Sampling Continuous Determinantal Point Processes with ApproxFun

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Outline of Presentation

- Lanczos Algorithm
- DPP Sampling
- Sample Some Familiar DPPs
- Sample Some Less Familiar DPPs

Lanczos Algorithm

- The main loop of Lanczos does the following for j from 2 to n:

$$\beta_j = |w_{j-1}|$$

$$v_j = \frac{w_{j-1}}{\beta}$$

$$\bar{w}_j = Av_j$$

$$\alpha_j = \bar{w}_j \cdot v_j$$

$$w_j = \bar{w}_j - \alpha_j v_j - \beta v_{j-1}$$

- If all the inner products are switched for

$$< f, g >= \int f(x)g(x)w(x)\partial x$$

- Then we can build orthogonal polynomials orthogonal to whatever $w(x)$ is

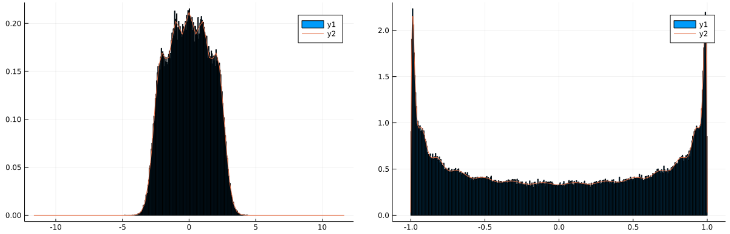
DPP Sampling

- The output of Lanczos is going to be Y a collection of n orthogonal polynomials
- $Y'Y = I_n$
- We square each polynomial individually and average them
- Weight them with the $w(x)$ and that is the probability space

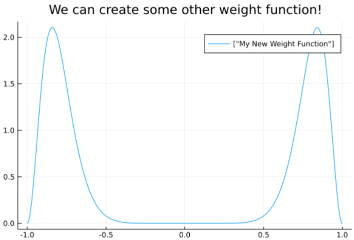
DPP Sampling

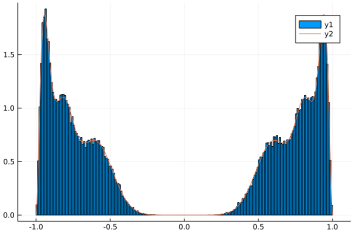
- The first task in sampling the DPP is to construct the density function and draw a random sample x_0
- The next task is to alter the density function so that we will not draw this sample again
- The idea is the following construct the discrete vector v_n of polynomial values evaluated at x_0
- Get the Q from qr of this vector. All the rows and columns of q are orthogonal to v_n besides v_n itself
- Contract $Q_{kj} Y_j$ to get new Y_k
- All these new polynomials (besides one of them) will be exactly zero at x_0
- Drop the one polynomial Y_k which is nonzero at x_0
- Repeat this process using Y_k. Continue until we run out of polynomials

Some Familiar DPP



Some Less Familiar DPP

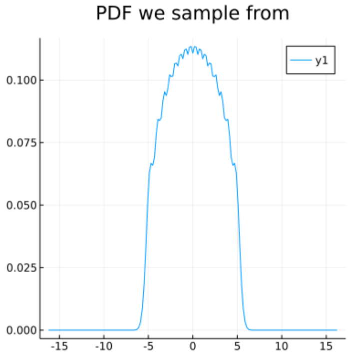




$$w(x) = \sin\left(x^4 \pi\right)$$

https://approximatelyfunctioning.blogspot.com/2020/09/quasi-matrices-orthogonal-polynomials.html

Sampling Algorithm



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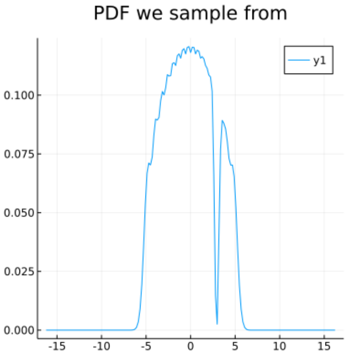

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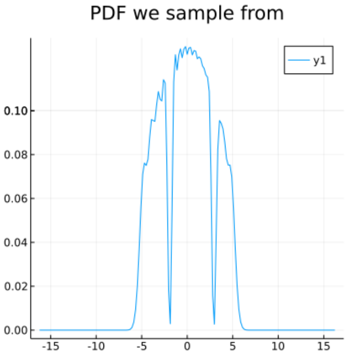

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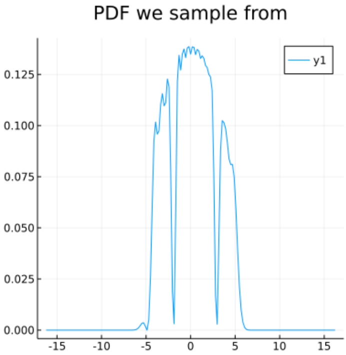

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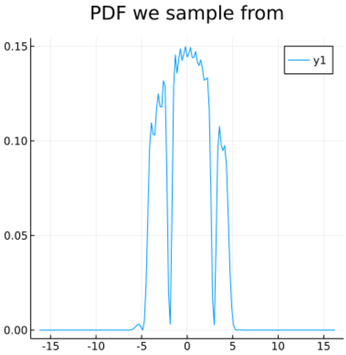

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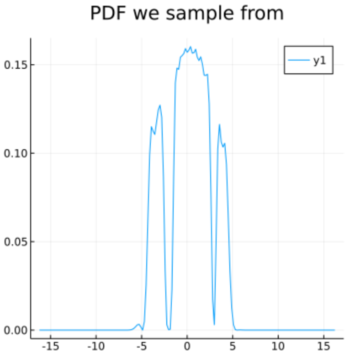

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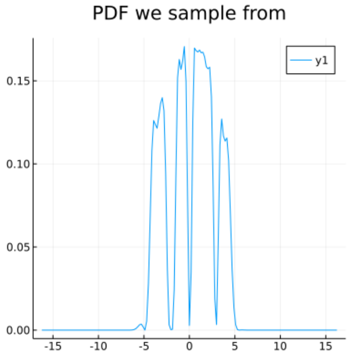

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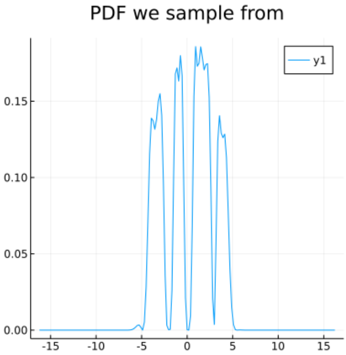

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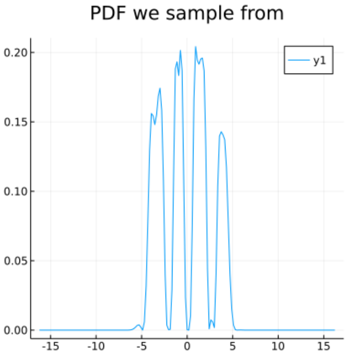

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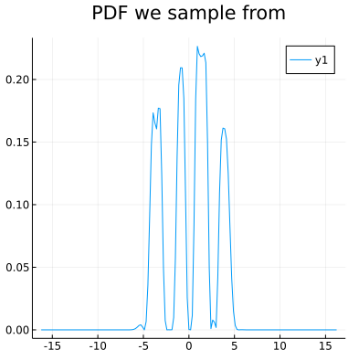

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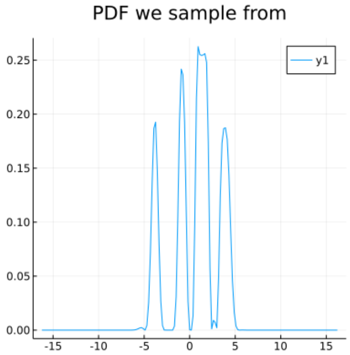

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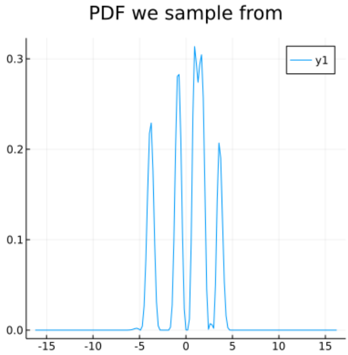

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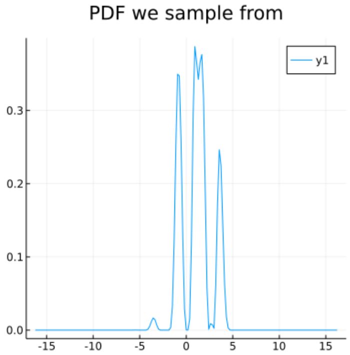

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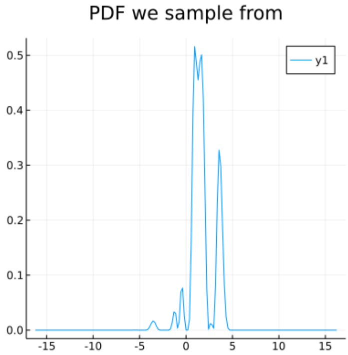

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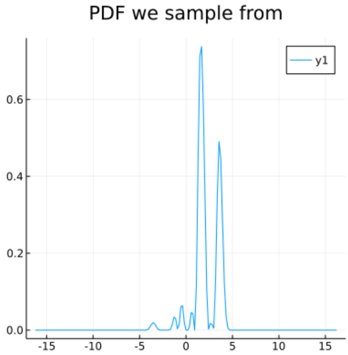

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