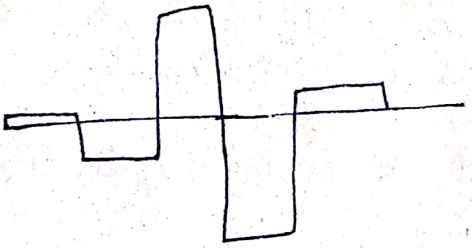
Unlike Sinusoids which extends to ∞ a wavelet exists for a finite duration.

→ Wavelet come in different sizes & shapes. The availability of wide range of wavelets is a key strength of wavelet analysis
To choose the right wavelet you will need to consider the application you will use it for

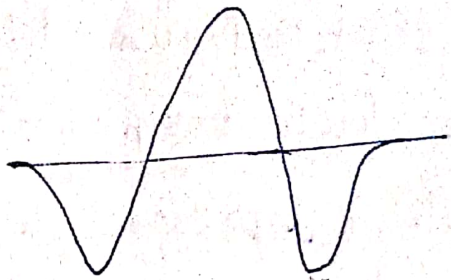
Morlet



Daubechies.



Biorthogonal



Mexican Hat

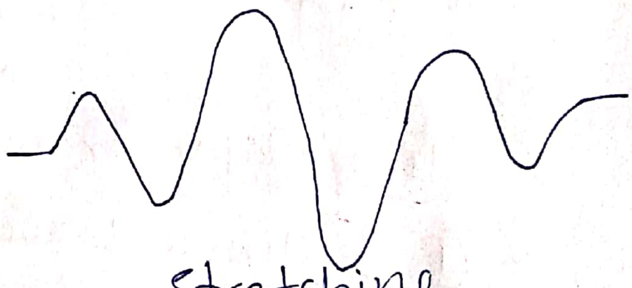
→ Wavelet analysis allow us to isolate & manipulate specific types of patterns hidden in masses of data.
eg :- Our eyes can pick out the trees in a forest, Our ears can pick out the flute in a symphony

Key Wavelet Concept :-

(i) Scaling (ii) Shifting

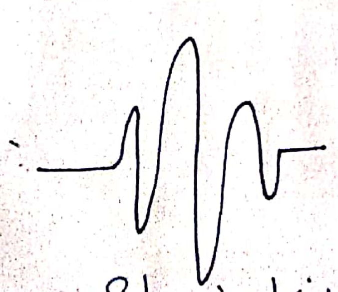
(i) Scaling :- Scaling refers to the process of stretching or shrinking the signals in time which can be expressed using this equation $\psi\left(\frac{t}{s}\right)$ $s>0$

→



Stretching

↓



Shrinking.

Smaller scale = Shrunken wavelet which corresponds to a high frequency.

S is the scaling factor which is a (+)ve value & corresponds to how much a signal is scaled in time.

larger scale factor results in Stretched wavelet which corresponds to lower frequency.

→ A stretched wavelet helps in capturing the slowly varying changes in a signals. while compressed wavelet helps in capturing the abrupt changes.

(ii) Shifting :- Shifting a wavelet simply means delaying or advancing the onset of the wavelet along the length of the signal.

→ A shifted wavelet represented using this notation means that the wavelet is shifted & centered at K. $\phi(t-k)$

→ We need to shift wavelet to align with the feature we are looking for in a signal.

Two Major Transforms in Wavelet Analysis.

① Continous Wavelet Transform

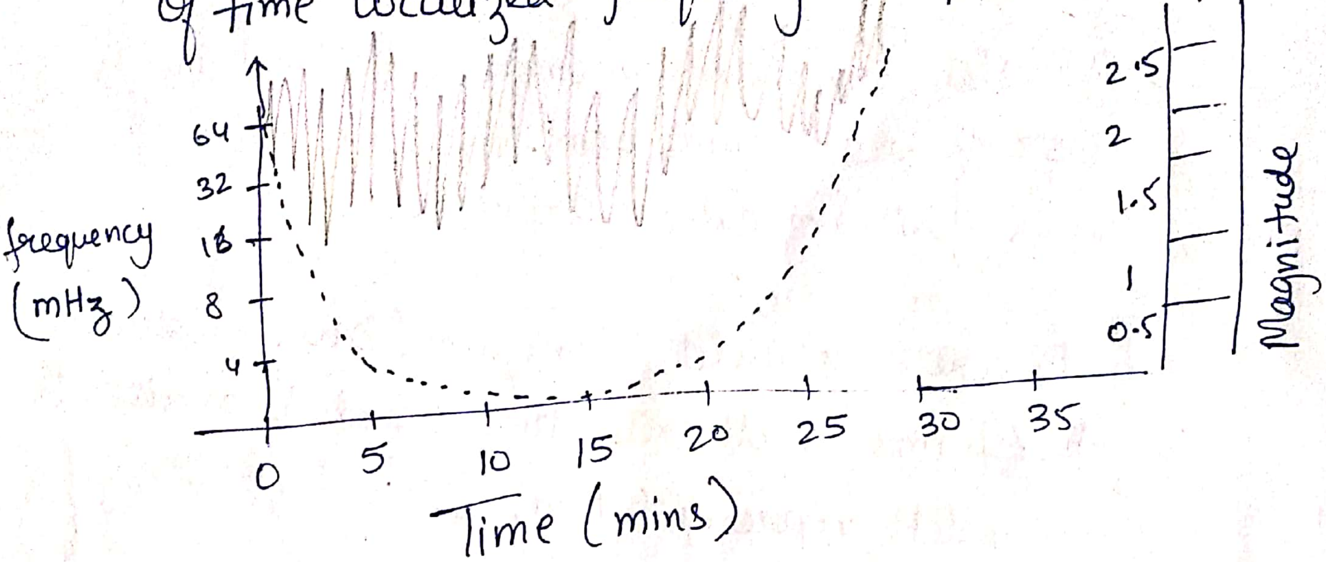
② Discrete Wavelet Transform.

→ These transforms differ based on how the wavelets are scaled & shifted.

Scanned by CamScanner

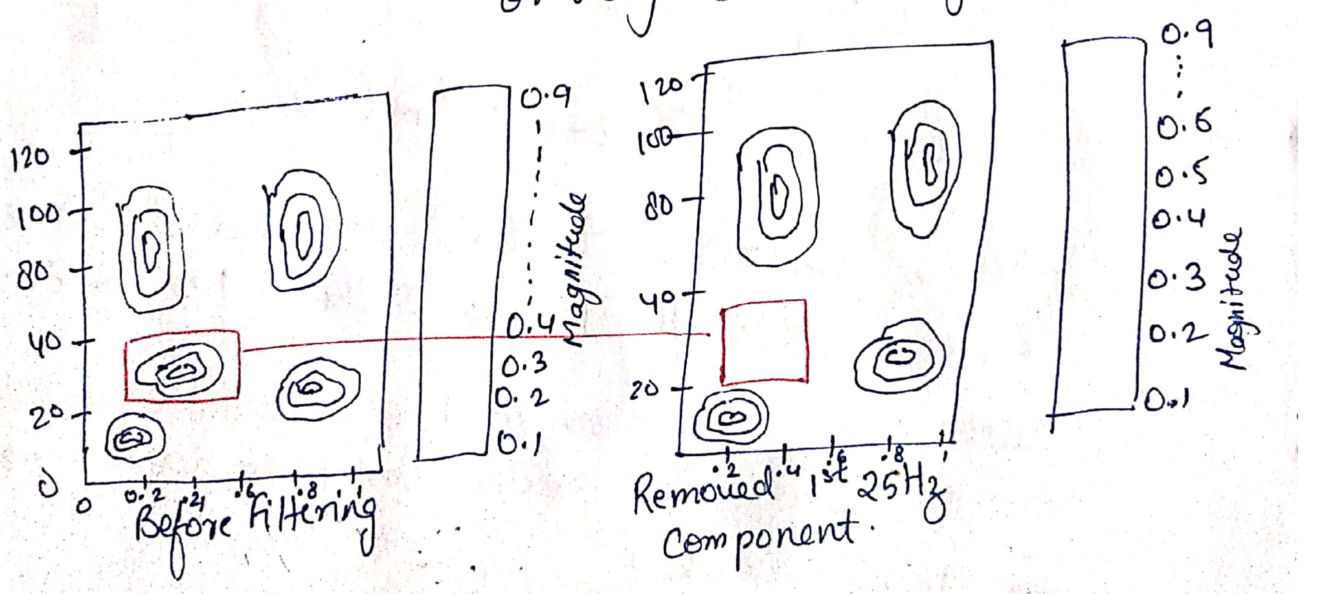
① Continuous Wavelet Transform

→ Key applications of continous wavelet analysis are time frequency analysis and filtering of time localized frequency components.



Magnitude Scalogram

// Magnitude of image → If the image is bright it means a big change in the initial image.
If it is dark it means no change or very little change.



② Discrete Wavelet Analysis.

→ The key applications for discrete wavelet analysis are denoising & compression of signals & images.

→ Wavelet transform decomposes a signal into a set of basis ~~transform~~ functions. These basis funⁿ are called wavelets.

→ Discrete Wavelet Transform (DWT), which transforms a discrete time signal to a discrete wavelet representation.

→ DWT converts an image series $x_0, x_1, x_2 \ldots x_m$ into one high pass wavelet co-efficient series & one low pass wavelet co-efficient series (of length $n/2$ each) given by :-

$$H_i = \sum_{m=0}^{k-1} x_{2i-m} \cdot s_m(z)$$

$$L_i = \sum_{m=0}^{k-1} x_{2i-m} \cdot t_m(z)$$

where, $s_m(z)$ & $t_m(z)$ are called wavelet filters, K is the length of the filter & $i = 0, 1 \ldots \frac{n}{2} - 1$

→ Such transformation will be applied recursively on the low-pass series until the desired no. iteration is reached.