

Generator & Parity Check Matrix of Cyclic code

① Cyclic Code Generator Matrix for non-systematic cyclic code

$$\begin{cases} (7,4) \text{ cyclic code} \\ g(x) = 1 + x + x^3 \end{cases}$$

Solution

$$G = \left[\begin{array}{ccccccc} & & & & & & \end{array} \right]_{k \times n}$$

4×7

k steps

- ① $g(x) = 1 + x + x^3$
- ② $xg(x) = x(1 + x + x^3) = x + x^2 + x^4$
- ③ $x^2g(x) = x^2(1 + x + x^3) = x^2 + x^3 + x^5$
- ④ $x^{n-k}g(x) = x^3g(x) = x^3(1 + x + x^3) = x^3 + x^4 + x^6$

$$G = \begin{bmatrix} 1 & x & x^2 & x^3 & x^4 & x^5 & x^6 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}_{k \times n}$$

4×7

$$g(x) = \frac{x^4 + x^3 + 1}{x^4 + x^3 + 1}$$

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}_{4 \times 7}$$

$$G = [I_k \ P]$$

Example $d = 1010$, $g(x) = 1 + x + x^3$
 $c = ?$

① $c = d \cdot g$

$$= [1010] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [1110010]$$

② $c(x) = d(x) \cdot g(x)$

$$= (1+x^2)(1+x+x^3)$$

$$= 1+x+x^3+x^2+x^3+x^5$$

$$c(x) = 1+x+x^2+x^5$$

$$C = \begin{matrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & x & x^2 & x^3 & x^4 & x^5 & x^6 \end{matrix}$$

Parity Parity Check Matrix H

We know that $g(x)h(x) = 1+x^n$

here $(7,4) \Rightarrow n=7$

$$g(x) = 1+x+x^3$$

then $h(x) = \frac{1+x^7}{1+x+x^3}$

$$= \frac{(1+x)(1+x+x^3)(1+x^4+x^5)}{(1+x+x^3)}$$

$$h(x) = 1+x^2+x^3+x^4$$

$$h\left(\frac{1}{x}\right) = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^4}$$

$$H(x) = x^{n-1} h\left(\frac{1}{x}\right)$$

$$= x^{7-1} \left[1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^4} \right]$$

$$= x^6 \left[1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^4} \right]$$

$$= x^6 + x^5 + x^4 + x^2$$

$$H(x) = x^2 + x^4 + x^5 + x^6$$

$$H = \left[\right]_{m \times n}$$

$$H = \begin{bmatrix} 1 & x & x^2 & x^3 & x^4 & x^5 & x^6 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}_{3 \times 7}$$

Sys:

- ① $d(n)$
- ② $g(n)$
- ③ $c(n)$
- ④ $z(x)$
- ⑤ $s(n)$
- ⑥ $e(n)$

⑦ $d \rightarrow c$

⑧ $c \rightarrow d.$

⑨ $z \rightarrow e \rightarrow c \rightarrow d$

⑩ G, H

Systematic Cyclic Code

$C =$



Steps

- ① $d(x)$
- ② $a(x) = x^{n-k} d(x)$
- ③ $b(x) = x^{n-k} d(x) \bmod g(x)$
- ④ $c_s(x) = a(x) + b(x)$
- ⑤ C_s

Example (7,4) cyclic code with $g(x) = 1+x+x^3$
data = 1010, find systematic cyclic code.

Solution

(i) $d = 1010$

(ii) $d(x) = 1+x^2$

(iii) $a(x) = x^{7-4} d(x) = x^3(1+x^2) = x^3 + x^5$

(iv) $b(x) = a(x) \bmod g(x)$

$$\begin{array}{r} x^2 \\ x^3+x+1 \overline{) x^5+x^3} \\ \underline{x^5+x^3+x^2} \\ x^2 \end{array}$$

$b(x) = x^2$

(v) $c_s(x) = a(x) + b(x)$

$= x^3 + x^5 + x^2$

$= x^2 + x^3 + x^5$

$d = 1100$
 $C_s = 0$

(vi)

$C = \begin{array}{ccccccc} 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ \hline 1 & x & x^2 & x^3 & x^4 & x^5 & x^6 \end{array}$