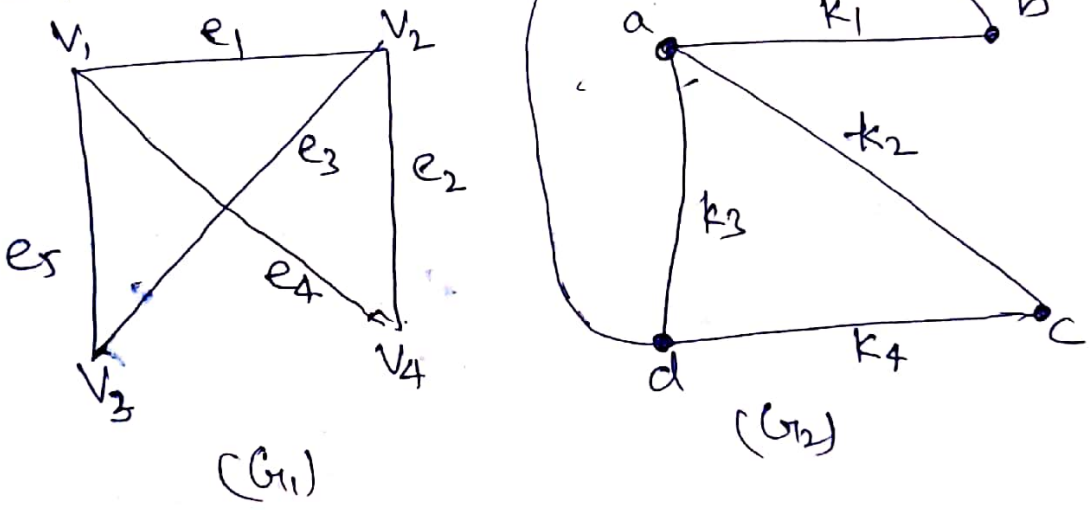
Isomorphism

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. G_1 is said to be isomorphic to G_2 if there exists a one-to-one correspondance $f: V_1 \to V_2$ and a one-to one correspondance $h: E_1 \to E_2$ such that for any edge $e_k \in E_1$, vertices v_i & v_j are end vertices of e_k in G_1 if & only if $f(v_i)$, $f(v_j)$ are end vertex of $h(e_k)$ in G_2.

Example



No. of vertices in G_1 & G_2 are same.

No. of edges in G_1 & G_2 are same.

degree sequence of G_1 is 3, 3, 2, 2 and G_2 is 3, 2, 2, 3

Total degree is same. then.

$f(V_1) = d$ $\quad f(V_3) = b$

$f(V_2) = a$ $\quad f(V_4) = c$

$g(e_1) = k_3$

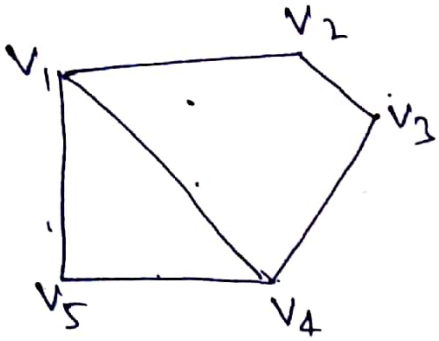
$g(e_2) = k_1$

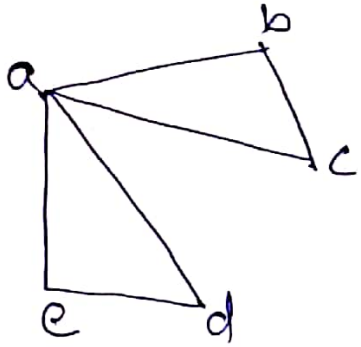
$g(e_3) = k_2$

$g(e_4) = k_5$

$g(e_5) = k_4$

Example Determine whether the following graphs are isomorphic or not.





no. of edges = 6 No. of vertices = 5

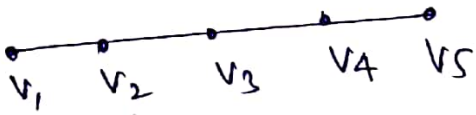
degree sequence of vertices in G_1 = {3, 2, 2, 3, 2}

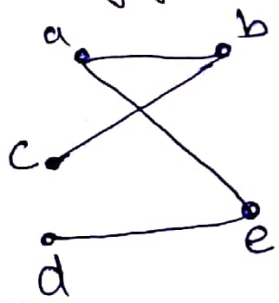
degree sequence of vertices in G_{12} = {4, 2, 2, 2, 2}

Since degree sequences for both the graphs is not same, G_1 is not isomorphic to G_{12}.

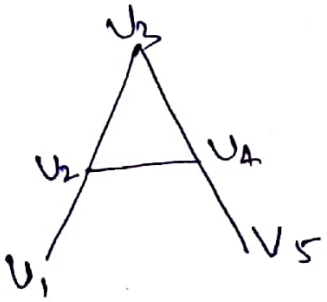
Q Determine whether the given pair of graphs is isomorphic or not.

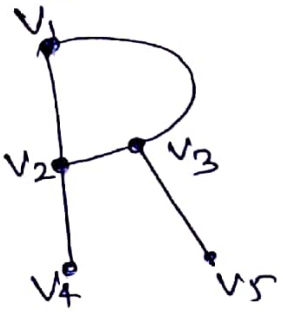
(a)



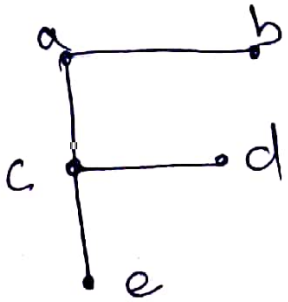


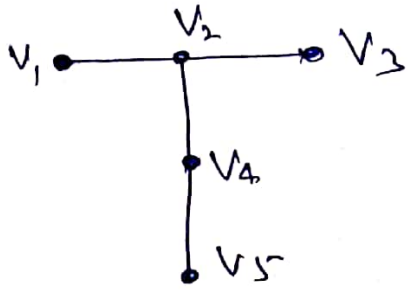
(b)





(c)





Eulerian circuit & Path

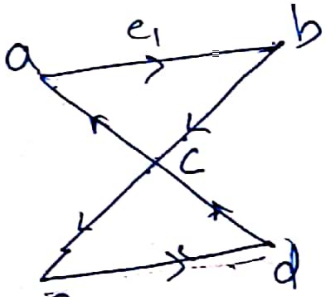
A path in a connected ~~path~~ graph G, is called Euler path if it includes every edge exactly once. Since path contains every edge exactly one it is also called Euler trial.

An Euler path that is a circuit called Euler circuit i.e. a closed Euler path is Euler circuit.

Note:
Eulerian graphs can be traced without lifting pen and without retracing an edge.

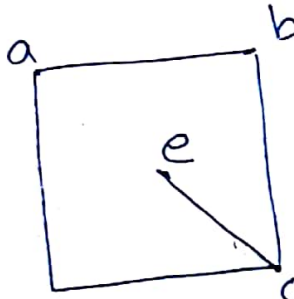
Q Which of the following graphs has Euler path or circuit.

(a)



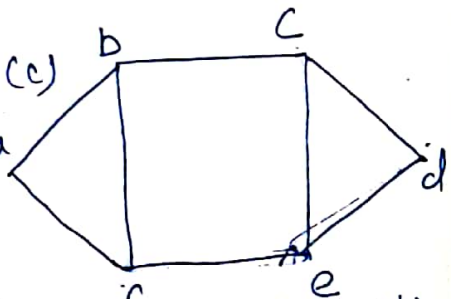
(all vertices & are even)

(b)



(2 vertices are odd & all other vertices are even)

(c)



(4 vertices & odd which > 2)

(a) c a b c d e is euler path and Euler circuit

(b) e, c, d, a, b, c. euler path but not euler circuit

(c) ~~f b a c~~

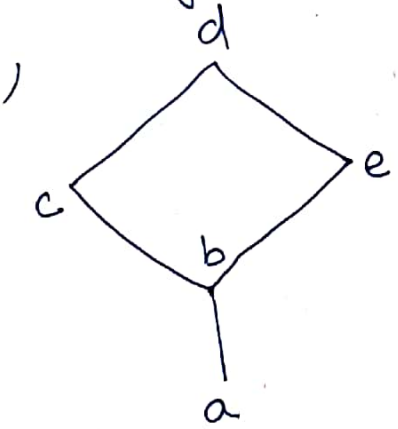
Hamiltonian Path & Circuit

Hamiltonian path : A path is connected graph G is called Hamilton path if it includes every vertex exactly one.

Hamiltonian circuit : A cycle in a connected graph G is called Hamiltonian cycle if it contains every vertex of G exactly once except the starting & ending vertex which are same.

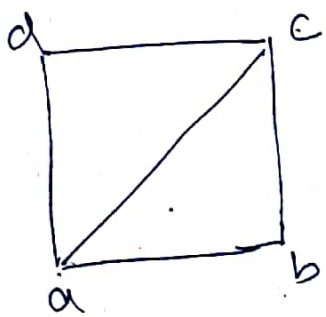
Q Which of the following graphs has an Hamiltonian path or cycle.

(i)



Hamiltonian path
a, b, c, d, e.
but not Hamiltonian cycle.

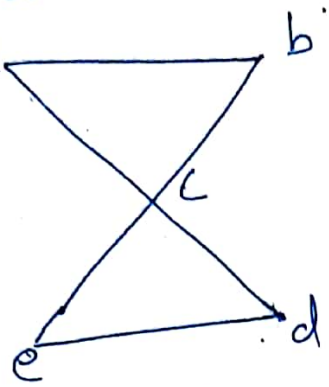
(ii)



Hamiltonian cycle.
a, b, c, d, a.

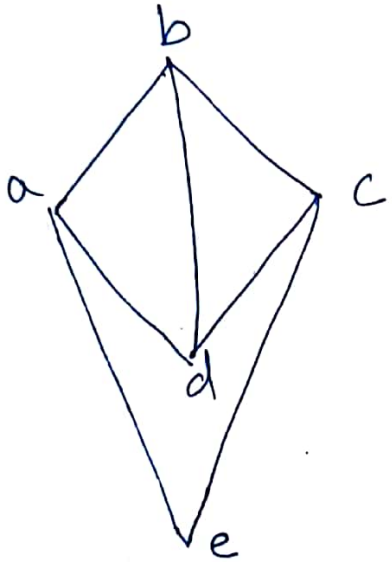
~~Hamilton~~

(iii)



no Hamiltonian path
no Hamiltonian cycle

(iv)



Hamiltonian cycle
a, e, c, d, b, a.

Fleury's Algorithm

This Algorithm to print Eulerian trail or cycle.

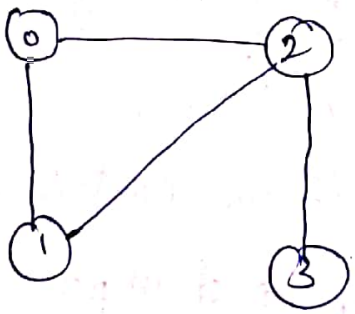
Step1 : Make sure the graph has either 0 or 2 odd vertices

Step2 : If there are 0 odd vertices, start anywhere. If there are 2 odd vertices, start at one of them.

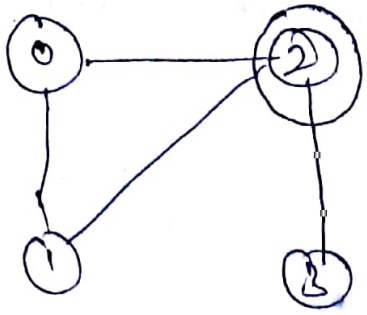
Step3 : Follow edges one at a time. If you have a choice b/w a bridge & non-bridge, always choose non-bridge.

Step4 : Stop when you run out of edges.

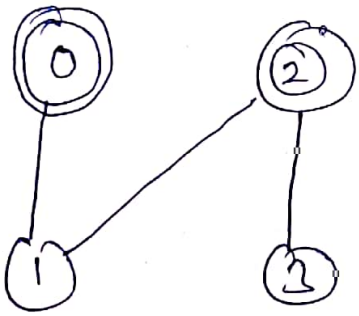
Example



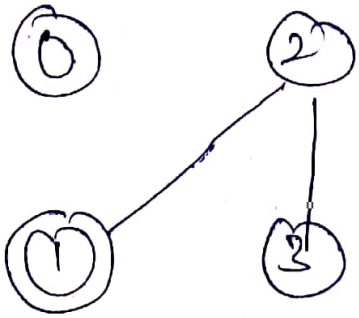
There are two vertices with odd degree 2 and 3. We can start path from any of them. Let us start tour from vertex '2'.



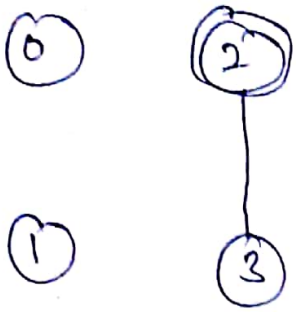
There are three edges going out from vertex 2. ~~i.e. We can pick any of the~~ We don't pick edge 2-3 because that is bridge (we won't be able to come back to '3'). We can pick any one of the remaining two edges. Let us we pick '2-0'. We remove this edge and move to vertex '0'.



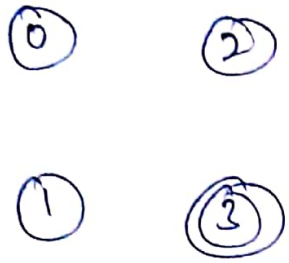
2-0



2-0, 0-1



2-0, 0-1, 1-2



2-0, 0-1, 1-2, 2-3.