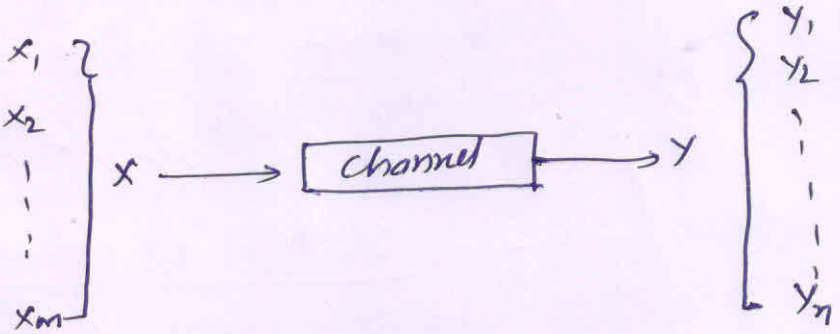
Discrete Memoryless Channel

DMC channel accepts an input symbol from X and in response it generate an output symbol from Y.



Channel Matrix

$P(y_j|x_i)$ = conditional probability of obtaining output y_j given that the input is x_i

$$P(Y|X) = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_n|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \cdots & P(y_n|x_2) \\ \vdots & & & \\ P(y_1|x_m) & P(y_2|x_m) & \cdots & P(y_n|x_m) \end{bmatrix}$$

$$P(X) = [P(x_1) \quad P(x_2) \quad \cdots \quad P(x_m)]$$

$$P(Y) = [P(y_1) \quad P(y_2) \quad \cdots \quad P(y_n)]$$

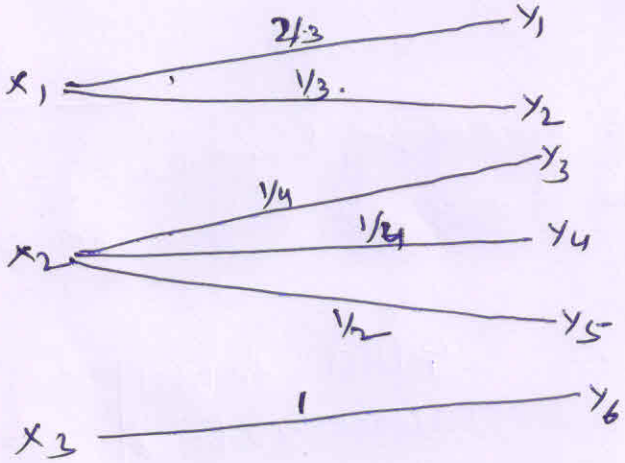
$$P(Y) = P(X)\,P(Y|X)$$

$$P(X)_d = \begin{bmatrix} P(x_1) & 0 & 0 & 0 \\ 0 & P(x_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & P(x_m) \end{bmatrix}$$

$$P(XY) = [P(X)]_d\, P(Y|X)$$

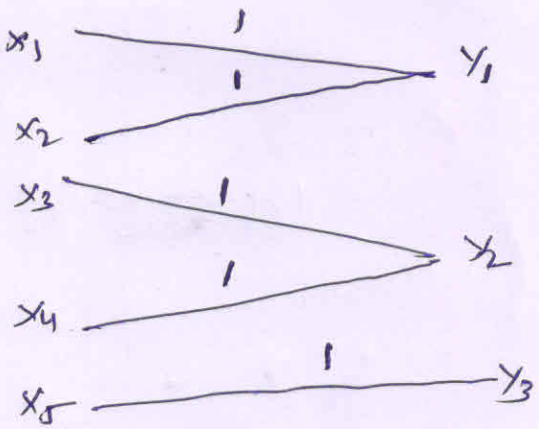
Classification of Discrete Channels

1. **Lossless channel** : the transmitted message is determined by the received message from the channel and hence that no transmission error can occur. The channel matrix of such a channel consists of only one non-zero element in each column.



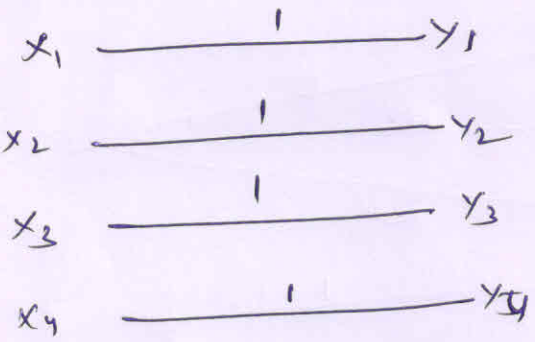
$$P(Y|X) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2. **Deterministic Channel** : output is determined by the input. The channel Matrix of such channel consists of one non-zero element in each row.



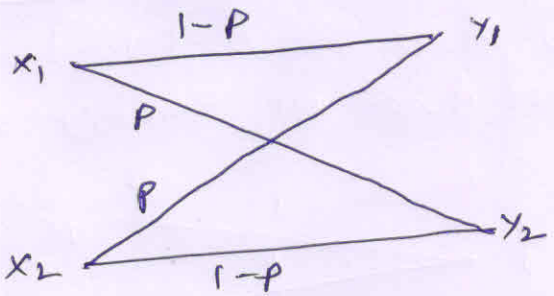
$$P(y|x) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

3. Noiseless channel : it is both lossless & Deterministic. The channel Matrix has only one element in each row and in each column. and this element is unity. It is square matrix as the number of input and output are equal.



$$P(y|x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Binary Symmetric Channel



$$P(y|x) = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$