The PIGEONHOLE PRINCIPLE → (RTU 2014 / 2017)

The pigeonhole principle is nothing more than the obvious remark: if you have fewer pigeon holes than pigeons and you put every pigeon in a pigeonhole, then there must result at least one pigeon hole with more than one pigeon. It is surprising how useful this can be as a proof strategy.

In other words

if k is a positive integer and $k+1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

Eg:- Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

eg:- In any group of 27 english words, there must be atleast two that begin with the same letter, because there are 26 letters in the english alphabet.

Generalized pigeonhole principle → (RTU-2017)

If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

eg: $N = 27$, $K = 26$ [eg 2] (english word's = 27) (letter's = 26)

$$\left\lceil \frac{N}{K} \right\rceil = \left\lceil \frac{27}{26} \right\rceil = \lceil 1.038 \rceil = \underline{2}$$

It mean's that, there must be atleast two $[\lceil N/k \rceil]$ word's start with one $\underline{[box]}$ letter.

eg: What is minimum number of Student's needed ~~to ensure that~~ in a discrete mathematics class to be sure that at least six will receive the same grade, If there are 5 possible grades. A, B, C, D and F?

Solution → The minimum number of Student's needed to ensure that at least six student's receive the same grade is the smallest integer N such that $\lceil N/5 \rceil = 6$. where $\boxed{\lceil N/K \rceil < (N/K) + 1}$ Condition should satisfy. The smallest such integer $N = 5 \cdot 5 + 1 = 26$. If you have only 25 students, it is possible for there to be 5 who have received each grade. Thus 26, the minimum number which ensu