18.338 Eigenvalues of Random Matrices Fall 2023

Problem Set 1

Due Date: Wed September. 20, 2023

Instruction

Please submit your homework via canvas.mit.edu. If you are submitting .jl or .ipynb files, you must additionally submit .html or .pdf file that captures running notebook or code.

Reading and Notes

Read chapter *5, 10, 11* of the class notes, and comment on `nb.mit.edu` (The lecture notes and the link to `nb` can be found in Piazza: `http://piazza.com/mit/fall2023/18338`.). Please give your feedback especially high level style and where things did not make sense, in addition to spelling or technical errors.

Homework

Do at least four out of the following problems (Computational/Mathematical problems are denoted as C/M). Exercise with numbers and pages are from the class notes.

1. (C) Exercise 5.3 (p96) (You don't have to do the Monte-Carlo simulation. Let's only work on theoretical derivation of the Level densities of Laguerre and Jacobi, $\beta = 2$ case. You can use orthogonal polynomial recurrence relationships or any code to plot the "Theoretical Level density" of complex Laguerre and Jacobi in this problem. No randomness needed for this problem.)
2. (C) Implement your own Codes 5.2, 5.3 and 5.4 and compare them with your favorite package in Julia that evaluates orthogonal polynomials.
3. (C) Exercise 5.6 (page 97) (Please use Julia for this problem. For evaluating Hermite polynomials, `SpecialPolynomials.jl` is an option.)
4. (C) Exercise 5.7 (page 97) (This part we will concentrate more on the random part. You don't need to work with the orthogonal polynomials, but rather work with complex Laguerre and Jacobi ensembles. Sample random complex Laguerre and Jacobi ensembles and plot their eigenvalues. You can use singular value or gsvdvals format if you want.)
5. (M) Exercise 10.1 (p181)
6. (M) Exercise 10.2 (p182)
7. (M) Exercise 10.3 (p183)
8. (M) Exercise 11.1 (p216)
9. (M) Exercise 11.2 (p216)
10. (M) Exercise 11.4 (p217)
11. (M) Exercise 11.5 (p217)