parity check Matrix H.

$$H = [P \quad I_m]_{m \times n} \qquad \underline{m = n - k}$$

$$H^T = \begin{bmatrix} P^T \\ I_m \end{bmatrix}$$

Relationship between G and H

$$G = [I_k \quad P^T]$$

$$H = [P \quad I_m]$$

Now $GH^T = [I_k \quad P^T] \begin{bmatrix} P^T \\ I_m \end{bmatrix}$

$$= P^T \oplus P^T$$

$$= 0$$

Syndrome Decoding

c : trasmitted code

r : received code

$r = c + e$ e error

we evaluate rH^T

$$= (c \oplus e) H^T$$

$$= cH^T \oplus eH^T$$

$$= dGH^T \oplus eH^T$$

$$= d(0) \oplus eH^T$$

$$= 0 \oplus eH^T$$

$$= eH^T$$

$$= s$$ syndrome of r.

we can identify the error position by comparing s to the rows of H^T

$$H = [P \quad I_m] \qquad m = n - k = 6 - 3 = 3$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Q.1 For a (6,3) systematic linear block code, the three parity-check bits c_4, c_5 and c_6 are formed from the following equations.

$$c_4 = d_1 \oplus d_3, \quad c_5 = d_1 \oplus d_2 \oplus d_3, \quad c_6 = d_1 \oplus d_2$$

(a) Write down the generator matrix G
(b) Construct all possible code words
(c) Suppose that the recived code is 010111, Decode it

solution

(a)
$$P = \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$G = [I_3 \quad P^T]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(b) $c = dG$

data	code
000	000000
001	001 110
010	010 011
011	011 101
100	100 111
101	101 001
110	110 100
111	111 010

(c) $r = \cancel{010011}$ 010111.

$S = r H^T$

$$= [0\ 1\ 0\ 1\ 1\ 1] \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ \ 0\ \ 0]$$

s is equal to fourth row of H^T, an error is at fourth bit.

$r = 0\,1\,0\,1\,1\,1$

$c = 0\,1\,0\,0\,1\,1$

$d = 010$

<u>Q-2</u> A parity check code has the parity check Matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) Determine the generator matrix G.

(b) Find the code word that begins with 101_ _ _

(c) Let recived code word is 110110.
decode this recived word.

sol

$\because H = [P \; I_m]_{m\times n}$

$m=3$
$n=6$
$k = n-m$
$= 6-3$
$= 3.$

$$P^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) $G = [I_k \; P^T]$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(b) first k bit of code is data

$c = 101---$

$d = 101$

Now $c = dG$

$$= [1\;0\;1]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= [1\;0\;1\;0\;1\;1]$$

(c) $s = rH^T$

$$= [1\,1\,0\;1\,1\,0]\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [0\;1\;1]$$

s is equal to 2^{nd} row of H^T
so error in its second bit
$r = 110110$
$c = 100110 \Rightarrow d = 100$