Ques Define a normal distribution of height of 300 students are randomly distributed with mean 64.5 inches and standard deviation 3.3 inches. How many students have heights.

(i) less than 5 ~~heig~~ feet

(ii) b/w 5 feet & 5 feet & 9 inches.

Sol^n Given,

mean $(\mu) = 64.5$ inches.

S.D $(\sigma) = 3.3$, $N = 300$

(i) Less than 5 ft.

$x = 5\text{ ft} = 5 \times 12 = 60$ inches.

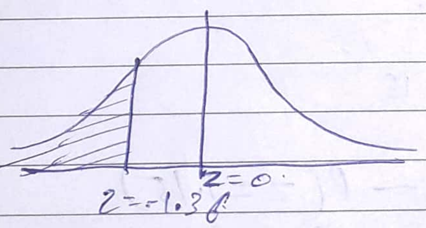
So,

$$z = \frac{X - \mu}{\sigma} = \frac{60 - 64.5}{3.3}$$

$$z = -1.36$$

Now,

$$P(x < 5) = P(z < -1.36)$$



$$P(z < -1.36) = .5 - P(z = -1.36)$$

$$= .5 - .4131$$

$$P(z < -1.36) = .0869$$

no. of students having heights less than 5 ft

$$= N \times P(z < -1.36)$$

$$= 300 \times .0869$$

$$= 26.07 = 26$$

(ii) b/w 5 ft & 5 ft & 9 inches

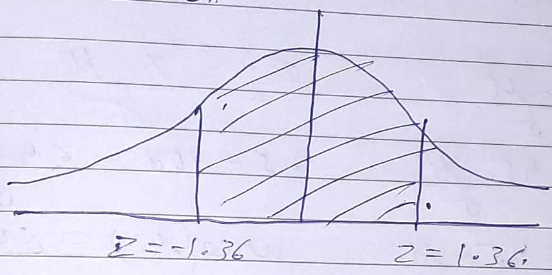
$x = 5 ft = 60$ inches.

$$Z = \frac{60 - 64.5}{3.3} = -1.36$$

$x = 5 ft\ 9 inch = 69$ inches.

$$Z = \frac{69 - 64.5}{3.3}$$

$Z = +1.36$



$P(-1.36 < z < 1.36) = 2P(Z = 1.36)$

$= 2 \times 0.4131$

$= .8262$

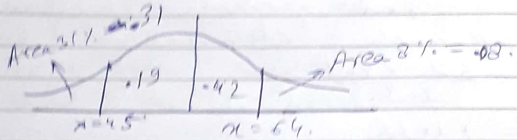
no. of students $= 300 \times .8262$

$= 247.86 = 248$

Imp

Ques. In a normal distribution 31% items are under 45 & 8% are over 64. Find the mean and std. deviation of the distribution.

Sol^n



$Z_1 = \frac{x-\mu}{\sigma}$ $\qquad$ $Z_2 = \frac{x-\mu}{\sigma}$

$(.5 - .31) = \frac{45-\mu}{\sigma}$ $\qquad$ $.5 - .08 = \frac{64-\mu}{\sigma}$

$area(.19) = \frac{45-\mu}{\sigma}$ $\qquad$ $area(.42) = \frac{64-\mu}{\sigma}$

By normal table.

for area(.19), $z = .5$ & for area(.42) $= 1.41$

$.5 = \frac{45-\mu}{\sigma}$ $\qquad$ $1.41 = \frac{64-\mu}{\sigma}$

$\Rightarrow 45 = \mu + .5\sigma$ —(1) $\qquad$ $64 = \mu + 1.41\sigma$ —(2)

Solving (1) & (2)

$\mu = 50$, $\sigma = 10$ (after round off)

In case of arrays always default constructor is invoked
Eg - Sample ob[100];

Polymorphism

Mathematical Expectation

If X is any random variable having values $x_1, x_2, \ldots$ with the corresponding probabilities $p_1, p_2, \ldots$ then the mathematical expression of X is defined by

$$E(X) = p_1 x_1 + p_2 x_2 + \cdots = \sum p_i x_i$$

Ques Thirteen cards are drawn simultaneously from a pack of 52 cards of ace count 1, face cards 10 & others according to their domination. Find the expectation of the total score in 13 cards.

Sol^n Acc. to Ques.

R.V X having values

$x_1 = 1,\ x_2 = 2,\ x_3 = 3,\ x_4 = 4,\ x_5 = 5,\ x_6 = 6,$
$x_7 = 7,\ x_8 = 8,\ x_9 = 9,\ x_{10} = 10,\ x_{11} = 10,\ x_{12} = 10,$
$x_{13} = 10$

$$P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$$

$$P(2) = P(3) = \cdots = P(10) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{Jack}) = \frac{12}{52} = \frac{3}{13} = P(\text{Queen}) = P(\text{King})$$

$$E(X) = \sum p_i x_i$$
$$= \frac{1}{13}\left[1+2+3+4+5+6+7+8+9+10+10+10+10\right]$$
$$= \frac{85}{13}$$

$$E(X) = 6.54$$

Imp.

Ques Find the expected values of the sum of numbers of points on them, if 2 unbiased dice are thrown.

* In case of arrays always default constructor is invoked.
Eg. Sample ob[100];

Q) Polymorphism (constructor overl...

numbers

ce

Moments

The word moment is familiar mechanical term which refers to the measure of a force with respect to its tendency to provide rotation and the strength of tendency depends on the amount of force and distance from the origin of the point at which the force is exerted.

The r^{th} moment about any point 'a' is defined as

$$\mu_r' = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^r$$

The r^{th} moment about mean $\bar{x}$ is defined as

$$\mu_r = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^r$$

Pascals Law

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

Relation b/w first four moments about point 'a' & first four moments about mean 'μ'

$$\mu_0' = \frac{1}{N} \Sigma f_i (x_i - a)^0$$

$$\mu_0' = \frac{1}{N} \Sigma f_i = \frac{1}{N} \times N = 1$$

* In case of arrays always default constructor is invoked.
Eg- Sample ob[100];

Polymorphism (constructor overl...

$$\mu_1 = \frac{1}{N} \Sigma f_i (x_1 - \bar{x})^1$$

$$= \frac{1}{N} \Sigma f_i x_i - \frac{1}{N} \Sigma f_i \bar{x} = \bar{x} - \bar{x} = 0$$

$$\mu_2 = \mu_2'$$

$$\mu_2 = \mu_2 - 2\mu_1'\mu_1' + (\mu_1')^2$$

$$\Rightarrow \boxed{\mu_2 = \mu_2' - (\mu_1')^2}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' - 3\mu_1'(\mu_1')^2 - \mu_0'(\mu_1')^3$$

$$\boxed{\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 4\mu_1'(\mu_1')^3 + \mu_0'(\mu_1')^4$$

$$\Rightarrow \boxed{\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4}$$

Karl Pearsons Coefficient.

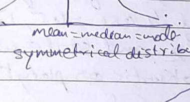
Karl Pearson defines the four coefficients which are based on the first four moments.

(i) $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$ (ii) $\beta_2 = \frac{\mu_4}{\mu_2^2}$

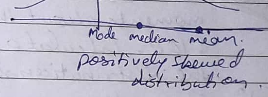
(iii) $\gamma_1 = \sqrt{\beta_1}$ (iv) $\gamma_2 = \beta_2 - 3$

These are very important coefficient giving information about the shape of distribution of the curve. Coefficients β_1 & β_2 measure for skewness and Kurtosis.

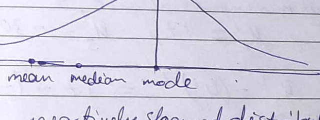
Skewness – it is the deviation from symmetry



mean = median = mode
symmetrical distribution.



Mode median mean.
positively skewed distribution.



mean median mode

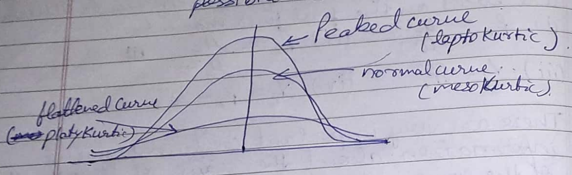
negatively skewed distribution.

Delhi Public School Jodhpur
Constructors and D...

* In case of arrays always default constructor is invoked.
Eg- Sample ob[100];

Q) Polymorphism (constructor...

Kurtosis: It is the flatness or convexity of the curve. There are three possibilities.



If $\beta_2 = 3$ then the curve will be mesoKurtic.

If $\beta_2 > 3$ then laptoKurtic
If $\beta_2 < 3$ then platyKurtic.

Ques1) In a certain distribution, the first four moments about the point 4 are -1.5, 17, -30 & 108. Calculate β_1 & β_2 and state wether the distribution is laptoKurtic or platyKurtic.

Sol^n Given, four moments about 4

$\mu_1' = -1.5$, $\mu_2' = 17$, $\mu_3' = -30$, $\mu_4' = 108$

Now,

$$\mu_2 = \mu_2' - (\mu_1')^2 = 17 - (-1.5)^2$$

$$\mu_2 = 14.75$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' - 2(\mu_1')^3$$

$$\mu_3 = 39.75$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$\mu_4 = 142.3125$$

Now,

$$\beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3} = 0.492.$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 0.654.$$

$\because \beta_2 < 3$ so the distribution will be platyKurtic

Ques2. The first four moment of a distribution about the value 5 of the variable are 2, 20, 40 & 50. Find β_1 & β_2. Comment upon the nature of distribution.

Scanned with CamScanner